The Design of Switching Circuits
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To

N., J., and J.
The present is an excellent time for this book to appear. In the past, general interest in the switching art and knowledge of its unique techniques were limited to a few quarters where complex control mechanisms such as telephone switching systems were developed and used. Now, however, the situation is changing rapidly and radically. College professors, research engineers, scientists, and mathematicians are now aware of and keenly interested in the subject. This change was brought about, or at least greatly accelerated, by the appearance of automatic digital computing systems. Fundamentally, such systems employ essentially the same techniques as those useful in telephone switching. However, their ability to perform complicated mathematical calculations furnished a brilliant and readily understandable demonstration of the fact that switching circuits can be designed to accomplish, mechanically, at least certain sorts of operations which are normally associated with human mental effort. For example, they can follow trains of orders, they can make use of tables, they can make deductions following formal logical rules. Of course, in other respects such mechanisms may be sadly lacking in comparison with human intelligences. The abilities they do have, however, seem great enough to promise the elimination of much of the mental drudgery required by our modern world. It is this glimpse of a coming age of "mechanized intelligence" that recently caught the imagination of professional people and caused the upsurge of interest in switching. If this textbook succeeds in nourishing that interest, and if it helps its readers to work toward the ultimate realization of such a significant goal, the effort that went into its preparation will indeed have served a very worthy purpose.

The book is a natural product of the Bell Telephone Laboratories, since the design of switching circuits to establish telephone connections is a basic and vital activity of this organization.

When we make a telephone call, we spin our dial a few times, and — in a matter of seconds — the particular phone that we want among millions of others is ringing. This happens with such regularity and such reliability that we have come to take it for granted. Most of us never try, or for that matter are not able, to visualize the series of intricate events that take place automatically in a modern dial telephone office before that called phone is rung.
In simple terms, what happens is that at the instant we lift our handset, a group of switching circuits springs obediently to our service. These switching circuits identify our line as the one in need of attention, connect themselves to it, give us the signal to proceed, count our dial pulses, memorize the series of digits, determine our call's destination, locate and test the available paths, select the most preferred combination, join the links end-to-end, and finally, when the connection is all set, ring the wanted phone.

All this is accomplished for us in seconds by the switching circuits of the dial central office. Just for perspective, it is also worth noting that while one group of switching circuits works on our call, additional groups, in perfect teamwork, are taking prompt care of the random requests of thousands of other customers.

Obviously, then, switching circuits used in dial central offices are swift and versatile mechanical servants; and even if they did no more than establish connections, when we are once aware of their existence, we are likely to be intrigued by them. However, they go further in their performance. To facilitate proper charging for their services, these switching circuits also record who calls whom, and when, and how long each conversation lasts. Such competence on their part certainly increases our respect for them.

Even this is not all. As a little extra service during their daily work, these central office circuits make a second attempt at setting up a connection in case their first attempt fails; or they locate alternative routes to a destination if the preferred one is not available, or furnish maintenance men with precise information regarding troubles, and so on.

This roll-call of events in a dial central office conveys, perhaps, an adequate sense of the type of actions that switching circuits can perform. It also reveals how far the ingenuity, imagination, and precise logic of design engineers have advanced the art in a particular field of application. Such a revelation may induce readers of this textbook to go beyond its Foreward. Those who do, will find a wealth of switching fundamentals, succinctly rationalized and presented by the authors.

Since the material for this book was prepared as part of a post-war training program for young engineers at the Bell Telephone Laboratories, it is appropriate to close this Foreward with a simple tribute to the Laboratories' management, particularly Mr. A. B. Clark, Vice President, for initiating and firmly supporting such a training program on the basis of its long-term merits, when there was intense
need for manpower in more immediately profitable switching development projects. In years to come, the training program will bring a harvest in the form of technical contributions by the young engineers whose minds have been alerted to the challenges of the switching art. For the present, it is hoped that this book, a direct outcome of the training program, will be a tangible source of satisfaction to those whose official support it represents.

JOHN MESZAR

July 1951
This book is not a text on telephone systems. It is concerned, rather, with the basic techniques of switching circuit design: techniques which are applicable to digital computers and other complex control systems as well as telephone switching systems. The writing of this text was started soon after the end of World War II as a series of lecture notes for use in training new engineers in the Switching Systems Development Department of the Bell Telephone Laboratories. During the succeeding years the notes were revised and the methods of instruction were improved. In 1950, with general interest in the subject of switching increasing, arrangements were made with the Massachusetts Institute of Technology to give to graduate students a one-semester course in switching circuit design. The text for this course was based on the material used for training within the Laboratories. The present volume is a final edition of the text used in the M. I. T. course, revised to take advantage of the academic experience gained there.

The objective of this book is to present the fundamental principles underlying the design of switching circuits rather than to describe the operation of specific circuits, except as examples. The material is planned to be complete within itself, no previous knowledge of switching being assumed. For this reason explanations are often given in considerable detail. The book is suitable for individual study as well as for a one-semester or two-semester course of formal instruction. The only prerequisites for its use are a logical mind and a knowledge of elementary electrical circuit theory.

The subjects of the chapters range from elementary concepts to the design of circuits containing a relatively large number of switching devices. Chapter 1 serves as an introduction to the subject. Chapter 2 discusses the physical and electrical characteristics of the various devices used for switching. Chapters 3 through 10 give the fundamentals of switching circuit design, including the logical concepts involved and the form of Boolean algebra which has been given the name "Switching Algebra". Chapters 11 through 21 discuss "unifunctional circuits", which are aggregates of switching devices performing single specific switching actions. These are the building blocks of large switching systems. The last three chapters contain a brief treatment of the methods of synthesizing such basic functional circuits into practical
working systems. Principles and techniques are illustrated primarily by relay switching circuits. However, electronic techniques employing tubes, rectifiers, and transistors are discussed and many examples are given. Of particular interest in this regard are Chapter 10, which discusses electronic circuit fundamentals, and Section 21.7 of Chapter 21, which illustrates the application of switching algebra to the design of electronic switching circuits.

On the basis of teaching experience, it is felt that design practice is the best way to gain facility in the application of principles. Following most chapters is a set of suitable problems which have been tested by actual classroom use. Since almost all switching problems have a number of satisfactory answers, the solutions are sometimes difficult to judge. As a guide, the approximate quantity of apparatus required for a good solution is given in many of the problems. This is not always the minimum possible apparatus but represents a reasonable point to which a circuit may be reduced. In learning the subject, it is also helpful for the student actually to build circuits from original designs. Many of the problems are suitable for this activity. Particular attention is called to the special problem given at the end of Chapter 11.

The basic ideas presented in this text have, for the most part, been distilled from the accumulated work of practicing circuit design engineers. A number of people have contributed directly to the work leading up to the publication of this book. Mr. A. E. Joel was associated with the authors in the early stages of the teaching program and contributed valuable ideas, particularly in the organization of functional circuits. Messrs. G. R. Frost and W. S. Hayward prepared the first drafts of several chapters. Messrs. R. W. Roberts and H. N. Seckler assisted in the details of producing the final text. The book was designed by Mr. K. M. Collins, who not only handled all problems of publication and printing, but made available to the authors a wealth of information and experience in the fine art of preparing a book for public presentation. Finally, the authors wish to express their appreciation for the friendly advice, encouragement, and discerning criticism of Mr. John Meszar in all of the activities leading up to the production of this book.

WILLIAM KEISTER
ALISTAIR E. RITCHIE
SETH H. WASHBURN

July 1951
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Chapter 1

FUNDAMENTAL CONTROL CONCEPTS

From a technological point of view, one of the most striking aspects of the present period of human endeavor is the emergence of machines that handle information automatically. In more graphic but less accurate terms, these are machines that "think". With every passing day, not only scientists and engineers but the general public as well are becoming more aware of complex devices or systems that can accept information in one form or another, correlate and digest this information, and arrive at logical conclusions. Mechanisms of this nature are broadly classified as "automatic control systems."

Automatic control systems break down into two distinctly different subdivisions: those that function on the basis of continuous input data, or data that vary smoothly from one value to another; and those that function on the basis of data that vary in discrete steps. The subject of this book is the design of circuits for systems of the latter class, properly known as "switching" systems. Among the more spectacular and well-known examples of such systems are the modern large-scale digital computers and the automatic telephone switching systems. Both of these systems accept information in numerical form and produce output results that are precisely related to the input data.

The nature of a switching system and its component circuits can perhaps be more clearly illustrated than defined. Modern dial telephone systems are an example. The machinery of a central office contains a network of switching devices by means of which subscriber lines and trunks can be interconnected. Centralized control circuits are provided to control the actions necessary in establishing a connection. These control circuits may range in size from less than a dozen switching elements to a thousand or more. The system is controlled by instructions from the subscriber which consist of a series of pulses generated when a number is dialed. These signals start an intricate train of reactions which are determined in part by the information received from the subscriber, in part by information collected within the system concerning busy and idle path conditions, and finally by built-in knowledge. This knowledge is placed there by the designer who analyzed a large number of control situations, determined suitable plans of action, and developed switching circuits to carry out these plans.
A variety of actions are performed. The dial pulses are counted as they arrive and the information is stored as numerical digits in coded form, to be used as required. Various circuit units are interconnected with each other to cooperate in the control. Trunks and interconnecting linkages are selected in groups and tested for busy and idle conditions. On its own initiative a control circuit can pick an idle path from a large group or, if all are busy, can choose an alternate group or make other disposition of the call, such as returning a busy signal. Centralized control circuits can operate any switch in the office used for interconnecting subscriber lines and trunks, and can locate suitable switches for a given call. After a control circuit has completed its action, it will restore itself to a normal condition and be available for use on another call.

During this entire process, information, either received as input data or held as stored data, is recorded, translated, manipulated, transmitted, etc., as discrete digital items. For any set of inputs, there is a corresponding logical and invariable output condition or group of alternative output conditions (for instance, a connection to a busy-tone source instead of the called line, if the line is busy).

Other examples can be cited to illustrate the varying facets of switching systems, ranging from relatively simple machine controls up through such complex mechanisms as automatic accounting systems and inventory control systems. The latter type of system, for instance, can store information on the complete inventory of a business enterprise, be it a department store, factory, warehouse, or airline. Each inventory item is assigned an identifying code, and with it is associated the present status or quantity of the item. The system can answer requests from key equipments at local or remote points as to the current state of any stock item, and can change information automatically, by subtraction in accordance with filled orders or by addition in accordance with cancellations or new purchases.

In all these switching systems, the internal circuitry that relates input to output consists of paths interconnecting discrete-valued apparatus elements. Well-known examples of this type of apparatus are electromagnetic relays, electromagnetic switches, rectifiers, gas-filled electronic tubes, magnetic tapes and drums, mercury delay-lines, and certain arrangements of vacuum tubes. The action of any of these elements in switching circuits is to open or close, or switch, interconnecting paths, or to establish definite conditions over them, in predetermined logical patterns which are ordered by the input information. The outputs generated by a system as a whole may consist of pure information, as a printed page of figures in a computer; control signals, as the motor on-off signals in an elevator system; or a particular
state of the system, as the establishment of a new talking channel in a telephone office.

The concept of logic is the most noteworthy aspect of switching circuits; in fact, the schematic representation of a relay circuit is a symbolic expression of a set of conditions and their logical conclusion. For example, the configuration of a circuit is largely determined by logical relationships of the form: output X should be closed when inputs A and B occur simultaneously; output Y should be closed when input A occurs before input B; output Z should be closed when either input A or input B occurs. Methods of determining logical relationships from statements of requirements, and the developing of circuits to conform to the logic, form a principal part of this volume.* Furthermore, it will be seen that the terminology and procedures of formal logic are of great assistance in designing circuits. From this point of view, it is not difficult to see that there is some validity to the feeling that machines designed on such a basis "think." Note, however, that the thought processes are in all cases built into the machine and that the machine at most merely reproduces the original plans of the designer.

The preceding discussion indicates that there is a vast ultimate field of application for switching circuits, for not only are many control situations subject to logical treatment, but they are inherently discrete or numerical in nature, or can be transposed into this form by suitable detecting and converting devices.

The ability of switching circuits to perform complex as well as simple functions by means of relatively simple apparatus elements is based upon the combinations into which the apparatus elements of the circuits can be manipulated. Any individual combination of several elements can be made to give a unique indication by suitable interconnections. If apparatus elements that can assume any one of five states or "values" are used, one element can indicate five conditions; two elements, twenty-five conditions; three elements, one hundred and twenty-five conditions; etc. If apparatus elements with only two values are used, the same effects are evident. For example, a group of twenty such two-valued elements can be set into more than a million unique combinations.

It can be seen almost intuitively that the simplest and most elemental control conditions are two-valued, that is, they are "on" or "off", "positive" or "negative", "open" or "closed". Indeed, in a sense, it is straining for complication to go beyond two values for control purposes,

* An understanding of the electrical basis of circuits is assumed, for the most part, and this phase of the subject is not stressed.
since marginal conditions and tolerance limits increasingly difficult to meet are thereby introduced. Since it is natural to match the apparatus to the most effective control conditions, the use of two-valued apparatus for switching applications is clearly indicated. Even with this restricted number of values or states per apparatus element, the rate at which the distinct combinations of elements increases with the number of elements permits the handling of the most complex requirements with a relatively small quantity of apparatus.

Multi-valued apparatus, such as a multi-terminal electromagnetic switch, is useful in certain special applications in switching circuits. Although there may be a multiplicity of paths through the device, individually the paths are either opened or closed during operation and the apparatus may be analyzed essentially on a two-valued basis. Control of the device is also usually two-valued, consisting either of a repetition of signals over one lead or combinations of on-off signals over several leads. The multi-valued device can normally be considered as an aggregation of two-valued devices, constructed in a package for reasons of economy or convenience. Since it is designed for specialized applications, it does not offer the circuit flexibility inherent in two-valued apparatus. Therefore the major effort in this volume is devoted to the design of circuits employing two-valued elements.

An important characteristic of switching circuitry is that it can be made to hold or retain a given state after the excitation that produced the state has passed. This is the factor of memory which permits switching circuits to respond to a sequence or time-order of input intelligence, and eliminates the necessity of an input path to each apparatus element in a circuit. An example of the utility of this concept is furnished in the association of a telephone switching office and a subscriber by means of a pair of wires. Over this path, by a sequence of identical signals, the subscriber can control the connection of his line with any one of millions of other subscribers.

The remarks up to now have been general and do not give a clear picture, in a detailed sense, of what switching circuits do. And in this introduction it is not essential to point out the many simple control functions, where effect is obviously related to cause, that circuits of this type can be designed to handle effectively. However, large circuits or systems, involving dozens or hundreds of apparatus elements and performing intricate combinations of control functions, deserve further consideration. Circuits of this type are inevitably composed of many interrelated sub-circuits, each of which performs one or a small number of individual functions. These sub-circuits can be considered as the true building blocks of major switching circuits. Design of a large circuit or a system, then, consists of determining from the requirements
of the problem the basic and individual functions which are necessitated by circuit logic; designing circuits to perform these basic functions; and finally, integrating or coordinating the functional circuits into the complete system. In the final system, information received from the outside over input channels is recorded and interpreted, decisions are made as to what functional circuits should be called into play, internal information is passed back and forth between circuits, checks are made as to accuracy and validity of performance, and finally output signals or actions appear.

Surprisingly, even in a large and complete switching system the majority of switching circuit requirements can be met by a relatively small number of types of functional circuit. Within each type, of course, there are many variations, some due to detailed requirements and some to the whim of the designer. The names of the types of circuit give an immediate clue to the kind of action each performs. Only a few examples are mentioned here; later chapters describe most of the types of circuit in detail.

A simple basic circuit is the "register" circuit which records and holds information until required by another functional circuit. The particular configuration of the register circuit depends upon the form or code in which the information is made available. Often associated with the register circuit is the "translating" circuit which converts information from one code to a different code. It is often convenient to receive or record information in a particular code, whereas a different form is required for use or transmittal to other parts of the system.

Information is often sent on a repetitive pulse basis, and to receive it a "counting" circuit is necessary. The counting circuit is actuated by successive pulses and can be arranged to count either a definite number of pulses or to give a distinct indication corresponding to each individual pulse. This circuit is also of use in keeping track of a series of events.

In most multi-functional circuits, there is need for "connecting" circuits to associate functional parts for short periods or to connect to circuit inputs and outputs. There are many types of connecting circuits, with the differences dependent upon the number of connectable circuits, the duration of individual connections, and the number of leads required.

Often associated with connecting circuits are the "locating" circuits which are used to choose one item out of many. Classified under locating circuits are "selecting" circuits, which pick one item from many on a predetermined basis; "hunting" circuits, which pick any available one from several identical items; and "finding" circuits, which identify a particular item that requires service.
The "lock-out" circuit is used to regulate the flow of traffic among the parts of a complex system. Its usual function is to permit but one out of several competing items to act or to have access to a common part of the system.

Functional circuits that have less general application than the preceding ones are the "calculating" circuits. These are circuits which perform the basic arithmetical functions of adding, subtracting, multiplying, and dividing. Their chief application is in calculating or computing systems.

This partial list of basic functional switching circuits serves to give an idea of the types of action of which switching circuits are capable in control applications. As mentioned, these circuits have little utility individually, but are the functional building blocks from which complete systems are constructed.

It has already been stated that two-valued apparatus elements are at present considered the best devices for general use in automatic control circuits of the switching type. Where multi-valued elements are used, it is in specific applications where economy or efficiency can be achieved by, in effect, combining several two-valued elements into a single piece of apparatus. Among the two-valued elements, the leading contenders for use in existing switching circuits are electromagnetic relays, hot- or cold-cathode gas tubes, and vacuum tubes connected to act as two-valued devices. Other discrete-valued devices which are appearing in switching circuits at an accelerating rate are varistors, transistors, and magnetic recording devices.

Each of these devices has specific characteristics which peculiarly suit it to certain applications in the switching field. An instance of this appears in the requirement for extreme speed, in the microsecond range, which sometimes arises. This inevitably drives the designer to the use of vacuum tubes or the semiconductors. However, as a general tool for the majority of switching applications, the electromagnetic relay has as yet no peer.

Of major importance in this respect is the manner in which the relay indicates its state. This is by means of contacts that may be used in circuit paths which are completely independent, electrically, of the relay operating path. These contacts may be open or closed when the relay is unoperated, and will be in the inverse state when the relay is operated. By placing the contacts of several relays in series, any conclusion requiring a coincidence of conditions can be indicated. By connecting contacts in parallel, a conclusion dependent upon the existence of any one or more conditions out of many can be indicated. In logic, these two alternative sets of conditions would be known as "and" and
"or" requirements. By suitable interconnections, any conceivable combination of "and"-"or" relationships can be expressed with a relay contact network. Of equal importance with this concept is the fact that use of normally open and normally closed contacts permits specifying not only the inclusion, but also the exclusion, of dependent conditions.

These considerations make the relay an excellent instrumentality for expressing the logical relationships which are the very basis of switching problems. A secondary factor, which has considerable practical importance, is the fundamental simplicity of the relay as a circuit element. Although the interconnecting paths of a multi-relay circuit viewed as a whole may become highly complex, the control network for each individual relay forms a simple open or closed path which rarely requires additional apparatus for correct functioning. Physically, the relay is easy to comprehend, even though a great deal of knowledge and ingenuity has entered into its design. From the circuit point of view, in most applications the relay is simply a device which opens or closes a multiplicity of paths when power is applied to it, and performs the inverse operation on the paths when power is removed.

The importance of this simplicity should not be underestimated. It enables the circuit designer to concentrate on the logical aspects of any set of circuit requirements rather than be diverted by essentially extraneous problems of making the circuit elements perform in a particular manner. A natural consequence of this is that the designer has greater freedom in dealing with the logical aspects of a switching problem when working with relays than with any other type of circuit element. This is a general statement, and does not exclude the choice of other apparatus elements when their characteristics match the requirements of a particular problem.

The relay in a general sense has other attributes which contribute to its utility and flexibility as a circuit tool. Its operating time, depending upon construction, ranges from a minimum of a few milliseconds up to about one second, with more extreme values available in special relays. It has seen such extensive use over many years that it is today a highly-developed and reliable device. Of utmost importance is the fact that it is widely available in a great variety of types that offer almost complete freedom in the choice of the correct relay for each particular application.

In view of these considerations, the major portion of this volume is devoted to the design of typical relay circuits. There is less loss of generality in this treatment than might appear, however, since the same fundamental logic is applicable to all types of two-valued switching apparatus. The differences in approach to electronic circuits, for instance, arise in the application of detailed design techniques rather than in
general principles. In order to indicate the nature of these differences, selected examples of electronic switching design techniques are incorporated in the text, and tube circuits performing functions comparable to relay circuits are presented. Some basic circuits for control of multi-terminal switches are also included for the same reason.

In presenting the subject of this volume, the objective is to progress from the basic design concepts up through the design of relatively complex multifunctional circuits. Although no formal division is made, the text can be considered as falling into three parts: basic relay circuit design fundamentals; the design of single-function circuits; and the planning and design of multifunctional circuits.
Chapter 2

SWITCHING APPARATUS

It has been noted in Chapter 1 that the design of switching circuits consists primarily of analyzing the functional circuit requirements in terms of two-valued conditions, and devising logical interrelationships of circuit paths and apparatus to meet the conditions. The choice of apparatus to use in the circuit is in a sense a separate problem which can often be met after the basic circuit design has been completed. This is particularly true with relay circuits, and to a lesser extent with circuits using electronic apparatus.

Since this volume is primarily concerned with the fundamentals of relay circuit design, little detailed study of apparatus is included. However, this chapter will discuss the basic characteristics of relays as circuit elements in order that the circuit design chapters need not be interrupted by digressions on the physical aspects of apparatus. Also, the schematic conventions and the vocabulary used in relay circuit design will be given. On the other hand, it will be more convenient to give the comparable description of switches and electronic circuit elements in the chapters devoted to their circuit use. This will be done since the circuit design using such apparatus is more intimately related to the apparatus than is true with relays.

In addition, this chapter will mention some types of apparatus such as keys, plugs, and jacks, which are useful adjuncts to circuits. In all cases descriptions will be general since detailed information can be found in manufacturers' specifications and catalogues.

A relay is essentially a remotely controlled switch which can open or close contacts when suitable electrical conditions are met. From a circuit point of view, relays can be divided into two broad classes, known as general-purpose relays and special-purpose relays. The general-purpose relay, often called a telephone-type relay, is the work-horse in relay switching circuits around which is constructed the major portion of switching circuitry. In its usual form it is a direct-current relay operating in a voltage range of a few volts to approximately 100 volts*. In order to be a flexible tool, the general-purpose relay provides a basic structure on which can be placed a variety of

* The most common voltage in telephone central offices, for instance, is 48 volts.
contact combinations and a variety of winding combinations to meet different operating conditions. The contacts in general carry only moderate currents (1.0 ampere or less), primarily for controlling other relays. The acting times of these relays range from a few milliseconds to several hundred milliseconds.

The special-purpose relay, on the other hand, is usually designed for a particular type of application, and will vary in construction and appearance depending upon its function. Its attributes may be extreme speed, high sensitivity, polarized operation, operation on alternating current, contacts of very heavy load-carrying capacity, etc.

2.1 GENERAL-PURPOSE RELAYS

All electromagnetic relays are composed of the same three basic parts: a magnetic structure or framework, a winding, and a set of contacts mounted on springs. The winding and the magnetic structure convert electric power into mechanical action; and the set of contacts, driven by the resultant motion, open and close external circuit paths. The general relation of parts is shown on Fig. 2-1.
The magnetic structure consists of a core, one or two pole-pieces, and a movable armature hinged to the rear pole-piece, often called the heel-piece. All these parts are made of soft iron or other magnetic material, and form a closed magnetic circuit except for an air gap between the free end of the armature and either a front pole-piece or the core itself. When the relay is energized, the pull is concentrated in the air gap, and the armature moves to close the gap.

The relay winding (or windings) is placed on the core of the structure, and the winding ends are brought out to connecting terminals. The pull developed at the armature is a function of the current, the number of winding turns, the air gap, and the magnetic material and structure.

The relay springs carrying the contacts are usually clamped to the structure at one end and are free to move at the other. There are two sets of springs, one set relatively stiff and fixed in position, the other set relatively flexible and linked to the armature through insulating studs or a comb arrangement. Precious-metal contacts are attached to the mating surfaces of the springs. When the armature moves, the contacts either open or close, depending upon their initial condition.

Although all general-purpose relays conform in these fundamental aspects, the construction and appearance of different relays vary in accordance with the manufacturer’s design objectives and concepts of efficiency and performance. Two representative types of general-purpose relays are shown on Fig. 2-2. In service, they mount in horizontal positions on flat metal plates, usually with the moving parts in a vertical plane to avoid the effects of gravity.

In considering the general purpose relay from the circuit point of view, the important aspects of the device are: the contact or spring...
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combinations; the winding combinations; the power requirements; the acting speeds; and the contact current load capabilities. The magnetic structure of the relay is of primary interest only in an indirect sense, because of its effect upon the power and speed capabilities. However, modifications of the basic relay structure to affect its speed characteristics are important. These various aspects of the relay will be taken up in the following paragraphs.

Spring Combinations. The two basic relay spring combinations are the "make" and the "break" combinations. These are illustrated in Figs. 2-3A and 2-3B in two standard schematic forms. In this text, the first form shown, which is the communications standard, will be used exclusively. On the figure, alternate acceptable designations are also given in addition to make and break. Two other very common spring combinations, the "transfer" and the "continuity-transfer" or "continuity", are shown as Figs. 2-3C and 2-3D. The transfer normally has the property of opening the break contact before closing the make contact, whereas the continuity insures a make-before-break contact sequence.

Fig. 2-4 Specialized Spring Combinations
Examples of more specialized spring combinations are shown on Fig. 2-4. In general, these are less universally used than the combinations of Fig. 2-3.

Several spring combinations may be grouped on an individual relay in one or more assemblies. The maximum allowable number of individual springs varies with the type of general purpose relay, but usually does not much exceed 24. A large number of different spring arrangements utilizing the basic combinations are, of course, possible. A typical example in two separate spring assemblies is shown associated with the schematic representation of a relay core on Fig. 2-5. The individual springs are often numbered in accordance with either the order of wiring terminals in back or the order of springs viewed from the front.

Relays are also available with a large number, of the order of 50 or 60, of single-type springs, such as make contacts. These are somewhat out of the category of general purpose relays, and are often designated multi-contact relays.

Windings and Winding Arrangements. General-purpose relays are available with a variety of different windings and combinations of two or more separate windings. The effect of the number of turns and the resistance of particular windings will be discussed in the section on operating characteristics; in this section, only the various winding arrangements will be covered.

The most common winding arrangement is the single winding shown on Fig. 2-6A. The semi-circular dot on the winding symbol indicates the inner end, or the end of the winding closest to the core. Following the single winding in utility is the double winding, shown on Fig. 2-6B, in which the two windings are often designated primary and secondary. In this and other multi-winding cases, the dots can be interpreted to give the directional sense of the windings in order to simplify the making of aiding or opposing connections. Double windings are prepared in a variety of ways, of which the simplest is to place one winding directly on top of the other. This permits the utmost flexibility in providing windings completely independent in characteristics. Other methods of construction may be employed when it is necessary to
achieve a high degree of balance between the two windings. An example of this is the sandwich winding, where part of one winding is laid on the core, the second winding is next added, and finally the remainder of the first winding is completed. Another example is the parallel type where the wires of the two windings are laid concurrently. In these cases, the designations \( P_1 \) and \( P_2 \) are sometimes applied to the two windings instead of \( P \) and \( S \).

Other winding arrangements of a much more specialized nature are shown on the remainder of Fig. 2-6. The arrangements shown in Figs. 2-6E, 2-6F, and 2-6G include resistors wound non-inductively on the cores. These serve no purpose that could not be achieved by external resistors, but it may be convenient and economical to include them in the relay structure. The specialized arrangements have little general utility, but may be of use in particular applications.

Relay Operating Characteristics and Adjustments. The force which pulls the relay armature to actuate the contact springs is developed by current flowing through the winding. Since the winding is inductive, the initial current does not immediately jump to a value determined by the relay resistance and circuit voltage, but rises exponentially at a rate determined by the time constant of the winding and certain other electric and magnetic properties of the structure. Thus, the contacts do not move until the current has risen to a level where the magnetic flux generated across the armature air gap provides sufficient force to overcome the initial back tension of the armature. When the armature starts to move, the back tension increases as the contact springs are flexed to a greater and greater extent. At the same time, the operating force increases as a function both of the continuing
rise in current and the decreasing reluctance in the magnetic circuit as the air gap closes. The operating force must increase more rapidly than the armature back tension for the relay to operate completely.

For a particular type of relay, the force or pull developed across the armature air gap is a function of the product of the current and the number of turns of the winding. This product is generally known as ampere-turns. It can be seen that, for a specific required pull, there must be a balance among the winding turns, the winding resistance, and the voltage level for which the relay is designed. The resistance and the voltage level are related to the normal power-handling capacity of the relay, which, for general-purpose relays, is usually in the range of one to ten watts.

When the circuit to the relay winding is opened, current flow ceases rather abruptly, unless the winding is shunted, and the relay starts to release. The decay of magnetic flux is affected by eddy currents in the core, by the magnetic material of the relay, by the residual air gap left when the relay is fully operated, and by any conducting paths encircling the core. All these factors, plus the time for mechanical motion, determine the release time of the relay.

A set of mechanical adjustments is specified for any relay. These have to do with unoperated armature air gap, residual armature air gap (in some cases), direction and limits of spring tension, contact pressure, etc. As determined by the tolerance range of these mechanical adjustments, there are a set of four basic electrical requirements that can be applied to a relay and which determine its operation in a circuit. These are the operate, hold, non-operate, and release requirements, as defined below. The principal mechanical variable which can be changed to make the relay meet the requirements is the spring tension against the relay armature. The requirements are normally specified in terms of current flow.

Operate Requirement: The current level at which the relay will definitely operate. The current at which the relay can operate may extend well below this level.

Hold Requirement: The level to which the current may be reduced, after the relay has operated, which will insure that the relay remains fully operated.

Non-Operate Requirement: The current level at which an unoperated relay definitely will not operate. The relay may operate at any current level above the non-operate value.

Release Requirement: The level to which the current through an operated relay can be reduced with assurance that the relay will completely return to an unoperated condition.
Since the same mechanical condition must permit the relay to meet all four electrical requirements, or all that are specified in a particular case, the electrical requirements are necessarily closely interrelated. Setting any one requirement value, such as operate, automatically establishes limits within which the others must fall. If a relay is adjustable, the spring tension is regulated to a compromise value which permits meeting all specified electrical requirements. The status of the relay is then as shown on Fig. 2-7. In practice, however, it is seldom that all four requirements are placed upon a relay. For normal straightforward operation in open and closed circuits, the operate requirement alone may be specified. The others are necessary only to meet marginal conditions, or to hold the relay to special speed requirements.

In order for a relay to function satisfactorily in a circuit, some margin must be left between the circuit current under adverse conditions and the corresponding relay current requirement. Otherwise, a slight change in the mechanical condition caused by temperature or humidity changes or by wear may cause the relay to fail. A margin of 10% is usually considered adequate to take care of this situation. In practice, the worst circuit current is computed under extreme conditions of voltage and resistance limits, and a check made that the resultant value exceeds the operate or hold requirement by 10% or more. For release, the worst circuit current should be less than the
release relay requirement by 10% or more, and should be less than the non-operate requirement by a greater amount if possible.

Operating and Release Times. General-purpose relays are obtainable with acting times ranging from less than five milliseconds up to approximately a half-second. Although acting times above the order of fifty milliseconds require special methods of construction which will be discussed in a later section, the remarks here will apply to relays in general.

For a given mechanical structure, neglecting the effect of any special time-increasing features, the operating time of a relay is a function of several variables. Of these, the most important for present purposes are the spring load on the armature, the unoperated armature air gap, and the power absorbed by the relay winding. For a given winding, the operating time will increase as the spring load on the armature increases and as the unoperated armature air gap increases. The air gap is fixed to a great extent by the spring combination and is not, strictly speaking, an independent variable. For a given spring load, the operating time will decrease as the power to the winding increases. Therefore, the higher the circuit voltage and, assuming the use of well-designed coils, the lower the coil resistance, the faster will be the relay. These facts are important to the circuit designer since, where speed of operation is important, he must make a careful choice of the winding coil and must not place a large number of contact springs on the relay. The choice of coil is influenced by the circuit voltage, which is generally set by other considerations, and the maximum power that can be applied to the relay without causing excessive heating.

The release time of a relay, again neglecting the effect of any special time-increasing features, is primarily a function of operated spring load and operated armature air gap, the time of release increasing as either of these factors decreases in magnitude. It is worth noting that, to a certain extent, the factors which aid in gaining low operate time tend to increase the release time, and vice versa.

It is difficult to attain a high degree of precision in the operate and release times of general-purpose relays. These times are affected both by the structural variations in the relays and by electrical variations in the circuit. The result of this is that individual relays of a particular type, meeting all mechanical and electrical adjustment limits, may differ in operate or release times by as much as two or three to one. The designer must constantly keep this in mind when attempting to meet delicate circuit time requirements.

An aspect of the time characteristics of relays is the phenomenon of contact stagger. Since a relay does not operate or release instantaneously, there is a finite armature transit time during which contacts
may open or close more or less in a random sequence. Thus, several milliseconds may elapse between the closure of one make contact on a relay and another make contact in the same spring pile-up. The magnitude of the stagger usually increases with the acting time of the relay. Although definite contact sequences can be imposed, either by use of certain spring combinations or by adjustment, it is generally preferable to design the circuit in such a way that it is not affected by random stagger.

**Relay Construction for Slow Action.** The acting time of a relay, either on operate or release, can be increased considerably by any means which delays the build-up of magnetic flux (operate) or retards its decay (release). This can be done most simply by encircling the relay core by a conducting path. When the circuit is closed to the winding, induced currents in this path will generate magnetic flux to oppose the operating flux. When the circuit to the winding is opened, induce currents will generate flux in the core in a direction which tends to hold the relay operated. The conducting path has no effect when the steady state of the relay has been reached.

The most frequently used method of achieving these effects is to place a highly conducting sleeve, usually of copper, over the core. The sleeve may extend over the length of the core, or may be concentrated at either end of the structure. With end sleeves, operating time is increased to a much greater extent when the sleeve is located at the front end of the relay, close to the armature air gap, than when the sleeve is displaced toward the rear of the relay. Release time is affected to a much lesser degree by sleeve position.

The effectiveness of sleeves is much greater in increasing the release time than the operate time of a relay. The maximum reliable operate time that can be achieved with a general-purpose relay is well under 0.1 second, while the maximum release time may run as high as 0.5 second or more. It is also true that release time can be more closely controlled than operate time in manufacture. As a result, it is generally preferable to use the release of a relay for delayed action in circuits.

Aside from the use of sleeves, the acting time of relays can be increased to some extent by circuit means. A short-circuited secondary winding, for example, will increase the time of operation and release, particularly the latter. The release of a relay by short-circuiting the operating winding will increase the release time appreciably. Finally, as indicated in the preceding section, the operate time of a relay becomes greater as the circuit current is allowed to approach the operate adjustment value.
Switching Apparatus

When slow-acting relays, either operate, release, or both, are used in a circuit, it is customary to indicate the nature of the relay on the schematic drawing. This is done by placing code letters such as SR (slow release) in a box, drawn as part of the relay core. Illustrations of this are shown on Fig. 2-8. On the same figure is a list of the more common abbreviations used in similar fashion for indicating other special operating features of relays.

**Relay Contacts.** The function of a contact on a general-purpose relay is to provide a reliable low-resistance circuit path for its load under specified conditions of current flow, type of load, and total number

- AC – alternating current
- D – differential
- DB – double biased (biased in both directions)
- DP – dashpot
- EP – electrically polarized
- FO – fast operate*
- FR – fast release†
- MG – marginal
- NB – no bias
- NR – non-reactive
- P – magnetically polarized, using biasing spring, or having magnet bias†
- SA – slow acting
- SO – slow operate
- SR – slow release
- SW – sandwich wound to improve balance to longitudinal currents
- TS – two step

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**ANY COMBINATION OF THESE SYMBOLS MAY BE USED WHERE A RELAY POSSESSES MORE THAN ONE SPECIAL FEATURE.**

*Used where unusually fast operation or fast releasing is essential to the circuit operation.
†The proper poling for a polarized relay shall be shown by the use of + and – designations applied to the winding leads. The interpretation shall be that current in the direction indicated shall move or tend to move the armature toward the contact shown nearest the core on the schematic. If the relay is equipped with numbered terminals, the proper terminal numbers shall be shown.

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Fig. 2-8 Symbols Used to Indicate Slow Action and Other Special Features of Relays
of operations. To this end, contact metal in a variety of shapes and materials is fastened to the mating surfaces of the relay springs. Three common shapes of contact are shown on Fig. 2-9. Any of these may be mounted either singly or, in order to minimize troubles due to dirt, in twin form on independent or bifurcated springs. The contact metals most frequently employed are silver, platinum, palladium, and several alloys based on these and other metals. The precious metals are used to minimize the effects of corrosion and tarnish.

When properly used in circuits and when maintained in a clean atmosphere, the contacts of well-designed relays are capable of giving good service for many millions of operations. Improper use results in accelerated erosion of the contact metal. In addition, exposed contacts are always subject to the deposition of foreign material (film or dust) on the current-carrying surfaces. The effect of these trouble sources is modified by mechanical considerations such as contact pressure, contact chatter during operation, and the wiping action that often takes place when contacts open and close. Contact troubles manifest themselves in service by high or variable contact resistance, a completely open circuit, or mechanical locking of the contacts.
Contacts are usually rated in terms of maximum steady-state current, maximum current to be made or broken, circuit voltage or power, and type of contact load.

Contact erosion is accelerated by a reactive load, but the effect of such a load can be substantially reduced by the use of suitable circuit measures. An inductive load, such as a relay winding, for example, produces a high voltage by self-induction when the circuit to it is broken. This, in an unprotected circuit, causes sparking and rapid deterioration of the contact breaking the circuit. One form of contact protection consists of a resistor-capacitor network bridged across the contact or the load to absorb the high-voltage surge. A typical application is shown on Fig. 2-10. The optimum value of resistance and capacitance is a function of such variables as the inductance, circuit voltage, and load resistance.

If the load is capacitative in nature, the charging current, when the contact is made, may be high enough to damage the contacts. In this case, series resistance in the contact path will reduce the initial current to safe proportions.

2.2 SPECIAL-PURPOSE RELAYS

The function of special-purpose relays is implied directly by their name. They are the relays which are used to meet operating conditions outside the capabilities of general-purpose relays. Special-purpose relays are used primarily to respond to unusual input signals, or as the terminal equipment in signaling systems working over a distance. In some circumstances, special-purpose relays may be required as output apparatus - to handle very heavy currents, for example. Except for performing certain checking functions, they are rarely used within the main body of switching circuits.

Since the major portion of this volume is directed toward the logical aspects of circuit design in which special-purpose relays have little part, only a cursory review of their characteristics will be given here. It is safe to say, however, that in practice the circuit designer will find it possible to locate a choice of commercial relays to meet almost any reasonable extremes of operating conditions.

Polarized Relays. Polarized relays respond to current direction in addition to current magnitude. This effect is most frequently achieved by a permanent magnet incorporated in the relay structure, although some types achieve a similar result by electrical means. Many polarized relays are characterized by high sensitivity in addition to their primary attribute.
The polar relay, in general, operates in one direction to close one contact or set of contacts when current of a particular polarity is connected to its winding; and operates in the opposite direction to close a different contact or set of contacts when the current flow through the winding is reversed. On sensitive polar relays, usually no more than a single transfer contact is provided. The relay may remain in its last operated position when the circuit is opened, or it may be provided with a spring bias or a separate biasing winding to restore it to a known position. Some varieties of polarized relay assume a neutral position when no current is flowing through a winding.

Alternating-Current Relays. Although d-c relays will respond to alternating current, the armature has a strong tendency to chatter or buzz at twice the frequency of the applied voltage. In effect, the relay starts to release each time the current passes through zero. This difficulty can be eliminated in relays designed for a-c use by special construction of the relay core. The usual method is to split the core where it faces the armature and encircle one prong with a copper ring. The copper ring introduces a phase shift in a portion of the magnetic flux flowing through the magnetic circuit of the relay so that at no time does the total pull on the armature decrease to the point where the relay can start to release.

Extremely Slow Relays. Operate and release times attainable by the use of conducting sleeves can be greatly exceeded by other special methods of construction. Two of these techniques will be mentioned here.

The dashpot relay must force a volume of oil in a chamber through a small aperture before it can actuate its contacts. It can be arranged for immediate release to provide for rapid recycling. This construction permits delays with fair accuracy from a few seconds up to several minutes.

The thermal relay employs a heating coil and an element which expands or shifts position under application of heat. The element, usually a bimetallic strip, actuates the contacts. Means for compensating for ambient temperature changes are normally incorporated in the structure. This type of relay can provide delays up to thirty seconds or more, but it is inherently incapable of rapid recycling.

Sensitive Relays. Very sensitive relays can be built by special methods of construction, involving reduction in mass and inertia of moving parts, improved balance, light contact pressure, and close mechanical tolerances. As opposed to general-purpose relays which require operating power of the order of one to ten watts, sensitive
relays are available which operate on a few milliwatts. Such relays have a minimum of contacts, but usually offer high speed in addition to sensitivity.

**Marginal Relays.** With general-purpose relays, it is difficult to achieve a reliable non-operate current level much higher than 50% of the operate requirement value. However, by special construction, non-operate percentages as high as 90 are possible.

**Mechanically Locking Relays.** Relays are available with a latching arrangement which holds the armature in an operated position when the circuit to the operating winding is broken. Release is by means of an additional electromagnet, with a separate winding, which opens the latch to restore the armature. Relays of this type have little general utility, but may be of use when a circuit requires relays to remain operated for exceptionally long periods of time.

**Heavy Contact Load Capacity Relays.** Relays sometimes must switch relatively large amounts of power as, for example, in controlling motors. For such purposes relays are manufactured with special heavy-duty contacts, or are arranged to operate mercury switches.

**Miscellaneous Types.** In addition to the above types of relay, many other varieties with special features are available. There are vibration-resistant and shock-resistant relays for use where such conditions may be encountered in service. Midget and light-weight relays are designed for use where size or weight is an important consideration. Special construction is often necessary to achieve an exceptionally high number of operations or a very long life. For use under adverse weather or temperature conditions, relays with special finishes and coil treatments are manufactured.

Although this is but a partial listing of available relay structures and types, it should give an indication of the wide variety of relays and relay-like devices available to the circuit designer. However, it is worth pointing out again that the use of the special-purpose relays is limited, and that, in the field of control-circuit design covered in this volume, the general-purpose relay is the type required almost exclusively.

### 2.3 ELECTRONIC DEVICES AND ELECTROMAGNETIC SWITCHES

As pointed out earlier in this chapter, the description of electronic tubes, semiconducting devices, and electromagnetic switches is more conveniently given in chapters dealing with their circuit usage. The discussion of tubes and semiconductors is included in Chapter 10; the comparable description of switches is in Chapter 9.
2.4 MISCELLANEOUS SWITCHING APPARATUS

In addition to the basic apparatus described or mentioned previously in this chapter, switching circuits often require miscellaneous components to some extent. Many of these, such as resistors, capacitors, inductors, are standard electrical apparatus and require no discussion. Many other items are highly specialized types of equipment which have limited use in particular applications. There is nothing to be gained in attempting to catalogue them here.

However, three types of apparatus which have frequent and important use in switching circuits are manual switches or keys, jacks, and plugs. A brief description of their functions and characteristics will be given.

A switch or key is a manually operated set of contacts used to impart "on" or "off" information to a circuit, or to set up connections to or within a circuit. Although the designations "switch" and "key" are both in good standing for apparatus of this nature, the word "key" is used more frequently in discussing switching circuits.
Keys are available in non-locking types which restore when manual control is removed, and locking types which hold the position to which they are set. The contacts and springs of many types of key are similar to the contacts of general-purpose relays. The number of contact springs ranges from a single make or break to elaborate combinations of different contact arrangements. Typical examples of key contacts, illustrating the standard schematic conventions, are shown on Fig. 2-11.

Keys are most often actuated by a lever, a push-button, a turn-button, or a knob. When the device is set by the rotary motion of a knob, the designation "switch" is used more frequently than "key". The number of separate key positions, each with its own contact set, depends upon the type of key or switch. Push-button keys usually have two positions; lever-operated keys, two or three positions, as shown on Fig. 2-11; and rotary switches may have up to twelve or more. The latter switches operate on a different principle from lever or push-button keys, usually rotating brush or wiper contacts over wafers bearing fixed contacts or contact strips. Examples are shown schematically on Fig. 2-12.

Jacks and plugs are used for setting up temporary connections on a flexible basis. For example, a number of circuit paths may be connected to a field of jacks, and any desired interconnection between these circuit paths may be established manually by inserting cord-connected plugs in the corresponding jacks.

Standard plugs, which carry two or three conductors, are shown schematically on Fig. 2-13. The corresponding jacks are shown on Figs. 2-14A and 2-14B. In many circuits utilizing this type of connection, it is desirable to have the insertion of a plug open or close circuit paths that are independent of the paths through the plug. For this purpose, auxiliary springs may be supplied on the jacks, as shown on Fig. 2-14C.
2.5 BATTERY AND GROUND CONVENTIONS ON CIRCUIT DRAWINGS

Before leaving the subject of switching apparatus, the battery and ground conventions that are in general use on circuit drawings and that will be used in this book should be explained. Several of those conventions are illustrated by Fig. 2-15.

On Fig. 2-15A is shown the standard battery symbol with polarity indicated. The battery voltage may be shown where necessary. In text illustrations, the positive and negative battery terminals will be indicated solely by the long and short lines, respectively, of the symbol.

It is customary in most circuit applications to connect one side of the battery supply to the earth, or ground, in order to insure that the system is tied to a fixed potential. The symbol for ground is shown in Fig. 2-15B, and the symbol for a grounded battery (positive side grounded) in Fig. 2-15C.

Using this symbolism, the complete operating circuit for a relay may be shown as in Fig. 2-15D. However, if it is understood that a common buss connection makes available the same ground potential at all parts of the circuit, the drawing may be simplified as shown in Fig. 2-15E. Because of the resultant reduction in the number of lines on the circuit drawing, this convention of separated battery and ground will be used throughout the text.
Chapter 3

BASIC RELAY CONTROL PATHS

A relay circuit is an interrelated set of relay structures with input and output paths and internal configurations of contact networks. The relays are controlled both by the input paths and the contact networks on the relays themselves, whereas the output paths are activated for the most part only by the contact networks. Additional elements such as resistors, capacitors or varistors may occasionally be introduced into the relay circuit to achieve particular results.

Although the term "circuit" is sufficiently general to include groups of relays ranging in number from one to several thousand, the relays of a circuit usually are related functionally. If larger circuits are well designed, they break down naturally into smaller functional units each of which has all the attributes of a complete circuit, that is, inputs, outputs, and contact networks. A further breakdown of a circuit yields the actual control paths which operate and release each of the relays at the proper time. Although these control paths may be complex networks involving perhaps dozens of individual contacts, they have the common characteristic that they may occupy but one of two states at a time, that is, closed or open. The condition where an impedance is inserted in a control path is considered a special case which will be treated separately.

The methods of designing contact networks to meet circuit requirements are taken up in the next two chapters. Before the implications of contact network design can be adequately comprehended, however, it is necessary to understand the various ways in which a relay can be operated and released by an open or closed path. This chapter discusses these fundamental factors in relay circuit design.

3.1 BASIC CONTROL METHODS FOR RELAYS WITH SINGLE WINDINGS

There are four basic methods for controlling a relay with a single winding. Two of these, each the inverse of the other in effect, permit operation and release of the relay by closing or opening of a path. The other two cannot operate the relay by themselves, but permit holding it in an operated or released state by closing a path.
Direct Control. Fig. 3-1A shows the simplest and most common relay control path. Relay (X) is under direct control of contact network A: when A is closed, (X) is energized and operates; when A is open, (X) is de-energized and releases. Although Fig. 3-1A and Fig. 3-1B are the same in effect, the arrangement in Fig. 3-1A is generally preferred in practice since false grounds on the contact network will not blow a fuse. The arrangement shown in Fig. 3-1B is sometimes desirable, however, when other requirements of a circuit are considered. It is important to remember that in this and all other relay circuits there is a time delay between path closure and relay operation while the magnetic field is being built up and the moving parts of the relay are in transit. A similar but not necessarily equal delay exists between the opening of the path and the release of the relay.

Shunt Control. In some cases it may be desired to operate a relay when a path opens. The circuit of Fig. 3-2 does this by using a shunt path. When A is open, (X) is energized; when A is closed, (X) is de-energized.

Although this arrangement is sometimes very useful, it has several disadvantages. In relay circuit design, much effort is spent to keep a circuit economical and reliable. The resistor shown in Fig. 3-2 is necessary to limit current through the shunting path, but adds to the
expense of the circuit. The current drain through the shunt path performs no useful work. In addition, the release time of the relay is increased since current in the winding decays exponentially through the shunt path, instead of terminating abruptly.

![Fig. 3-3 Lock-up](image1)

**Fig. 3-3 Lock-up**

**Fig. 3-4 Variation of Basic Lock-up**

**Lock-Up.** In the previous control methods, the controlled relay had no part in its own operation. In a "locking" circuit, a relay affects its own control path. Thus, in Fig. 3-3, the path which operates relay (X) is through network A. Thereafter the relay can hold through network B, although prior to the operation of (X), network B has no effect upon the circuit. Relay (X) is said to have locked-up to B through its own contact. It is this circuit that imparts to relays the quality of "memory." That is, the relay is enabled to hold operated after the operating condition has disappeared. Fig. 3-4 shows a similar locking circuit where A and B control the relay in a slightly different manner.

![Fig. 3-5 Continuity Lock-up](image2)

**Fig. 3-5 Continuity Lock-up**

Another method of lock-up using a continuity spring combination is shown in Fig. 3-5. Once (X) is locked up under control of B, A is free to control other paths without being affected by the ground on the winding of (X). It is evident that in this arrangement B must always be closed before A, or the act of (X) operating will break its own operating path and release itself. Relay (X) will then buzz* until B is closed, locking it up. Other combinations of lock-up and operate paths which have different effects in the circuit are shown on Fig. 3-6.

*If the back-contact of a relay is included in its own operating path and no early-closing locking path is provided, the relay will alternately operate and release at a rapid rate when current is applied. This is known as buzzing.
Lock-Down. Lock-down is a scheme whereby a relay enters into its own control to keep itself from being operated by one path until another path has opened. As seen in the two arrangements in Fig. 3-7, network A cannot operate relay (X) unless network B is open. Once (X) has been operated by A, however, B may open and close with no effect upon (X). This circuit requires a current-limiting resistor, and it will drain current while the relay is shunted down. The acting time of the relay may be affected slightly by addition of the current-limiting resistor; the shunt lock-down path, however, does not change this acting time.

3.2 SUMMARY OF THE FOUR BASIC CONTROL METHODS

Fig. 3-8 shows the four basic control paths applied to one relay. The paths are the direct D, shunt SH, lock-up LU, and lock-down LD. Analysis will show the interrelationships between paths. For example, the relay can never operate if D is open, or SH is closed, or LD is closed; SH can always release the relay; LD is without control after the relay has operated, and so forth. In circuit design it is often necessary
to interchange control paths by contact network manipulation in order to achieve the desired operating characteristics.

3.3 RELAYS WITH SPECIAL CHARACTERISTICS

The preceding paragraphs have dealt with the basic ways of controlling a single-winding, general-purpose relay. Often situations arise within a switching or control circuit that can be met more economically by relays with special characteristics than by relays controlled only by the paths described above. In addition, a restrictive design condition, such as a limited number of leads available for control, can sometimes be met only by use of relays with special characteristics. The rest of this chapter deals with the control of such relays.

Relays with More Than One Winding. The most common use of a relay with several operating windings is to permit two or more operating paths of this relay to be isolated. A simple case is shown in Fig. 3-9A. Either or both paths will operate relay (X) without shunting or otherwise interfering with each other. This scheme can be used to isolate completely an operating path from a locking path without the strict time sequence required of a continuity lock-up, as in Fig. 3-9B.

A useful arrangement employing a double-winding relay is the differential circuit. In this circuit, the two windings are connected in opposition so that when both are energized, the net flux through the core is insufficient to operate or hold the relay. As shown in Fig. 3-10, A or B alone can operate relay (X); but if neither or both are closed, relay (X) is released. With this circuit, the two windings must be closely equal in turns and resistance, or external resistance must be added to balance the effect of the windings. A common
use for this arrangement is to indicate if two circuit paths such as A and B do not open and close at the same time. Another use is to permit releasing a relay by closing instead of opening a path. The operating path must remain closed until the release path is closed, and then the two paths must be opened simultaneously.

A variation of this scheme is to use an opposing release winding which is more effective than the operate winding in developing a magnetic field. By this means the total flux in the relay core can be driven rapidly through zero, producing a fast release action. In order to insure that the current paths are opened before the relay can re-operate on the reversed field, a make contact on the relay in question should be included in the circuit as shown on Fig. 3-11.

Marginal Relays. A marginal relay is one that will not operate on currents below a definite value through its winding, but will operate on currents over this value. It is often used in arrangements such as Fig. 3-12 where, in combination with a sensitive series relay, two conditions over a single lead can be detected. The sensitive relay operates on current below the non-operate current of the marginal relay. Thus A alone closed operates (S), but resistance R keeps the current too low to operate (MG). B closed operates both (S) and (MG). This circuit is one of the very few in which appear control paths with more than two conditions.

Theoretically a number of marginal relays could be used in series, operating successively as the current is increased in steps. However, variations in apparatus, adjustments, heating, battery voltage,
and so forth, make close margins hard to meet; practical circuits are seldom found having more than one marginal condition.

**Polar Relays.** Polar relays have two characteristics which make them valuable in switching and control circuits. First, they react to current direction; second, they are quite sensitive and capable of operating at high speed. The spring combination of many polar relays is limited to a maximum of a single transfer, and, unless biasing springs or windings are provided, there is no assurance which side, if either, of the transfer will be closed when the winding is de-energized.

Because of their sensitivity, polar relays are often used as marginal or sensitive relays to meet critical circuit conditions. An arrangement is shown in Fig. 3-13 where the relay has two windings, one of which is used for biasing. The biasing winding is poled to oppose the operating winding, and when energized alone, it holds the armature against contact No. 1. When the current in the operating winding exceeds that of the biasing winding by a small amount, the armature will operate to contact No. 3.

![Fig. 3-13 Polar Relay with Biasing Winding](image)

When the relay is used for its polar characteristics, some means of reversing the flux must be provided. Fig. 3-14A shows an easy way of doing this if two battery supplies are available. Two voltages of different amplitude but the same polarity achieve the same result, as shown in Fig. 3-14B. The source of lower voltage might be a resistance voltage divider.

**Slow-Acting Relays.** Slow-acting relays include a deliberate factor of time added to the operation or release of a relay. The slow action may be desired for either of the following two reasons:
1. To produce or measure a definite time interval.
2. To insure correct sequence of operation in a multi-relay circuit.

Slow-acting relays are, in general, controlled in exactly the same manner as standard relays, and can be incorporated into relay circuits with no difficulty. When a slow-release relay is used for timing, it is necessary to pre-operate the relay sufficiently in advance of the timing period to insure that current through the winding can attain full value before the relay control path is opened.

PROBLEMS FOR CHAPTER 3

3-1 A relay (R), equipped with only a single transfer-contact, operates when a key (A) is operated and remains operated when the key is restored to normal. The relay may then be released by operating a key (B), and will remain released when B is restored to normal until (A) is again operated.

Show two circuits for each of the four following conditions where the springs of the keys are limited to a single make or break on each key. For one circuit, one contact spring of both keys must be permanently connected to ground; for the other circuit, a contact spring of one key only may be permanently connected to ground. Use only single-winding relays.

(a) Key (A): BREAK  Key (B): BREAK  
(b) Key (A): MAKE  Key (B): MAKE  
(c) Key (A): BREAK  Key (B): MAKE  
(d) Key (A): MAKE  Key (B): BREAK

3-2 Three relays (X), (Y), and (Z) are controlled by keys (A), (B), (C), and (D) as follows: relay (X) operates when key (A) is operated and releases when key (A) is released; relay (Y) operates when either key (A) or key (B) is operated and remains operated until key (D) is operated; relay (Z) operates when any of keys (A), (B), and (C) are operated, and remains operated until key (D) is operated. Key (D) is only operated when all other keys are released. Keys (A), (B), and (C) are each provided with a single make-contact and key (D), with a single break-contact.

Design a circuit fulfilling these requirements and using no more than one make contact per relay.

3-3 (a) Two non-locking keys, (A) and (B), each with two make-contacts, are operated at random. Design a circuit for operating a relay (X) when either key is operated, but not when both are operated. The relay is to remain operated until both keys are operated, at which time the relay is released.

(b) Repeat part (a), assuming that each key is equipped with two break-contacts only.

3-4 In Fig. 3-10, assume that the contacts A and B controlling relay (X) are make contacts on relays (A) and (B). Design a circuit for controlling a single-winding relay (X) by contacts on (A) and (B), so that (X) operates in the same manner as in Fig. 3-10. Additional contacts may be used on relays (A) and (B) if necessary.
3-5 Two relays are selected to have an operate time of 0.2 second and a release time of 0.3 second.

(a) Design a continuously running interrupter circuit which will give repetitive output pulses grounded for 0.6 second and open for 0.4 second.

(b) Arrange the above circuit so that operation of a control key will change the output pulses to ground for 0.5 second, open for 0.5 second.

3-6 Three keys, (A), (B), and (C), control a relay (X). Relay (X) should operate when any one key is operated or when all three keys are operated. Under all other conditions, the relay should be released. Design the circuit, using no more than one transfer contact on each key.

3-7 In the diagram given below, indicate the positions in which lamps a, b, c, and d light as switch (SW) is turned from 1 to 8. Relay (B) will not operate or remain operated in series with relay (A) and a resistor (R) on 24 volts. Indicate also what relays are operated for each position of (SW).
A relay contact network is an arrangement of a number of individual contacts jointly controlling circuit paths. It is convenient to divide contact networks into two general types: two-terminal networks with a single input and a single output terminal; and multi-terminal networks, usually with a single input and a plurality of outputs. The sequence of operation of the individual contacts in the network may or may not be of importance, depending upon the application of the network. In this chapter, problems concerned with the configuration of contact networks are considered, without regard to sequence of operation. The network manipulations and simplifications discussed are of the most elementary variety. The problem of systematic network rearrangement is taken up in much greater detail in Chapter 5.

4.1 LOGIC OF CONTACT NETWORKS

The requirements which a contact network must fulfill may be formulated by stating those conditions under which the network will be open and those under which it will be closed. An analysis of the statement from the standpoint of logical validity is equivalent to an investigation of the network configuration itself. Thus, the tools and methods of logic underlie the analysis and solution of network problems.

Any two-terminal contact network can be considered to be a two-valued device. At any given instant it presents either an open (infinite impedance) or closed (zero impedance) path to electric current. Thus, the network can be represented by a single contact which is open or closed under conditions identical to those imposed on the network. Because the network, like the contact, can only be either closed or open, it is similar to a proposition in logic which is either true or false depending upon the truth or falsity of the various conditions of the proposition. The closed condition of a contact or network can be considered the equivalent of a true condition in logic. Then a proposition which is true only if conditions A and B are both true corresponds to a network which is closed only if contacts A and B are both closed.

Each condition upon which the proposition relies corresponds to an individual contact in the network. The design of a contact network to
perform a certain function under given conditions corresponds to the problem in logic of synthesizing a complete proposition from individual true or false statements. Moreover, the arrangement of the individual contacts in the network is similar to the relation of the various statements whose truth or falsity make the proposition true. If the synthesis of the proposition is not completely sound, the corresponding contact network will not function in the required manner. This similarity between contact networks and propositions in logic will be more fully brought out in succeeding sections.

4.2 "AND-OR" AND SERIES-PARALLEL RELATIONSHIPS

The most basic of all two-terminal networks are the simple series circuits and simple parallel circuits shown in Figs. 4-1 and 4-2. The network of Fig. 4-1 is closed, and relay (R) operates, only when both A and B are closed; whereas in Fig. 4-2 the circuit is closed if A or B (or both) are closed. Fig. 4-1 represents a proposition which is true only when both conditions A and B are true. If only one of the two conditions is true, the proposition is not true, and the circuit in Fig. 4-1 would not be closed.

A proposition which is true if condition A or B is true is represented by Fig. 4-2. It is evident that the conjunction "and" is equivalent to a series contact arrangement, and that the conjunction "or" is equivalent to a parallel contact arrangement. For example, consider the circuit shown in Fig. 4-3. Relay (R) operates if A and (B or C) are closed. The equivalent statement in logic would be: "The proposition is true if conditions A and (B or C) are true." Similarly, in Fig. 4-4 the circuit is closed (or the proposition is true) if A or (B and C) are closed (true). Although for more intricate circuits the punctuation of the statement might become highly complicated, it is possible to describe
completely the conditions for circuit closure for any series-parallel network by indicating parallel combinations by "or" and series combinations by "and".

It should be noted that in the phrase "A or B", the two components are not mutually exclusive. The network in Fig. 4-2 is closed if A or B or both are closed. In dealing with contact networks, the term "or" should always be interpreted to imply "either or both" unless otherwise explicitly stated.

Rather than stating a set of requirements in terms of open and closed contacts, it may be more convenient to state it in terms of relays operated and released. If a circuit is to be closed when a relay is operated, a make-contact is indicated; if the closure results from the release of a relay, the condition requires a break-contact. The relay circuit in Fig. 4-5 is equivalent to the network in Fig. 4-4 since the operating path for relay (R) is closed when relay (A), or relays (B) and (C), are operated. Another representation is shown in Fig. 4-6, where relay (R) operates when relay (A) is operated, or when (B) is released and (C) is operated.

4.3 THE NEGATIVE RELATION

In logic the "negative" of a proposition is a proposition which is false for those conditions, and only those conditions, for which the original proposition is true. The original proposition and its negative are therefore mutually exclusive. Similarly, the relation between a contact network and its negative is that one is closed when the other is open. It follows that, if network A is the negative of network B, B is
also the negative of A. The most elementary example of a contact network and its negative is a make-contact and a break-contact on the same relay. When a circuit through one contact is closed the other is always open*.

It is often possible to determine the negative of a particular network by inspecting the statement of conditions for closure for that network.

![Figure 4-7](image1.png)  
A Simple Network and Its Negative

![Figure 4-8](image2.png)  
Another Network and Its Negative

The circuit in Fig. 4-7A is closed only when relays (A) and (B) are both operated. The negative of this network would then be one which is open only when relays (A) and (B) are both operated, and closed when (A) or (B) is released. In this final phrase, the "or" suggests parallel contacts, and the fact that the relays are released to close a circuit suggests that the contacts be breaks. The circuit is shown in Fig. 4-7B. A second example is that of Fig. 4-8A where relay (A) must be released and either (B) or (C) operated to close the network path. Brief consideration indicates that the circuit in Fig. 4-8B is the desired negative.

From examination of the above networks and their negatives, it appears that to replace a network by its negative, it is only necessary to substitute for each contact in the network its negative and to place these negatives in a network in which series and parallel connections have been interchanged. In actuality, this method of obtaining negatives holds for any two-terminal network consisting of only series and parallel connections†. A somewhat more complicated network and its negative, illustrating the method, are shown in Fig. 4-9.

* This, of course, is not strictly true with commercial relays. During the acting time of these relays one contact may close before or after another opens.

† The proof justifying this method will be indicated in Chapter 5.
4.4 BASIC SIMPLIFICATIONS IN TWO-TERMINAL NETWORK DESIGN

Designing a network to meet requirements by substituting a contact for each logical condition often results in a redundancy of contacts. After the two-terminal contact network has been designed for closure under given combinations of relays operated and released, it may be possible to simplify the network by combining certain contacts and, perhaps, omitting others. First inspection often indicates simplifications which may be made at once. In the network of Fig. 4-10A, the circuit is closed if relays (A) and (C) are operated, or if relay (A) is operated and (D) is released. It can easily be seen that an equivalent
but simpler statement of the same conditions would be: relay (A) operated, and (C) operated or (D) released. The network in Fig. 4-10B, then, is equivalent to that of Fig. 4-10A. A similar example appears in Fig. 4-11. Here it is again possible to combine the two make-contacts on relay (A) into a single make.

![Network Diagrams](image)

**Fig. 4-12 Network Simplification Illustrating an Invalid Step**

When simplification of the type illustrated above is done, it is well to check that the final network contains all those paths and only those paths that appear in the original network. Fig. 4-12A shows a network, and Fig. 4-12B, a simplified equivalent to that network. An attempt to simplify further the network of Fig. 4-12B might suggest combining the two makes on relay (C), resulting in the circuit of Fig. 4-12C. This last manipulation, however, cannot be justified since a path not inherent in the original circuit is present in Fig. 4-12C. (The network of Fig. 4-12C is closed when B alone is closed, which is not true of Fig. 4-12A).

Other basic simplifications become evident when the actions of networks are closely analysed. For example, requirements might state that a network should be closed when contacts A or (A and B) are
closed. If the network is drawn and examined, it can be seen that the network always is closed when A is closed, no matter what the condition of B is. Therefore the expression "or (A and B)" is redundant and the network reduces to a single make contact, A. A similar result is obtained by analysis of the network to be closed when contacts A and (A or B) are closed.

Another example of permissible simplification occurs when a contact of a network is in series or parallel with a portion of the network including the negative of the first contact. In this class is the

![Diagram A](image1)
![Diagram B](image2)
![Diagram C](image3)
![Diagram D](image4)

Fig. 4-13 Simplification Employing Negative of Network
network closed when relay (A) is operated or (relay (A) is released and relay (B) is operated). This network is always closed when (A) is operated, and, when (A) is released, can only be closed by operation of (B). Therefore the break-contact on (A) representing (A) released is redundant, and the network reduces to A or B. This type of simplification will be discussed more fully in Chapter 5.

An indirect method of simplification, which in some cases may indicate steps not otherwise obvious, involves deriving the negative of the network, simplifying the negative, and finally taking the negative of this simplified circuit. In accordance with previously defined relationships, the negative of the negative of a network is exactly equivalent to that network. To illustrate, the negative of the network in Fig. 4-13A is determined by inspection. This negative circuit, shown in Fig. 4-13B, is simplified by inspection to give the network of Fig. 4-13C. Observe that the uppermost branch in Fig. 4-13B is unnecessary since it is paralleled by a branch of series breaks on relays (A) and (C) which is unaffected by the condition of (B). Then, the negative of this simplification is drawn as in Fig. 4-13D. Examination of Fig. 4-13D shows that it contains all those paths contained in the circuit of Fig. 4-13A and no others.

![Fig. 4-14 Basic Bridge Network and Its Series-Parallel Equivalent](image)

### 4.5 BRIDGE-TYPE NETWORKS

Not all two-terminal networks are composed of only series and parallel elements. Sometimes a circuit arrangement will include cross-connecting paths between parallel paths of series elements. Such networks are referred to as bridge-type networks.

The fundamental bridge-type network, shown in Fig. 4-14A, is conveniently analyzed by tracing all possible paths between input and output terminals. These paths indicate all the conditions for closure for the network and may be sketched as in Fig. 4-14B. Any bridge-type network may be transformed into a series-parallel arrangement by this
The process of investigating all paths. The number of paths increases with the complexity of the bridge design. For example, the bridge in Fig. 4-15 has thirteen possible paths between input and output terminals.

It is evident that the bridge circuit may afford some economy in number of contacts. Thus, such a network may be superior in this respect to its series-parallel equivalent. However, very few series-parallel arrangements have an equivalent bridge and, moreover, the equivalent bridge may be very difficult to derive.

Contact network simplification may, if the designer is not on guard, lead to a bridge-type circuit which is not exactly equivalent to the original network. An illustration of such inappropriate simplification is shown in Fig. 4-16. Although the bridge network of this figure contains all the paths present in the original network, it also is closed for the combination of relay (B) and (C) operated and (E) released, a path not existent in the unsimplified circuit. Of course, the bridge network might be acceptable if such a condition never exists.
4.6 MULTI-TERMINAL NETWORKS

The most common form of multi-terminal network is that of a single input which may be connected through the network to two or more outputs. The design of such a network may be carried out as the design of several inter-related two-terminal networks. An elementary case is illustrated by Fig. 4-17 where two make-contacts on relay (A) are combined into a single make. The resulting circuit is a three-terminal network, one terminal connected to ground and the other two to relay windings. However, care must be exercised in combining contacts to avoid "sneak" paths between outputs. Such a sneak condition is introduced in Fig. 4-18, and consists of path B and C and D in Fig. 4-18B which does not exist in the unsimplified network of Fig. 4-18A.

It should be noted in the example in Fig. 4-17B that if relays (B) and (C) are operated, the windings of relays (X) and (Y) are tied together, a situation which cannot occur in the circuit of Fig. 4-17A. Whether this situation is undesirable depends upon the requirements for the circuit and possible conditions at the network terminals. For example, the addition of a locking contact on relay (X), as in Fig. 4-19, would permit a sneak path through the make-contacts on relays (C) and (B) to hold relay (Y) operated after relay (A) has released. The effect of the sneak path is to add a second condition for the operation of relay (Y): relays (B) and (C) and (X) operated; whereas the original operating path for (Y) was through operated relays (A) and (C).
4.7 TRANSFER CONTACTS IN NETWORKS

If two paths of a network pass through contacts on the same relay, one through a make-contact and the second through a break-contact, theoretically these paths can never be closed at the same time — assuming no contact stagger. Paths exhibiting this characteristic are said to be "disjunctive." By network manipulation it may be possible to arrange the make and break contacts so that each has a terminal connected to a common junction point; under this condition a transfer contact may be substituted for the make and break. Such a substitution is shown in Fig. 4-20.

While a make-and-break-contact on a relay and the corresponding transfer combinations are both inherently disjunctive devices, the transfer combination is often preferable since it employs one less spring than the make and break arrangement. Although contact stagger may allow independent make- and break-contacts to be closed simultaneously for a short period, the transfer permits only one of two paths to be closed at a given time. Thus, it can be used to insure disjunctivity between paths or outputs.*

* The designation "transfer" as used herein implies the definite break-before-make sequence. Some types of general-purpose relays employ the physical form of the transfer combination, and yet permit an indeterminate sequence. Use of such combinations must be avoided when simultaneous closure of make- and break-contacts can introduce hazards.
Relay Contact Network Configurations

Fig. 4-19 A Possible "Back-up" Situation

Fig. 4-20 The Transfer Contact Used to Isolate Outputs
Often it is not immediately recognizable that a transfer combination can be employed. An example is shown in Fig. 4-21 where the make- and break-contacts on relay (A) can be replaced by a transfer. Here, the transfer arrangement is probably preferable since definite sequence between make and break on (A) is established and, as a result, there can be no momentary undesirable path in the circuit due to contact stagger on relay (A), if the make closes before the break opens.

A more elaborate example of network manipulation is shown in Fig. 4-22. It is evident from this illustration that the disjunctive character of the transfer allows simplifications which, at first glance, seem invalid.

A transfer contact may sometimes be substituted for a make-contact or a break-contact in a network to permit simplification in another part of the network. The advantage gained from this substitution arises from the disjunctive aspect of the transfer. For example, in Fig. 4-18 a sneak path through contacts D, C, and B to the winding of (X) was introduced when the two A contacts were combined. In the original network, the path to (Y) is closed when contacts A and C, or contact D, are closed. Since the path through A and C is important only when relay (D) is released, a break contact on (D) may be inserted in series with A and C without disturbing the circuit. This permits arranging the circuit as shown in Fig. 4-23, where the sneak path no longer exists.

This simplification technique is applicable when it is desired to combine in whole or part one branch of paralleled groups of contacts with another part of the network. Care must be exercised in choosing the contact to be converted to a transfer, or the network may be completely changed in performance. The effect of the path discontinuity
introduced during the transit time of the transfer must also be considered. Methods for performing this type of network manipulation will be discussed more fully in the next chapter.

Fig. 4-22 More Elaborate Network Simplification

Fig. 4-23 Use of Transfer to Eliminate Sneak Path of Fig. 4-18
4.8 TRANSFER TREE CIRCUITS

A "transfer tree" is a particular type of multi-terminal network in which the single input may be connected to any one of m outputs according to the combination in which the relays actuating the network contacts are operated. The circuit of a four-relay tree is drawn in Fig. 4-24. There are sixteen outputs, each output corresponding to one of the sixteen possible combinations in which four relays may be operated, including the all-released condition. One output only can be connected to the input at a time, and, because of the transfer-contact combinations, the outputs are completely isolated under any condition. As well as being disjunctive, each path from input to an output in a true transfer

Fig. 4-24 The Basic Four-Relay Transfer Tree
tree must pass through a contact on each controlling relay once and only once.

Because of the triangular symmetry of a fully developed tree, there is a definite relationship between the number of controlling relays, the number of outputs, the maximum number of transfers on one relay, and the total number of transfers in the tree. The table at the right indicates this relationship.

A tree may not always be fully developed. For example, if outputs for the four operated relay combinations (B); and (B), (C); and (B), (D); and (B), (C), (D) are not desired, two transfers on (D), one on (C), and the make half of a transfer on (B) can be eliminated.

An obvious disadvantage of the tree circuit in the form shown in Fig. 4-24 is the very unequal distribution of the spring load on the relays. It is possible to obtain a more uniform loading on all the relays except the first by rearranging the circuit. There is no method of rearrangement which will place more load on the relay with the single transfer combination without greatly increasing the total number of springs in the circuit.

In approaching the problem of rearranging the tree circuit for relay spring load equalization, note that the circuit is composed of branches or minor trees. For example, the four-relay tree of Fig. 4-24 includes two three-relay trees involving relays (B), (C) and (D), or four two-relay trees involving relays (C) and (D) plus one two-relay tree on relays (A) and (B). Any of these minor trees may be manipulated without affecting the operation of the complete tree. To illustrate, consider the lower main branch connecting to the make contact of relay (A) of Fig. 4-24. This can be completely reversed on relays (B), (C) and (D) to give the circuit of Fig. 4-25. When the output leads are redesignated in accordance with the new configuration, the circuit is identical in operation with the circuit of Fig. 4-24, yet the contact distribution is one transfer on (A), five transfers on (B), four on (C) and five on (D).

This same type of rearrangement can be applied to minor branches individually or in successive stages to produce any desired distribution of contacts. In Fig. 4-26A is shown the uppermost minor branch of the transfer tree of Fig. 4-24. The two transfers on (D) may be placed on relay (C), and the transfer on (C) placed on relay (D), as in Fig. 4-26B. The output leads are redesignated, and the rearranged minor tree is

<table>
<thead>
<tr>
<th>Table 4-1 Quantities Involved in Fully Developed Trees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of relays in tree = n</td>
</tr>
<tr>
<td>Possible relay combinations = output terminals = m = 2^n</td>
</tr>
<tr>
<td>Maximum number of transfers on one relay = m/2 = 2^{(n-1)}</td>
</tr>
<tr>
<td>Total number of transfers in tree = m - 1 = 2^n - 1</td>
</tr>
</tbody>
</table>

Relay Contact Network Configurations

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reconnected to the rest of the tree. Fig. 4-26C shows that relay (D) now has seven transfers and relay (C), five.

The contact manipulation of the circuit of Fig. 4-26C may be continued as desired. In Fig. 4-27, the upper branch on relays (B), (C), and (D) of Fig. 4-26C has been rearranged by switching the transfers on (D) and (B). Note that the process of tree rearrangement is merely one of interchanging designations of the contacts composing a minor tree; in Fig. 4-27, designations B and D have been interchanged. If this rearrangement is replaced in the circuit of Fig. 4-26C, the load distribution is: relay (A), one transfer; (B), four transfers; (C), five transfers; and (D), five transfers. Thus, by continued rearrangement of branches, the load may be shifted as desired, remembering that the total number of transfers will remain $2^n - 1$.

Similar manipulations may be performed on incomplete trees. With these circuits, it is sometimes possible to reduce the total number of contacts by rearrangement. A three-relay tree with five outputs is shown in Fig. 4-28 together with a rearrangement of the tree. Here, the designations of relays (B) and (C) are interchanged and, as a result, the contact configuration has been simplified. Care must be taken to insure that the tree obtained by manipulation has the same outputs as the original tree. Also it should be kept in mind that, in an incomplete as
well as a complete transfer tree, each path from input to an output must pass through a contact on each controlling relay once and only once.

Fig. 4-26 Manipulation of Tree Contacts
4.9 TRANSFER CHAINS

Another method of disjunctively connecting a single input to a number of outputs is by means of a transfer chain circuit. There are two forms in which these chains may appear, as shown in Fig. 4-29;
one in which a path is closed to an output through the make half of the transfer, and the other in which the break half of the transfer closes the output path. For the first type, if a number of the relays (A), (B), (C), ..., (N) are operated, only the operated relay nearest to the input establishes an output path. Similarly, for the second type, only the released relay nearest to the input establishes an output path. Many functional circuits utilize a transfer chain of one or the other of the above types as an essential part of their control or output circuitry. For circuit reasons the transfers in the chains may be replaced by continuity-transfer contacts. The transfer chain is a specific example of a class of networks described in the next section.

Fig. 4-29 Transfer Chains

4.10 REITERATIVE NETWORKS: POSITIONAL AND SYMMETRIC NETWORKS

In the earlier sections of this chapter, any statement of the requirements for closure of a network path has itemized, in some form, the state of each relay involved in closing that path. In this section a class of networks is discussed in which the conditions for circuit closure may be expressed without designating particular relays as operated and unoperated. As a result of the regular repetitive pattern of the configurations that satisfy these requirements, these networks are designated reiterative networks.

A two-terminal reiterative contact network is normally composed of a consecutive chain of identical elements, the contact set on one or more relays comprising an element, as indicated by the diagram of Fig. 4-30. Each element in the chain is joined to the next by a number of connecting leads. The connection between two adjacent elements is usually identical to that between any other pair of elements. In many cases, however, the contact sets on the relays at the two ends of the
chain are somewhat different from the contact sets forming the elements in the center of the chain.

As an example of a reiterative network, consider the network of Fig. 4-31 which is closed when an odd number of relays in a group is operated. Each element of the network, indicated by the dotted-line blocks in the figure, is joined to the next succeeding element by two leads, "1" and "2". Taking the left-hand terminal of the network as a starting point, it is evident that a path is closed between this terminal and any particular "2" lead when the number of operated relays included between the terminal and the "2" lead is even. Similarly, a path is closed between the same terminal and any particular "1" lead when the number of included operated relays is odd.

Generalizing, it may be stated that the incoming leads to an element carry information concerning the state of the preceding relays. Identical output leads carry to the next stage similar information as modified by the state of the relay in question. A general method of design applicable to reiterative networks involves first, determination of the specific and pertinent information required by each element, including the number of necessary leads; and second, design of the modifying contact network for a prototype relay.

The contact configuration comprising an element depends upon the particular effect which the state of the corresponding relay must have on the paths from the incoming leads to the outgoing leads. In the

Fig. 4-31 A Reiterative Network Closed When an Odd Number of Relays are Operated
network of Fig. 4-31, when the (C) relay, for example, is unoperated, its incoming leads "1" and "2" are connected to its outgoing leads "1" and "2" respectively, since, no matter which of these leads is linked through preceding relays to the left-hand terminal, the situation preceding relay (D) is the same as that preceding relay (C). When relay (C) is operated, however, it connects incoming lead "1" to outgoing lead "2" since, if the number of operated relays preceding relay (C) is odd, the number of operated relays preceding relay (D) is even. Similar reasoning shows that incoming lead "2" must be connected to outgoing lead "1" when relay (C) is operated.

There are two general classes of reiterative networks: positional and symmetric networks. These two classes may be differentiated, one from the other, by the general type of statement which describes the conditions of closure of each. A positional network is one for which the conditions for closure may be expressed in terms of the relative physical position of operated and unoperated relays in a circuit. For example, a network, constructed on a line of relays, which is closed when two or more adjacent relays are operated and all other relays are released, is a positional network.

The requirements for closure of a symmetric network, on the other hand, may be given in terms of the number of relays operated. As an example, a network which is closed when any two out of five relays are operated is a symmetric network. Other examples are: three out of seven relays operated, all but one of a group operated, an even (or odd) number of relays operated (as in Fig. 4-31), and so forth. Series chains and parallel groups of make-or-break contacts are simple forms of symmetric networks, since they can indicate all relays operated or

![Fig. 4-32 "Both Operated or Both Released" Symmetric Circuits](Image)

![Fig. 4-33 "Only One of Two Operated" Symmetric Circuits](Image)
The Design of Switching Circuits

Fig. 4-34 Symmetric Circuits Based on Chains of Makes, Breaks, and Transfers
released, or at least one relay operated or released. A basic characteristic of the symmetric network is that the designations of relays controlling the network may be interchanged without affecting the condition for network closure. It is this characteristic which gives the class of symmetric networks its name.

Several examples of useful symmetric circuits are shown on Figs. 4-32, 4-33, and 4-34. The first of these figures shows two networks, each constructed on two relays, closed when both relays are operated or both released; the second shows two networks closed for "only one of two relays operated." Fig. 4-34 illustrates a number of additional symmetric networks together with the associated conditions for network closure. For some of these networks, two output terminals are shown, one for each closure condition for that network. The two outputs may be connected together if only a single indication is desired for both conditions.

As an illustration of the design of a reiterative network, assume the following positional requirements for closure: a two-terminal network is to be closed when, in a group of eight relays, a single set of one or more adjacent relays is operated and all other relays are released. The conditions for closure include all relays operated.

In solving this type of problem by the suggested method, the basic reiterative element for which a network in constructed may be established as one or more relays. In this particular case, trial indicates that a two-relay element is preferable. Designating the prototype relays as (C) and (D), it can be seen that there must be three information-bearing input paths to element (C-D), and three output paths from the element. This is shown on Fig. 4-35A. Input path (1) is activated when no preceding relays are operated; path (2), when an adjacent group including the immediately preceding relay is operated; and path (3), when an adjacent group not including the immediately preceding relay is operated. The output paths carry similar information as modified by element (C-D).

Analysis of the conditions under which the element must switch input to output paths indicates the following:

1 to 1: when (C) and (D) are both released.
2 to 2: when (C) and (D) are both operated.
3 to 3: when (C) and (D) are both released.

1 to 2: when (D) is operated, whether or not (C) is operated.
2 to 3: when (D) is released, whether or not (C) is operated.
1 to 3: when (C) is operated, and (D) is released.
Fig. 4-35 Design of a Positional Network Which Is Closed When a Single Group of One or More Adjacent Relays Is Operated
The contact networks corresponding to these conditions are shown on Fig. 4-35B. Combining makes and breaks on each relay permits simplification to the form shown on Fig. 4-35C. Analysis indicates that the path drawn with a dashed line is otherwise included and is therefore superfluous.*

The complete circuit constructed from four elements is shown on Fig. 4-35D. The end elements are suitably modified to provide the correct terminations for the network. Note that a slight adjustment at the output end will permit an odd number of relays in the network.

The method illustrated above is perfectly general and can be used to design reiterative networks, both two-terminal and multi-terminal, for any positional or symmetric requirements. However, for the often-used class of symmetric network in which a path is closed when "m out of n" relays are operated, it is more convenient to obtain the required

* The degree of simplification obtained is possible only because of the disjunctive character of the make and break contacts on an individual relay. Actually, by observing the junction points of make and break contacts on each of the (C) and (D) relays, the circuit can be further reduced to a transfer and a break per relay.
network by modifying the general symmetric lattice structure shown in Fig. 4-36. This figure shows the circuit in conventional notation and also in the symbolic form in which each contact is represented by a line segment. Inspection of the circuit indicates that the network is disjunctive and that the particular output cut through to the input terminal depends only on the number of relays operated. The designating numbers of the output terminals correspond to the number of operated relays.

The action of the circuit is more easily seen from Fig. 4-36B. If ground is applied to the input terminal and no relays are operated, ground will be connected through a chain of back contacts on level zero of the illustration to the output terminal O. If any one relay is operated, ground will pass along the zero level until it encounters the make on the operated relay, where it will be switched up to the next or first level. The ground then remains on the first level and will appear on terminal 1. If more than one relay is operated, the ground will be passed from one level to the next higher level at each operated relay. The highest level reached will correspond to the number of relays operated.

To satisfy a given condition with this network, it is only necessary to connect to the input and to the desired output terminal, and to eliminate all superfluous contacts. A circuit for "two out of five" relays operated is indicated in Fig. 4-37. The figure shows the circuit both as a lattice diagram and in more conventional form. Many of the junction points on the lattice diagram can represent transfers facing in either direction; an arrangement with fewer springs will result if the circuit is drawn from both ends with transfers facing toward the center.

Fig. 4-37 The "Two-Out-of-Five" Network
There is no known manner of redistributing the spring load of this type of symmetric circuit to give a more uniform spring load. Certain relays must necessarily carry more springs than others. However, since the circuit is symmetric, it makes no difference which particular relays in the group carry the heavy spring load, and if certain relays have a heavy load for other functions, they may be favored in the symmetric circuit arrangement.

When the circuit is to be closed for two or more combinations, the output points representing these combinations may be connected together and the superfluous parts of the circuit eliminated. An arrangement for closure when either one or four out of seven relays are operated is shown in Fig. 4-38 as an example.

Where the circuit is to be closed for several combinations and the number of relays operated form an arithmetic progression, or the numbers greater than a certain one form such a progression, the circuit can be simplified by "shifting down" a higher level to coincide with...
a lower one. For example, a circuit might be required to be closed when either none, three, or six of seven relays are operated. The third level of the symmetric diagram can then be shifted down to coincide with the zero level, as shown in Fig. 4-39. The network derived previously which is closed for an odd number of relays operated, is a shifted-down symmetric.

The design of symmetric networks is not merely an exercise in contact manipulation; such networks perform several useful functions in control or switching systems. One of the most important of these is a check function. It is often necessary to give an alarm unless a certain number of relays, no more and no fewer, operate at some given time. A suitable symmetric network controlled by these relays can give an indication by its closure that the correct number of relays is operated. The absence of this indication would denote an alarm or trouble condition.

In some circumstances it may be desirable to give a direct indication that an incorrect number of relays is operated. A network to fulfill this requirement is the negative of the network closed for legitimate conditions, and is actually a symmetric circuit closed for any number of relays operated except the specified number. A method of determining the negative of a symmetric circuit is given in the next chapter. The value of checks such as these will become more evident in later discussion of functional circuits.

**PROBLEMS FOR CHAPTER 4**

**4-1** A relay \(X\) is controlled by relays \(A\) and \(B\). Design suitable circuits for controlling \(X\) and lighting a lamp from a contact on relay \(X\), the lamp to light for each of the following conditions. Design a separate circuit for each of the conditions:

(a) When \(A\) is operated, and \(B\) is released (assume a single make-contact on \(X\)).

(b) When \(A\) is operated, and \(B\) is released (assume a single break-contact on \(X\)).

(c) When \(A\) is released, or \(B\) is released (assume a single transfer-contact on \(X\)).

(d) When \(A\) is released, and \(B\) is operated (assume a single make-contact on \(A\), a single break-contact on \(B\), and a single make-contact on \(X\)).

(e) When \(A\) is released, or \(B\) is released, but not both released (assume a single make-contact on \(X\)).

(f) When \(A\) is operated and \(B\) is released, or when \(A\) is released and \(B\) is operated (assume a single make-contact on \(X\)).
4-2 Combine and simplify the following two-terminal network. (It can be reduced to six springs). Draw the resulting two-terminal network and its negative.

4-3 (a) Simplify the following two-terminal network. (It can be reduced to fifteen springs).

(b) Derive the negative of the simplified network of part (a)

4-4 Simplify the following two-terminal network. (It can be reduced to six springs).

4-5 Design a multi-terminal network for controlling three relays (X), (Y), and (Z) as follows:

Relays (A) or (C) in operating normally operate relay (Y). Should relay (B) be operated, relay (A) in operating operates relay (X) instead of relay (Y), while relay (C) in operating still operates relay (Y); Should relay (D) be operated, relay (C) in operating operates relay (Z) instead of relay (Y). With (D) operated, relay (A) in operating operates relays (X) or (Y) depending upon the condition of relay (B). (This can be done with ten springs).
4-6 Combine and simplify the networks controlling relays (X), (Y), and (Z) as shown below. If possible, the final circuit should be such that relay (C) is equipped with only a single transfer and no other contact. (This can be done with fifteen springs)

![Diagram of switching circuits]

4-7 Design a transfer tree network for four relays, (A), (B), (C), and (D), to indicate each of the following ten combinations by lighting a corresponding lamp. For each combination given, the unlisted relays are released, and no lamp must light for any combination not shown.

<table>
<thead>
<tr>
<th>Combination</th>
<th>Lamp Lighting</th>
<th>Lamp Not Lighting</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>(A)</td>
<td>(A) and (B)</td>
</tr>
<tr>
<td>(A) and (B)</td>
<td>(A), (B) and (C)</td>
<td>(B) and (C)</td>
</tr>
<tr>
<td>(A) and (C)</td>
<td>(A), (B) and (D)</td>
<td>(B)</td>
</tr>
<tr>
<td>(B)</td>
<td>(A), (B), (C), and (D)</td>
<td>(C)</td>
</tr>
</tbody>
</table>

This design should be executed to distribute the contact spring load as evenly as possible, consistent with the lowest possible number of contact springs. (This can be done with a distribution 11-11-8-3 springs, not necessarily in that order).

4-8 Design a contact network to indicate by lighting a lamp when three and only three out of six relays are operated, with the following two exceptions: relays designated (A), (C), and (E) are never operated simultaneously, and relays (B), (D), and (F) are never operated simultaneously, under normal conditions. The lamp should not light if either of these two combinations of operated relays does occur inadvertently.

Draw the required contact network. (This can be done with thirty springs).

4-9 A circuit includes a group of six relays numbered in order from 1 through 6. Design a contact network which is closed when all relays above a certain one are operated and all others are released. The network should be closed when all relays are operated, but open when all are released. (This can be done with fifteen springs).

4-10 A circuit includes a group of eight relays numbered from 1 through 8. Design a network which is closed when any two adjacent relays are operated and all others are released. (This can be done with forty springs).
4-11 A circuit includes a large number of relays which operate and release in a random fashion, changing condition, however, only one relay at a time. It is desired to indicate by means of a ring of lamps whether the number of operated relays is increasing or decreasing. Only one lamp should be lighted at a time, and some lamp should be lighted under all conditions of the relays. An increase in operated relays is indicated by clockwise rotation of lighted lamps; a decrease in operated relays by counterclockwise rotation. Determine the minimum number of lamps necessary to achieve this effect and design a contact network to control the lamps. (A suitable network for ten relays can be constructed with eighty-one springs).
Chapter 5

SWITCHING ALGEBRA AND MANIPULATION OF RELAY CONTACT NETWORKS

In the design of circuits, rearrangements of contact networks are often necessary either to reduce to a minimum the number of contacts, or to meet requirements as to the number of springs permissible on a certain relay. These network manipulations and simplifications may be performed by methods of inspection similar to those indicated in Chapter 4. However, to carry out the manipulations of complex networks in this manner with any facility, considerable experience is necessary. The designer learns to recognize certain network configurations together with possible rearrangements, and develops his own favorite rules. During the design process the circuit must be redrawn repeatedly to afford a check on the validity of the manipulations employed. The examples quoted in the last chapter indicate that the simplification by inspection may become tedious and time-consuming.

In order to expedite relay contact network design and manipulation, an adaptation of the algebra of logic can be used\(^1\). The algebra of logic, created to introduce mathematical discipline into philosophical logic, is based upon the relationships among two-valued variables. The fundamental similarity between contact networks and propositions in logic was mentioned in Chapter 4. This similarity led to the publication by Shannon in 1938 of an algebraic method, developed from the algebra of logic, applicable to relay contact networks\(^2\).

Switching algebra offers, first, a very convenient notation for expressing contact networks. Utilizing this notation, the algebra then permits manipulation of any two-terminal series-parallel network into a variety of equivalent forms, including the simplest series-parallel form, with mathematical rigor. Switching algebra is an extremely useful design tool for setting up, simplifying, and combining complex networks. However, in its present form it takes no account of time or sequential relationships, and is of appreciable value only in the later stages of design, after the circuit plan has been clearly formulated.

\(^1\),\(^2\) See the references at the end of this chapter.
5.1 SYMBOLIC NOTATION IN SWITCHING ALGEBRA

In switching algebra as used in this volume, letters, in general, are employed as the symbols to indicate either two-terminal contact networks or single contacts of a relay. It is customary to designate a relay with a particular letter and thereafter represent each contact of the relay, and the two-terminal network controlling the relay, by the same letter. A primed letter indicates a contact (or a network) which is the negative of the contact (or network) designated by the same letter unprimed. For convenience, make-contacts are designated by unprimed letters. Finally, a series connection is indicated by a sign of addition, and a parallel connection by a sign of multiplication. As in ordinary algebra, the latter sign is usually omitted. To illustrate: \( A + B' \) is the notation for a make-contact on relay (A) in series with a break contact on relay (B); \( AB' \) indicates a parallel connection of the same contacts.

Fig. 5-1 Simple Networks and Their Algebraic Equivalents
The four networks shown in Fig. 5-1 are represented by the expressions $A(B + C)$, $A' + B'C$, $AB' + C'D$, and $(A + B) \left[ C' + D(E' + F) \right]$. It can be seen how the use of parentheses and brackets facilitates the expression of the more complicated networks. Any series-parallel two-terminal network may be represented in this fashion. The convenience of such notation is obvious.

The algebraic expression representing the conditions for closure of a contact network $X$ is usually spoken of as a function of $X$, written $f(X)$. Thus, if the conditions for operating relay (R) were relays (A) and (B) operated, the algebraic representation would be: $f(R) = A + B$. Although a contact on the controlled relay is often included in the network exerting the control, as in the expression for a circuit containing a lock-up path $[f(R) = (A + B)(R + C)]$, the arrangement of the expression is such that the use of the same letter to represent both a contact and a network should cause no confusion.

In numerical algebra the symbols may represent variables, each of which may assume one of a number of values in a particular set of circumstances. This is true also of switching algebra with the limitation that, since contacts and contact networks are two-valued devices, each variable can take on only one of two values, values corresponding either to an open or to a closed path. The quality which this value represents is defined as "hindrance", somewhat analogous to resistance. In the algebraic notation, the symbol 0 is arbitrarily assigned to a closed circuit, and 1 to an open circuit. That is, a closed circuit is said to have zero hindrance, and an open circuit, a hindrance of unity.*

### 5.2 THE POSTULATES OF THE ALGEBRA

As an essential preliminary to the formulation of a workable algebra, a set of postulates must be laid down, providing a suitable foundation. These postulates appear in Table 5-1. Note that the word "circuit" in the table refers only to contact networks and not to combinations of contacts and relay windings, since the winding of a relay has a hindrance neither of zero nor of unity.

In the algebra, it is assumed that all contacts on relay ($X$) act simultaneously. Each appearance of $X$ indicates a contact on ($X$), and it

* Since use of the + symbol of addition to indicate a series "and" connection provides a more natural representation of the physical reality of contact networks than use of the multiplication symbol, this convention has been adopted in switching algebra. The result of this convention, taken in conjunction with the use of the values 0 and 1 to represent "hindrance", is that the symbols for "and" (+) and "or" (•), and for closed-"true" (0) and open-"false" (1) as used in switching algebra, are the inverse of the conventions used in the algebra of logic. However, because of the perfect duality inherent in the algebra, the postulates and theorems are not affected.
is necessary that all similar appearances of X have the same value at the same time. Though this restriction is often of not too great importance, it should be kept in mind since the phenomenon of contact stagger can cause undesired effects in the circuit realization of an algebraic expression.

The postulates numbered (2) through (4) and the theorems to follow are arranged in pairs to indicate the duality that exists between the processes of multiplication and addition, and between the hindrance quantities zero and one. If, in any postulate, the zeros and ones are interchanged, and the multiplications and additions are also interchanged, the result will be the postulate which is the dual of the original postulate. Since the postulates exhibit that this duality is perfect, the dual of a theorem is proved if the theorem is proved.

5.3 THEOREMS

Theorems in switching algebra may be considered as statements of fundamental rearrangements which may be made in a circuit without affecting its action. Application of the theorems to two-terminal networks permits simplification and conversion into a variety of equivalent forms.
The Design of Switching Circuits

Theorems on Addition and Multiplication. The theorems in the following group indicate important properties of networks when the individual components are multiplied or added.

Theorem (1a) \( X + Y = Y + X \)
Theorem (1b) \( XY = YX \)
Theorem (2a) \( X + Y + Z = (X + Y) + Z = X + (Y + Z) \)
Theorem (2b) \( XYZ = (XY)Z = X(YZ) \)
Theorem (3a) \( XY + XZ = X(Y + Z) \)
Theorem (3b) \( (X + Y)(X + Z) = X + YZ \)
Theorem (4a) \( X + X = X \)
Theorem (4b) \( XX = X \)
Theorem (5a) \( X + XY = X \)
Theorem (5b) \( X(X + Y) = X \)

The first two theorems, (1a) and (2a), and their duals, (1b) and (2b), merely indicate that the grouping of symbols or the order in which they are taken has no effect upon the value of a symbolic equation, an obvious characteristic of contact networks. Theorems (3a) and (3b) permit reduction, expansion, or rearrangement of contacts in certain networks. Theorem (3b) is not true in numerical algebra though its dual, theorem (3a), is the familiar distributive law which makes the factoring process possible. Theorems (4a) and (4b) show that identical contacts on the same relay, or identical networks, connected in either series or parallel, are exactly equivalent to a single contact, or network. Theorems (5a) and (5b) often permit a considerable reduction in number of contacts.

To prove each of the above theorems, and those to follow, offers no particular difficulty. If a few basic theorems are proved from the postulates, the remainder can be proved in turn by manipulations of the basic theorems. However, such a method depends upon a different algebraic procedure for each theorem to be proved. A method of proof which may be used in identical fashion for every theorem is that of "perfect induction", testing the theorem by means of the postulates for all possible values of the variables. This procedure is practicable since each variable can have but two values.

As an illustration of the method of perfect induction, theorem (5b) will be verified. A table is constructed, as in Table 5-2, which gives each possible combination in which the variables can occur. For every combination, the values for each side of the theorem are entered. By
Switching Algebra and Manipulation of Networks

### Possible Values for X and Y

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Left Side of Theorem</th>
<th>Right Side of Theorem</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>X(X + Y)</td>
<td>X</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1(1 + 1) = 1 \cdot 1 = 1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0(0 + 1) = 0 \cdot 1 = 0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0(0 + 0) = 0 \cdot 0 = 0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5-2 Proof of Switching Algebra Theorem (5b) by Perfect Induction

means of the postulates, the left side of the theorem is simplified and shown equal to either 0 or 1.

By this system it is proved that, for every possible combination of values of the variables, the two sides of the theorem are equal, and hence, the theorem is a true statement under all conditions. Every theorem can be completely proved in this same manner.

Although the application of theorems (3), (4), and (5) can easily be visualized, some discussion of theorems (3a) and (3b) should be enlightening. Theorem (3a) may be applied in factoring an expression. For example:

\[
ABCD + ABE + AF = A(BCD + BE + F) = A[B(CD + E) + F]
\]

Circuit paths are shown in Fig. 5-2. Note that two A contacts and one B contact have been eliminated.

![Network Simplification by Factoring](image)

Fig. 5-2 Network Simplification by Factoring
Theorem (3a) also allows cross-multiplication as in numerical algebra. For example:

\[(A + B)(C + D) = A(C + D) + B(C + D) = AC + AD + BC + BD\]

The corresponding circuits are shown in Fig. 5-3. In this example, the final circuit contains more contacts than the original circuit. However, this particular manipulation has been helpful in that, while the original expression shows at a glance the conditions necessary for circuit closure, the final expression shows the conditions for opening the circuit. That is, the expression \((A + B)(C + D)\) indicates a closed circuit if \(A\) and \(B\), or \(C\) and \(D\), are closed; the expression \(AC + AD + BC + BD\) shows that the circuit will be opened if one of the following are open: \(A\) and \(C\), \(A\) and \(D\), \(B\) and \(C\), or \(B\) and \(D\). By the use of this theorem any circuit composed of series groups of parallel contacts can be put in the form of parallel chains of series contacts, and vice versa.

By means of theorem (3b) it is possible to "cross-add", which is the dual of cross-multiplication. This manipulation also indicates conditions for closing and for opening the circuit. To illustrate:


Fig. 5-4, showing the corresponding circuits, clearly indicates the duality between this and the process illustrated in Fig. 5-3.

The dual of the manipulation of Fig. 5-2 is given by the following example of simplification, illustrated in Fig. 5-5:

These two theorems, (3a) and (3b), perform operations similar to those applied in combining contacts as indicated in Chapter 4. However, the use of the theorems simplifies the process considerably.

Theorems (3), (4) and (5) complement each other to form a very flexible set of operations for the manipulation of algebraic expressions. For example, the expression \((X + Y) (X + Z)\) can be reduced to \(X + YZ\) either by the application of (3b) or by the successive applications of (3a), (4b), and (5a), as follows:

\[
\begin{align*}
(X + Y)(X + Z) & = XX + XZ + XY + YZ & (3a) \\
& = X + XZ + XY + YZ & (4b) \\
& = X + XY + YZ & (5a) \\
& = X + YZ & (5a)
\end{align*}
\]

In similar fashion, an expression \(XY + XZ\) can be reduced to \(X(Y + Z)\) through use of theorems (3b), (4a) and (5b) without recourse to theorem (3a). Thus, where either (3a) or (3b) may be applied, it is of little consequence which operation is used provided that, when indicated, it is followed by theorems (4) and (5) to eliminate surplus terms. However, a suitable choice between (3a) and (3b) will usually achieve desired simplification with a minimum amount of manipulation.
The Design of Switching Circuits

Theorems on Negatives. As discussed in Chapter 4, the negative of a contact network \( X \) is a network which is a closed path under all conditions for which \( X \) is open, and is open under those conditions for which \( X \) is closed. The basic relations between a contact network and its negative can be briefly expressed by the following two theorems:

Theorem (6a) \( (X)' = X' \)
Theorem (6b) \( (X')' = X \)

Theorem (6a) states that the negative of \( X \) is \( X' \); and (6b), that the negative of the negative of a contact network \( X \) is \( X \). It was also set forth

Fig. 5-6 A Network and Its Negative

\[
f(X) = AB' + C'D
\]

\[
(f(X))' = (A' + B) (C + D')
\]

Fig. 5-7 Another Network and Its Negative

\[
f(X) = A \left[ B' + C \left( D' + E'F \right) \right]
\]

\[
(f(X))' = A' + B \left[ C' + D' \left( E + F' \right) \right]
\]
in Chapter 4, without proof, that the negative of a network could be derived by taking the negative of each contact in the original network and substituting series paths for parallel paths and vice versa. Similarly, for obtaining the negative of an algebraic expression, substitute addition for multiplication and multiplication for addition, observing all brackets either present or implied, and replace for each variable its negative.

The theorems expressing this rule are:

Theorem (7a) \((X + Y + Z + \cdots)' = X' \cdot Y' \cdot Z' \cdots\)

Theorem (7b) \((X \cdot Y \cdot Z \cdot \cdots)' = X' + Y' + Z' \cdots\)

The proof of these expressions for two variables may be easily obtained by the method of perfect induction. The proof may then be extended to any number of variables by successive steps, utilizing the fact that a set of associated contacts is equivalent to a single contact. If the relations are assumed to have been proven for two variables, the extension to three variables is shown by the following expressions:

\[
(X + Y + Z)' = [(X + Y) + Z]' = (X + Y)' \cdot Z' = X' \cdot Y' \cdot Z'
\]

\[
(X \cdot Y \cdot Z)' = [(X \cdot Y) \cdot Z]' = (X \cdot Y)' + X' = X' + Y' + Z'
\]

As a demonstration of the application of the rule, consider the simple case shown in Fig. 5-6 for a network \(X\):

\[
f(X) = AB' + C'D
\]

\[
[f(X)]' = (AB')' \cdot (C'D')
\]

\[
= (A' + B) \cdot (C + D')
\]

A more elaborate example of a network and its negative appears in Fig. 5-7:

\[
f(X) = A[ B' + C \cdot (D' + E' F) ]
\]

\[
[f(X)]' = A' + B \cdot [ C' + D \cdot (E + F') ]
\]

Theorems on 0 and 1. There are certain theorems which indicate the behavior of contact networks when combined with their negatives or with open or short circuits. These theorems are tabulated below. As before, the symbol \(X\) represents any two-terminal network of one or more contacts.

Theorem (8a) \(X' + X = 1\)

Theorem (8b) \(X' \cdot X = 0\)

Theorem (9a) \(0 + X = X\)

Theorem (9b) \(1 \cdot X = X\)

Theorem (10a) \(1 + X = 1\)

Theorem (10b) \(0 \cdot X = 0\)
That theorems (8a) and (8b) are valid is evident by the definition of the negative of a network. A network in series with its own negative is always equivalent to an open circuit since at any time either the network or the negative is open. Similarly, a network in parallel with its negative presents a closed path at all times since one or the other is closed at any given moment. Theorems (9a) and (9b) and (10a) and (10b) represent relations obvious by inspection but necessary to the algebra.

As an example of the application of these theorems, consider the circuit of Fig. 5-8A:

\[
A(A' + B + C) = A'\bar{A} + AB + AC \quad (3a)
\]
\[
= 0 + AB + AC \quad (8b)
\]
\[
= A(B + C) \quad (9a), (3a)
\]

A similar example is the dual of the above, as in Fig. 5-8B.

As a more complicated example, consider the circuit of Fig. 5-9 which is simplified algebraically below. It would be an instructive demonstration for the reader to simplify the circuit by inspection before applying the methods of the algebra.

\[
A'(B+C')(A+B'C) = (A'B + A'C')(A + B'C) \quad (3a)
\]
\[
= A'AB + A'BB'C + AA'C + A'B'C'C \quad (3a)
\]
\[
= 0 + 0 + 0 + 0 \quad (8b), (10b)
\]
\[
= 0; \text{ a closed circuit.}
\]
This conclusion is not obvious from a superficial study of the circuit, although a detailed analysis will show it to be true. As in most cases, the final result could have been algebraically obtained by employing different combinations of theorems or the same theorems in a different order. The selection of the theorems to be used in simplifying a circuit is not critical; the desired simplification may usually be obtained in any of several different ways. The most rapid solution to this circuit would come from recognizing that \( A'(B + C') \) is the negative of \( A + B'C \).

![Additional Network Manipulation by Theorems on 0 and 1](image)

Two theorems that derive directly from the preceding group, (8) through (10), are the following:

Theorem (11a) \( (X + Y') \ Y = XY \)

Theorem (11b) \( XY' + Y = X + Y \)

In network simplification, these theorems frequently permit taking large steps without going through intermediate stages involving several other theorems. For example:

\[ (A + C')(B + A'C) = B(A + C') \]  \hspace{1cm} (11a)

This reduction can be performed in one step by noting that \((A + C')\) is the negative of \((A'C)\), and applying theorem (11a). The two theorems could also have been applied directly to the circuits of Fig. 5-8.

Another valuable use of theorem (11a) is to make two parallel branches of a network, \( X \) and \( Y \), disjunctive. This utilizes the theorem in the reverse direction from that employed in network simplification, and its applications will be discussed later in the chapter under the heading of multi-terminal networks. Theorems (11a) and (11b) also, at times, permit expanding a factor into a form where part of it can be combined with another factor, producing a net reduction in contacts.
Supplementary Theorems. Three additional theorems of some interest are as follows:

Theorem (12a) \((X + Y)(X' + Z)(Y + Z) = (X + Y)(X' + Z)\)
Theorem (12b) \(XZ + X'Y + YZ = XZ + X'Y\)
Theorem (13) \((X + Y)(X' + Z) = XZ + X'Y\)

These theorems are not as universally useful as those previously presented, but perform circuit simplification which it is difficult to otherwise obtain. Theorem (12b) is the dual of (12a) but is not so written above; the form shown brings out the equivalence of theorems (12a) and (12b) as indicated by theorem (13). Theorem (13) is its own dual.

Either (12a) or (12b) can be proved by perfect induction and the remaining two theorems derived from it. For (12a), note that, in the first two brackets of the expression to the left, the circuit is closed if both \(Y\) and \(Z\) are closed, regardless of the value of \(X\). Thus, the third bracket, \((Y + Z)\), is unnecessary.

A network susceptible to reduction by the use of theorem (12) is

\[ f(X) = A \cdot (B + AC')(C' + D)(B' + AD) \]

Although short-cuts can be taken if certain relationships between the factors of the equation are noted, a straightforward attack on the simplification would be first to multiply the expression out. Factors of the form \(XX'\) or \(X + X'\) are automatically dropped

\[ A(B + AC')(C' + D)(B' + AD) \]

\[ = (AB + AC')(C' + D)(B' + AD) \] (3a)

\[ = (ABC' + AC' + ABD + AC'D)(B' + AD) \] (3a)

The underlined factors drop out from application of theorem (5a)

\[ = AB'C' + AC'D + ABD \] (3a), (5a)

\[ = A(B'C' + C'D + BD) \] (3a)

\[ = A(B'C' + BD) \] (12b)

If, in the original expression, factor \(A\) and factor \((C' + D)\) had been multiplied together, the result would be

\[ (B + AC')(AC' + AD)(B' + AD) \] (3a)

By theorem (12a), this reduces to

\[ (B + AC')(B' + AD) = ABD + AB'C' \] (13)

\[ = A(BD + B'C') \] (3a)
A very important application of theorem (13) is to convert a circuit from a form in which a continuity or make-break transfer may be necessary, to a form in which an ordinary break-make transfer is satisfactory. The circuit of Fig. 5-10A corresponds to the expression \((X + Y)(X' + Z)\). It can be seen that in switching from one path to the other, there is necessarily a slight open interval while contact \(X\) is in transit. This circuit may cause a false release of \((R)\) unless a continuity-transfer is used for \(X\). However, the circuit of Fig. 5-10B, equivalent in accordance with theorem (13), does not suffer from this limitation. The included path, \(Y + Z\), covers \(X\) and \(X'\) in transit. By the use of this theorem, the necessity for a continuity-transfer can always be avoided in two terminals networks.

**5.4 GENERAL NOTES ON NETWORK SIMPLIFICATION AND MANIPULATION**

As can be seen, the use of switching algebra makes possible more rapid and often more reliable circuit manipulation than the inspection method. A disadvantage of the algebra is that it can be directly employed only for series-parallel two-terminal networks, although it is sometimes indirectly applicable to more involved circuit arrangements.

In using switching algebra, the attempt should continuously be made to note relationships among the terms of an expression which will permit the application of simplifying theorems. For example, if repetitions of a term or a factor can be discovered or can be arrived at by manipulation, it may offer a clue to eliminating unnecessary contacts. In the same sense, negative relationships between terms or between complex factors should be looked for. If such relationships can be ascertained, the use of theorems (8), (11), or (12) may be indicated.

If no direct steps suggest themselves in studying an expression, a good rule to follow is to multiply out all factors in accordance with theorem (3a). This will expand the expression or some portions of it,
and usually many terms will automatically drop out. The remainder may fall into a form which suggests additional simplification. In any expression in which contacts and their negatives occur, or several appearances of the same contact, it is worth while to derive several different forms of the expression before assuming that the simplest form has been found.

It frequently occurs that the number of appearances of either one of two variables may be reduced, but not appearances of both. To illustrate this point:

\[(A + B) (A + C)(D + C) = (A + B C) (D + C) \]  \hspace{1cm} (3b)

\[= (A + B)(A D + C) \]  \hspace{1cm} (3b)

Both these expressions are equivalent and require the same number of contacts. However, there may be contact requirements on the relays of the circuit that make one form more desirable than the other. This indicates one of the valuable uses of switching algebra: to manipulate a network into the form most suitable for integration into a relay circuit.

In this regard, it is always possible, for any two-terminal network, to reduce the number of contacts on any one relay to no more than one make and one break. If both occur, they may be combined into a transfer. This may be of value in situations where a special relay with limited contacts, for example, a polarized relay, is required in a circuit. However, it should be noted that this sometimes cannot be accomplished without a net increase in the total number of contacts.*

In analyzing circuits it is sometimes useful to be able to put an expression for a contact network into a form that will indicate at a glance all conditions for either opening or closing a path. As mentioned previously, these two forms are a product of sums (conditions for closing a path) and a sum of products (conditions for opening a path). Theorems for performing the transformations are (3a) and (3b), and the

* Two additional theorems that are useful in reducing contacts on a particular relay \((A)\) to no more than a make and a break, when contacts on \((A)\) are part of a network \(f(X)\), are as follows:

\[(14a) f(X) = A \cdot f(X)_{A=1, A' = 0} + A' \cdot f(X)_{A=0, A' = 1} \]

\[(14b) f(X) = \left[A + f(X)_{A=0, A' = 1}\right] \left[ A' + f(X)_{A=1, A' = 0}\right] \]

In applying theorem (14a), the term \(A\) is placed in parallel with \(f(X)\) and to this is added a second appearance of \(f(X)\) in parallel with the term \(A'\). Within each appearance of \(f(X)\), then, all terms \(A\) and \(A'\) are replaced by 1 and 0 as indicated by the theorem. Cancellation of factors in accordance with theorems (9) and (10) reduces the network to final form. Successive applications of the theorem can reduce appearances of a second relay \(B\) to no more than two makes and two breaks, etc. Theorem (14b) is handled in similar fashion.
technique for using them has already been discussed in Section 5-3, under the heading "Theorems on Addition and Multiplication".

There may be occasions where it is worth while to expand such expressions to include in each factor a symbol for all relays involved in the network. This will give a complete picture of the network for analytical purposes. In deriving a product of sums of this nature, the method is to add to each factor a term $XX'$ representing each missing symbol and then to continue the expansion. If a sum or products is desired, each factor is multiplied by terms $(X + X')$ representing each missing symbol, and the expansion is continued. To illustrate this with a simple example: a complete expansion of the network $A' + BC'$ is desired in terms of a product of sums.

$$A' + BC' = (A' + B)(A' + C')$$  \hspace{2cm} (3b)

$$= (A' + B + CC')(A' + BB' + C')$$

$$= (A' + B + C)(A' + B + C')(A' + B' + C')$$ \hspace{2cm} (3b)

$$= (A' + B + C)(A' + B + C')(A' + B' + C')$$ \hspace{2cm} (4b)

The final form shows that there are three distinct conditions of relays (A), (B) and (C) for path closure comprehended in the original expression. The complementary expression of a sum of products can be derived by similar means and will be found to include five terms.

5.5 MULTI-TERMIONAL NETWORKS

As has been explained previously, switching algebra is not directly applicable to the simplification and combination of multi-terminal networks. However, the notation of the algebra is useful in expressing the individual branches of a multi-terminal network in such a form that their combination is facilitated. Furthermore, the algebra is of value in converting the network branches to forms where their combination is possible.

Combining Elements in Series Networks. A common form of multi-terminal network which may often be developed with the aid of switching algebra is shown in Fig. 5-11A, where a single input is connected to several outputs by a contact network. One procedure for the design of such a network consists of determining the conditions for connecting the input to each output and deriving a two-terminal network for each of these paths, as in Fig. 5-11B. Often these two-terminal networks may then be combined into a single multi-terminal network, reducing the total number of contacts required.
Consider the following four two-terminal paths for controlling four separate relays at the outputs of these paths.

\[
\begin{align*}
  f(W) &= A + B + C + D \\
  f(X) &= A + B + EF \\
  f(Y) &= A + B (G + H) \\
  f(Z) &= C + F
\end{align*}
\]

In this set of equations having common elements, the common elements may be combined as shown. The solution is also drawn schematically on Fig. 5-12.

The method may be most easily applied when the individual two-terminal networks are expressed as a sum of products. Similar elements, such as A and B above, are collected to the left of the equations, the element appearing most often being first in order. In order to combine elements in a vertical line, these elements must be identical, not shunted by a factor which can be independently closed, and adjacent to a common junction point. It is evident that this method of reduction allows the connecting of two outputs together; isolation is not complete.
Rearrangement of a network may be necessary to permit combination with another network. For example:

\[ f(X) = (A + B'C) (A' + B) \]

\[ f(Y) = AB + D \]

By applying theorem (13a), \( f(X) \) may be rearranged and the paths combined:

\[ f(X) = (A + B'C) (A' + B) = AB + A'B'C \]

\[ f(Y) = AB + D \]

The final simplification appears in Fig. 5-13. Note the use of transfers on relays (A) and (B) to reduce the required number of contact springs.

Combining Elements in Shunt Networks - Disjunctive Paths. More difficulty may be experienced in combining elements which are part of parallel branches in two-terminal networks. This becomes clear in the simple case of the following two networks where it is desired to combine the A factors into a single contact.

\[ f(X) = AB \]

\[ f(Y) = A + C \]

If the attempt is made to combine these two networks directly in order to eliminate one appearance of A, the result is as shown in Fig. 5-14A. A sneak path is introduced whereby relay (Y) can operate through \( B + C \). The difficulty is that the parallel paths \( A \) and \( B \) of \( f(X) \) are not disjunctive, that is, they both can be closed at the same time. Therefore, connecting an independent path to the junction of \( A \) and \( B \) creates an unwanted path. The two factors, \( A \) and \( B \), can, however, be made disjunctive by the application of theorem (11a) as follows:

\[ f(X) = AB = (A + B') B = A(A' + B) \]

Since \( A \) is the combining term, the first of the two disjunctive expressions, that in which the additional contact is placed in series with \( A \), must be chosen. Otherwise the sneak path will not be eliminated. The
circuit form of the combined $f(X)$ and $f(Y)$ networks is shown on Fig. 5-14B. Note that the transit time of $B$ introduces a momentary open in the operating path of $(X)$. The effect of this must be carefully analyzed.

This method may be extended to more complicated situations. In general, with a set of two-terminal networks, all similar contacts on a particular relay may be combined into a single make or break, provided that each of these contact elements has been made disjunctive with respect to its shunting paths. By making both make and break contacts of a specific relay disjunctive, the spring load on that relay may be reduced to a single transfer, if the multi-terminal network of which the contacts now form a part is of the single-input type. This sometimes, however, takes heroic measures.

An example of this procedure is as follows: The paths for operating relays $(X)$, $(Y)$, and $(Z)$ in Fig. 5-15A are:

- $f(X) = (A + C) \ B$
- $f(Y) = A + D$
- $f(Z) = AE$

The elements $A$ of $f(X)$ and $f(Z)$ are made disjunctive:

- $f(X) = (A + C) \ B = (A + C + B') \ B \ (11a)$
- $f(Y) = A + D$
- $f(Z) = AE = (A + E') \ E \ (11a)$

and are combined to obtain the network of Fig. 5-15B.

Theorem $(11a)$ is also applicable to the simplification of two-terminal networks. Consider the network of Fig. 5-16A, symbolized by:

$$(A + B' + C) \ (A'B + D)$$

![Diagram](Fig. 5-14 Combining Shunted Paths)
Since the factors \((A + B')\) and \((A'B')\) are disjunctive, it should be possible to combine them in some manner. Applying theorem (11a),

\[ A'B' = A' (A + B) \]

then,

\[ (A + B' + C) (A'B + D) = [(A + B') + C] [A' (A + B) + D] \]

The network equivalent to the last expression is shown in Fig. 15-16B on the next page.
Combining a Network and Its Negative. The situation sometimes arises where, given a two-terminal network, it is desired to design a three-terminal network comprising the original network and its negative. This type of network is shown on Fig. 5-17. It can be demonstrated that, for two-terminal series-parallel networks, a combined network can always be designed using a transfer in place of each individual make and each individual break in the original network. The combining can sometimes be done directly or with simple manipulation; more frequently a transformation of one of the networks by theorem (11a) is necessary.

Fig. 5-17 Three-Terminal Network Comprising a Network and Its Negative

Fig. 5-18 Combining a Network and Its Negative: \( f(R) = X + Y \)
The method of combining applicable to the basic circuit $X + Y$ and its negative $X'Y'$ is shown on Fig. 5-18. In order to combine the circuits of Fig. 5-18A, the $X'Y'$ factor must be transformed to $(X + Y')X'$. This permits the circuit of Fig. 5-18B.

A more elaborate example is shown on Fig. 5-19A. The original network and its negative are:

$f(R) = AB' + C'D$

$[f(R)]' = (A' + B) (C + D')$

Treatment of this type of combination is facilitated by splitting the original network into two factors in product or sum form ($XY$ or $X + Y$). Each factor is then equivalent to contact $X$ or $Y$ in Fig. 5-18, and may be treated accordingly. The splitting process may be carried as far as is necessary beyond the first two factors. For example, in Fig. 5-19A the factor $AB'$ is considered as $X$ and the factor $C'D$ as $Y$. $AB'$ and its negative are then combined to give the equivalent of the $X$ transfer in Fig. 5-18B; and $C'D$ and its negative are combined to give the equivalent of the $Y$ transfer. These two circuits are then combined in accordance with Fig. 5-18B to give the circuit of Fig. 5-19B.

### 5.6 TABLES OF COMBINATIONS

A useful technique of setting up and simplifying two-terminal networks, based on switching algebra, is the method of tables of combinations. This method permits setting up in tabular form all conditions for closing a path. Most simplifications can be discerned directly from the table, and any remainder can be obtained by algebraic manipulation. The force of the method lies in the easy handling of many variables and in the mechanical assistance which helps assure that all conditions for path closure are included.
In a particular circuit of, for example, two relays (A) and (B), it is possible to have at any given moment one of four conditions. If an additional relay (C) is added, the same four conditions or combinations can obtain with (C) operated and also with (C) released to give a total of eight combinations. Each added relay will double the number of possible combinations. Thus, for n relays the total number of combinations equals \(2^n\).

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A'</td>
<td>+</td>
<td>B'</td>
</tr>
<tr>
<td>2</td>
<td>A'</td>
<td>+</td>
<td>B</td>
</tr>
<tr>
<td>3</td>
<td>A'</td>
<td>+</td>
<td>B</td>
</tr>
<tr>
<td>4</td>
<td>A</td>
<td>+</td>
<td>B'</td>
</tr>
<tr>
<td>5</td>
<td>A</td>
<td>+</td>
<td>B'</td>
</tr>
<tr>
<td>6</td>
<td>A</td>
<td>+</td>
<td>B</td>
</tr>
<tr>
<td>7</td>
<td>A</td>
<td>+</td>
<td>B</td>
</tr>
<tr>
<td>8</td>
<td>A</td>
<td>+</td>
<td>B</td>
</tr>
</tbody>
</table>

Table 5-3
Output Combinations for Three Relays in Algebraic and Simplified Notation

Obviously a separate network path can be closed for each combination of controlling relays operated and released. For three relays (A), (B), and (C), the eight possible different paths are as shown on the left side of Table 5-3.

Note that of these eight paths one, and only one, is closed at any given time, and that some one path is always closed. Any two-terminal network on three relays can be designed from the above table by connecting the desired combinations in parallel, omitting all undesired combinations.

For convenience, a similar table, employing a different notation, is used. Here, the symbol 1 represents the released condition of a relay and 0 represents the operated condition. (The assignment of these symbols is purely a matter of choice. The convention selected indicates the hindrance of a make contact of the associated relay when the relay is in an operated condition.) Such a table is written as on the right side of Table 5-3.

Here again each row represents a particular operated combination of the three relays. In row 5, for instance, the notation, 0 1 1, symbolizes the combination (A) operated, (B) released, and (C) released. From the standpoint of contact network paths, the symbol 1 represents a break, and 0 a make contact. It is important to avoid confusion between this notation and the 0 and 1 used in switching algebra.

To derive a two-terminal network, relay combinations for closure are selected from the table of combinations. Paths representing these combinations are placed in parallel and simplified. For example, a path is to be closed for the first two combinations from Table 5-3.
The algebraic notation is written and simplified:

\[(A' + B' + C') (A' + B + C') = A' + C' \]  

(3b), (8b)

For the first four combinations in Table 5-3, the combinations and circuit notation are as follows:

\[
\begin{array}{ccc}
A & B & C \\
1. & 1 & 1 & 1 \\
2. & 1 & 0 & 1 \\
3. & 1 & 1 & 0 \\
4. & 1 & 0 & 0 \\
\end{array}
\]

\[
(A' + B' + C') (A' + B + C') (A' + B + C) = A' + (B' + C')(B + C') (B + C) (B + C) = A' 
\]  

(3b), (8b)

Close examination of these two examples shows that the final network forms could have been predicted from inspection of the combinations used. In the first example a closed path is desired if (A) and (C) are released, no matter what the condition of (B) is. In the second example, circuit closure is to occur for (A) released, regardless of (B) and (C), since the latter two relays go through all possible combinations while (A) is released. A rule which immediately suggests itself, then, is that if, in any portion of a table of combinations describing a particular two-terminal network, all possible combinations of certain relays appear in conjunction with a single combination of the remaining relays, the network contacts on the former relays are redundant in that portion of the network. An alternative statement of the rule is as follows: when \(2^m\) different combinations of a total of \(n\) variables are identical in all but \(m\) columns, contacts on the corresponding \(m\) relays are not required.

As an illustration of the use of this rule, the following table of combinations describes conditions of closure for a certain network:

\[
\begin{array}{cccccc}
A & B & C & D & E \\
1. & 0 & 1 & 0 & 1 & 1 \\
2. & 0 & 1 & 1 & 1 & 1 \\
3. & 0 & 1 & 1 & 0 & 1 \\
4. & 0 & 1 & 0 & 0 & 1 \\
5. & 0 & 0 & 1 & 0 & 1 \\
6. & 1 & 0 & 1 & 0 & 1 \\
7. & 1 & 1 & 1 & 0 & 1 \\
\end{array}
\]
The Design of Switching Circuits

In rows 1, 2, 3, and 4, (A), (B), and (E) are identical, but (C) and (D) take on all possible combinations \((n = 5, m = 2, 2^m = 4)\). Rows 1, 2, 3, and 4, then, indicate \(2^m\) combinations of \(n = 5\) variables where all but \(m = 2\) of these variables are identical. Thus, contacts on the \(m\) relays (C) and (D) are not required for these four combinations. Likewise, in rows 3, 5, 6, and 7, the variables (C), (D), and (E) are identical, and contacts on (A) and (B) are unnecessary in these rows. Now all rows have been considered. Note that each row may be combined with others as many times as desired, but each row must be considered at least once. The desired circuit fulfilling all requirements of the seven combinations is expressed by

\[
(A + B' + E') (C' + D + E') = \\
E' + (A + B')(C' + D)
\]

Another example representing six conditions for network closure follows. In this example the reference numbers over the algebraic expressions indicate the combinations from which these factors were obtained:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2.</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3.</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4.</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5.</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>6.</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
(C' + D)(A + C') = C' + AD
\]

Referring again to Table 5-3, it is apparent that if the total number of possible combinations \((2^n)\) is divided into two parts, the circuit symbolized by one part is the negative of that symbolized by the second part. Therefore, if a network is closed for certain combinations of controlling relays, the negative of that network is closed for all remaining combinations, a corollary of the definition of the negative of a network.

This rule can be utilized in the simplification of two-terminal networks which are closed for a majority of the controlling relay combinations. To illustrate, consider a network closed for six of the eight combinations of Table 5-3. The combinations are numbered in accordance with the original table. This network can be simplified as already explained:
Switching Algebra and Manipulation of Networks

A B C
1. 1 1 1
3. 1 1 0 (1-3-5-7) (5-6-7-8)
5. 0 1 1 = B' A
6. 0 0 1
7. 0 1 0
8. 0 0 0

The result is AB'. However, it is more convenient than the above manipulation to note that the two unused combinations for relays (A), (B), and (C) are

2. 1 0 1 = A' + B
4. 1 0 0

The negative of this is AB' which, as can be seen, is equivalent to the other six possible combinations.

The practical utility of the columnar notation will be shown in succeeding chapters.

\[
\begin{align*}
\text{Fig. 5-20 Y-Delta Transformation}
\end{align*}
\]

5.7 DELTA-Y TRANSFORMATIONS

A transformation similar to the Y-Delta, or star-mesh, transformations in electrical network theory exists for contact networks. The Y-Delta transformation is shown in Fig. 5-20. The equivalence of the two forms may be indicated by considering all paths for all combinations of terminals taken two at a time. For example, the path between a and b in the Y form is \((X + Y)\), while the Delta arrangement for this path is

\[
(X + Y) (X + Z + Y + Z) = X + Y \quad (5b)
\]
Similarly, all other terminal-to-terminal paths may be shown to be equivalent.

The transformation from Delta to Y, in Fig. 5-21, is the dual of that from Y to Delta. Here again paths between terminals in the one form are equal to those in the other. Considering paths between a and b; in the Delta arrangement the paths are expressed by $X(Z + Y)$, and in the Y arrangement, by

$$XZ + XY = X(Z + Y) \quad (3a)$$

The Y-Delta transformation may be extended to junction points with more than three legs. This is illustrated by Fig. 5-22 in which the junction has four legs, a four-element "star". The equivalent mesh arrangement consists of a path from each terminal (a, b, c, and d) connecting to every other terminal; and each of these paths is composed of the sum of the two elements connecting to the corresponding terminals of the star. The transformation holds for a star of any number of elements, although the proof is not given herein. The rules for the dual transformation, from mesh to star, are not known for all cases.
5.8 RESOLUTION AND SYNTHESIS OF BRIDGE-TYPE NETWORKS

Although it is impossible to treat bridge circuits directly by switching algebra, since they cannot be described in their simplest form by the usual "and-or" relations, some rearrangements of such networks can be carried out by aid of the algebra. A bridge-type network, the basic form of which is shown in Fig. 5-23, may be expressed as a series-parallel configuration by any of three methods.

The first of these involves tracing out all possible paths from input to output terminal, as discussed in Chapter 4. Such a process results in a product of sums.

An alternative method is to determine all possible means by which the circuit may be opened by taking cuts across it, as in Fig. 5-24. The resulting equation is the sum of products:

\[ AB + BEC + AED + CD \]

In a complex network, consideration of all paths through the network becomes involved.

Another procedure for bridge reduction entails the use of Y-Delta or star-mesh transformations, and has the advantage over path-tracing of insuring that no paths have been overlooked. The transformations are employed to convert nodes or meshes of the bridge to a form which will combine with the rest of the bridge to give a series-parallel network. If the bridge is complicated, several successive transformations may be necessary in the reduction of the network.

Occasionally it is convenient to synthesize a bridge network from the equivalent series-parallel arrangement. To effect such a synthesis, the designer must recognize by inspection that such an equivalence exists, either from the configuration or from the algebraic expression of the network. For example, both the forms
(A + C) (A + E + D) (B + E + C) (B + D)

and

AB + BEC + AED + AD

indicate equivalence to the basic bridge network of Fig. 5-23.

If the bridge form to which a series-parallel network may be reduced includes any branches comprising a multiplicity of terms in series or parallel, it may be difficult to recognize that a bridge exists. In this case, the network equation can be manipulated by a systematic method into a form which will disclose the bridge relationship. The first step is to express the equation as a product of sums. A complete set of factors for each individual term is then developed and written as shown below. This particular set of factors is derived from the bridge of Fig. 5-23.

\[
\begin{align*}
(1) & \quad A + C + \frac{C}{D + E} \\
(2) & \quad B + \frac{D}{C + E} \\
(3) & \quad C + A + \frac{A}{B + E} \\
(4) & \quad D + \frac{B}{A + E} \\
(5) & \quad E + \frac{A + D}{B + C}
\end{align*}
\]

Since this set of factors in effect defines the simple bridge, if the equation under study can be shown to be identical in form to this set, it is thereby demonstrated to be a bridge.

When, in the actual case, any branch in the developed bridge contains multiple terms, it will be evident in the factor set how such terms can be combined and consolidated. For example, if the B branch in the simple bridge is replaced by \( P + Q \), the following factors among others would appear:

\[
\begin{align*}
P + \frac{D + Q}{C + E + Q} \\
Q + \frac{D + P}{C + E + P} \\
E + \frac{A + D}{C + P + Q}
\end{align*}
\]

These, respectively, are equal to:

\[
\begin{align*}
(P + Q) + \frac{D}{C + E} \\
(P + Q) + \frac{D}{C + E} \\
E + \frac{A + D}{C + (P + Q)}
\end{align*}
\]

Since the first two factors are identical, one drops out. The form of this combined factor indicates that the \( P \) and \( Q \) belong together, as suggested by the parentheses. By tying together the \( P + Q \) in all remaining factors, as indicated in the \( E + \) case, the reduction to the original form of the simple bridge is clear, and the circuit can be drawn. If, in an actual case, the \( P \) and \( Q \) could not be combined in all factors, the circuit could not be reduced to a bridge.
This method can be extended to apply to other types of bridge by developing the characteristic sets of factors with which correspondence can be established.

It can be seen intuitively that the negative of a bridge-type network should also be a bridge, probably with no more contacts than the original network. Since the bridge itself cannot be expressed algebraically, a direct transformation to the negative by theorem (7) is not possible. The indirect method of deriving the series-parallel equivalent to the bridge, taking its negative, and reconverting to another bridge, is generally unsatisfactory. Not only is the method time-consuming, but there may be considerable difficulty in obtaining the final bridge form.

However, a direct geometric technique, equivalent to duality transformations in transmission network theory, is available. This is applicable to any network that is "planar"; that is, one which may be drawn in a single plane without crossovers. To describe this process, the terms "node" and "mesh" will be used. A node is a junction point where three or more elements meet, whereas a mesh is a series of elements forming a complete closed loop. For example, in Fig. 5-23 the common junction point of A, C, and E, is a node, as is also the junction of the input lead and elements A and B. Elements A, E, and B form a closed loop or mesh. The input and output leads are considered to extend indefinitely so that the space above the diagram is bounded by a mesh and that below by a second mesh. Thus, the network contains four nodes and four meshes.

To illustrate this method of deriving the negative of a bridge-type network, consider the network of Fig. 5-25A, represented by Xab, which contains five meshes. A dot is placed in the center of each mesh, as in Fig. 5-25B, and the dots are connected by segments (shown by dotted lines in Fig. 5-25B) in such a manner that each segment crosses one and only one element of the original network. These dots form the nodes of a new network Xmn, and the connecting segments are the elements of this network. Each segment is made the negative of the element of the original network it crosses, as in Fig. 5-25C; and the new network Xmn is the negative of the original network Xab.

To justify this method, assume that Xab = 0, a closed circuit. Then, some path through Xab must be closed, and every element of this path is closed. However, this path completely cuts the network Xmn through elements of Xmn each of which equals 1 (open circuit). Therefore, Xmn must be an open circuit. Similarly, it may be shown that if Xab = 1, Xmn = 0. Thus, (Xab)' = Xmn; the network Xmn is the negative of Xab.

The above procedure, of course, may also be applied to series-parallel networks. As is evident from the method, the negative of a
planar network can always be realized with the same total number of makes and breaks as the original network. This is not generally true of non-planar networks, to which no convenient method may be applied to derive the negative.

Fig. 5-25 Graphical Method of Deriving Negative Network $(Xab)' = Xmn$
5.9 CONSIDERATION OF BASIC RELAY OPERATING PATH

In the design of relay circuits, the designer is not always at liberty to use make and break contacts as he wishes. For example, it may be required to operate a relay (X) when another relay (A) operates, yet only a break contact on (A) is available.

This requires the use of a shunt path to operate (X). The basic control paths for a relay were discussed in Chapter 3, and the relationship between direct and shunt control that permits this kind of rearrangement was pointed out. Further discussion of the subject is warranted at this time.

Fig. 5-27 Converting a Direct Control Path to Equivalent Forms: \( f(X) = A + B \)
The Design of Switching Circuits

\[ f(X) = A(B + X) = AB + AX \]

Fig. 5-28 Manipulations of Relay Control Paths: \( f(X) = A(B + X) \)

\[ f(X) = A + B(C + X) \]

Fig. 5-29 Manipulations of Relay Control Paths: \( f(X) = A + B(C + X) \)
If \( f(X) \) represents the operating condition for relay \((X)\), the normal control arrangement is to place \( f(X) \) in series with the winding of \((X)\). However, the same results, neglecting the time characteristics of the relay, are attained by connecting the negative of \( f(X) \), or \([f(X)]'\), in shunt with the relay. This is illustrated in Fig. 5-26.

If the operating path is made up of a sum of factors, for example:

\[
f(X) = A + B
\]

any one or more of the sum factors can be split off from \( f(X) \), and the negative of the split-off factors may be placed in shunt with the relay without affecting the control conditions. The following four arrangements, where subscripts \( d \) and \( s \) represent direct and shunt paths respectively, are equivalent for control of relay \((X)\):

\[
\begin{align*}
f_d(X) &= A + B \\
f_d(X) &= A, \quad f_s(X) = B' \\
f_s(X) &= A', \quad f_d(X) = B \\
f_s(X) &= A'B'
\end{align*}
\]

The four circuits are shown on Fig. 5-27.

Since \( A \) and \( B \) may represent networks instead of individual contacts, the relationship is perfectly general and permits any degree of manipulation, provided only that it is a sum factor that is split off for a shunt path. If \( f(X) \) is composed only of product factors that cannot be converted to a sum of products, the rearrangement is not possible unless the status of the relay is reversed, that is, the operated condition is considered normal. In the latter case, the negative of \( f(X) \), which is a set of series terms, becomes the direct operating condition.

Two additional examples of this type of manipulation are shown on Figs. 5-28 and 5-29.

5.10 RECAPITULATION OF THE POSTULATES AND THEOREMS OF SWITCHING ALGEBRA

The principles and relations used in switching algebra, as discussed in this chapter, are scattered over some thirty-odd pages of text. For the convenience of the reader in review and reference, the postulates and theorems have been tabulated in a concise form on the following page.
DEFINITIONS

Addition

(+) = AND = Series

Multiplication

(-) = OR = Parallel

Circuit States

0 = Closed Circuit
1 = Open Circuit

POSTULATES

(1) \( X = 0 \) or \( X = 1 \),
where \( X \) is a contact or a network

(2a) \( 0 \cdot 0 = 0 \)
(2b) \( 1 + 1 = 1 \)
(3a) \( 1 \cdot 1 = 1 \)
(3b) \( 0 + 0 = 0 \)
(4a) \( 1 \cdot 0 = 0 \cdot 1 = 0 \)
(4b) \( 0 + 1 = 1 + 0 = 1 \)

THEOREMS

Addition and Multiplication

Negatives, and 0 and 1

(1a) \( X + Y = Y + X \)
(1b) \( XY = YX \)
(2a) \( (X + Y) + Z = X + (Y + Z) \)
(2b) \( (XY) Z = X(YZ) \)
(3a) \( XY + XZ = X(Y + Z) \)
(3b) \( (X + Y)(X + Z) = X + YZ \)
(4a) \( X + X = X \)
(4b) \( XX = X \)
(5a) \( X + XY = X \)
(5b) \( X(X + Y) = X \)

(6a) \( (X)' = X' \)
(6b) \( (X')' = X \)
(7a) \( (X + Y + Z \ldots)' = X' \cdot Y' \cdot Z' \ldots \)
(7b) \( (X \cdot Y \cdot Z \ldots)' = X' + Y' + Z' \ldots \)
(8a) \( X' + X = 1 \)
(8b) \( X'X = 0 \)
(9a) \( 0 + X = X \)
(9b) \( 1 \cdot X = X \)
(10a) \( 1 + X = 1 \)
(10b) \( 0 \cdot X = 0 \)
(11a) \( (X + Y)' Y = XY \)
(11b) \( XY' + Y = X + Y \)

Supplementary Theorems

(12a) \( (X + Y)(X' + Z)(Y + Z) = (X + Y)(X' + Z) \)
(12b) \( XZ + X'Y + YZ = XZ + X'Y \)
(13) \( (X + Y)(X' + Z) = XZ + X'Y \)
(14a) \( f(X) = A \cdot f(X)_A = 1, A' = 0 + A' \cdot f(X)_A = 0, A' = 1 \)
(14b) \( f(X) = \left[A + f(X)_A = 1, A' = 0 \right] \left[A' + f(X)_A = 0, A' = 1 \right] \)
REFERENCES FOR CHAPTER 5

L. Couturat, "The Algebra of Logic" (Open Court, 1914).


PROBLEMS FOR CHAPTER 5*

5-1 (a) Sketch the contact networks represented by the following switching algebra expressions:

1. A + B + CD
2. AB (C + D)
3. (A + B)(C + D) E + F (GH + K)
4. A (BC + D) [(E + F) G + HK]

(b) Write the switching algebra expressions which represent the networks shown at the top of page 104.

5-2 (a) Express the following as a sum of products:
A (B + C) [D + E (F + G)] + HK

(b) Express the above as a product of sums.

(c) Simplify the following expressions:
1. (A + B + D)(E + C)(B + C + D)(A + E) (5 terms)
2. (B + D + E)(A + B + C + D)(A + D + E) (7 terms)
3. BDE + FA (C + F + GE) + BE (4 terms)
4. AB + BDE + CA + DCE (5 terms)
5. ABE + BCFG + B (D + G) + BCDF (5 terms)
6. AB + AC + BC (5 terms)

* In doing the problems for this chapter, show all steps in arriving at a solution.
5-3 (a) Sketch the circuits represented by the following expressions, showing transfer spring combinations where possible.

1. $A'B + A(B' + C)$
   (8 springs)

2. $(A + B + C')(A' + D + E)(F + G + C)$
   (16 springs)

3. $(A'B + B'CD + C'E)(A + E')$
   (14 springs)

4. $F[A(B + C) + D'E] (A' + D + E')$
   (15 springs)

(b) Write the negative of each of the above algebraic expressions.

c) Sketch the circuits represented by the negative expressions which you obtained in part (b), showing transfer spring combinations where possible.

5-4 Simplify the following contact networks by algebra.

(a) $(A + B + C)(A' + B' + C')(A + B' + C')(A' + B + C)(A + B + C')(A' + B + C')$ (2 terms)

(b) $ABC' + A'B'C + A'BC' + ABC + A'B'C + AB'C$ (2 terms)

(c) $(A + B'C)(A' + B' + C)$ (5 terms)

(d) $AB (C'D + A' + B) + B' (C + D')(A + B)$ (4 terms)

(e) $(A + D')(B'E) + A'C'DF + B'C'EF$ (8 terms)
5-5 Rearrange the following network to use break-make transfers instead of the make-break transfers (continuities) shown, without introducing the hazard of relay (X) releasing during the acting time of either transfer combination.

5-6 Five relays (A), (B), (C), (D), and (E) may be operated in any possible combination. Design a circuit with a minimum number of contact springs which lights a lamp (L) for all combinations except the following: (This can be done with 26 springs.)

\[(A + B + C' + D' + E)\]
\[(A' + B + C' + D + E)\]
\[(A + B' + C' + D + E)\]
\[(A' + B + C + D' + E')\]
\[(A + B + C + D' + E')\]
\[(A' + B' + C + D + E)\]
\[(A + B' + C + D' + E)\]
\[(A + B + C' + D + E')\]
\[(A' + B + C + D + E')\]

5-7 Simplify and draw the circuit for each of the following expressions:

(a) \[B + (D' + F)(E' + A + G)](B + F(D + A + E'))[A + G + C(D + E + F')]\[BD'F + DF'\]

(b) \[(A' + D' + B)(B + AC)(D + C + A')\]

(c) \[B'CE'(A' + D) + BD + AD'F\][F(CE' + A'B' + B'D) + B'(C' + E)(A' + D)]

(d) \[(W + XY')(X' + Y') + (Z' + XY')(X + Y) + (Y + W'Z) + (W' + Z') + (X' + W'Z)(W + Z)\]

5-8 Combine each of the following four sets of networks into single multi-terminal networks for operating the specified relays:

(a) \(f(X) = A + B + C\)
\(f(Y) = A + B + D\)
\(f(Z) = B + D + E\) (12 springs)

(b) \(f(X) = A' + B\)
\(f(Z) = (A + B + C)(C' + D)(C + E)\) (12 springs)

(c) \(f(W) = A + B'\)
\(f(X) = ABD\)
\(f(Y) = AB + C\) (13 springs)
\(f(Z) = (A + B')(A' + B)\)
5-9 Design and combine contact networks to light two lamps under the following conditions. If possible, use no more than one transfer per relay.

<table>
<thead>
<tr>
<th>Green Lamp</th>
<th>Red Lamp</th>
</tr>
</thead>
<tbody>
<tr>
<td>A + B' + C'</td>
<td>A + B' + C'</td>
</tr>
<tr>
<td>A' + B + C'</td>
<td>A' + B + C'</td>
</tr>
<tr>
<td>A + B + C</td>
<td></td>
</tr>
<tr>
<td>A + C</td>
<td></td>
</tr>
<tr>
<td>B + C</td>
<td></td>
</tr>
</tbody>
</table>

5-10 Simplify and draw the circuits corresponding to each of the following tables of combinations. Only the combinations shown in each individual table can be used in the simplification of the corresponding circuit.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(c)</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td>1</td>
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<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(b)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>(d)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>1</td>
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<td>0</td>
<td>1</td>
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<td>0</td>
<td>1</td>
<td>0</td>
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<td></td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

5-11 A circuit designer gathers from several sources the conditions under which red and green indicating lamps are to be lighted. The conditions are dependent upon the state of three leads, each of which can be connected to the winding of a relay. Design the simplest possible network on the three relays to control the two lamps. (This can be done with 8 springs.)

The red lamp lights when:

- Input lead b is grounded, lead c not grounded
- Input lead a is grounded, lead b is grounded
- Input lead a not grounded, lead c is grounded
- Input lead c is grounded, lead b not grounded
- Input lead b is grounded, lead a not grounded
- No leads grounded
- All leads grounded

The green lamp lights when:

- Input lead c is grounded, lead b not grounded
- Input lead b not grounded, lead a is grounded
- Only lead c is grounded
5-12 Develop a formal proof, based on the postulates and on theorems (1) through (11), of theorem (12a) and theorem (12b). Work from the left side of the theorem to the right, and do not use the method of perfect induction.

5-13 Design the most economical contact network for four relays to be closed when one, three, or four of the relays are operated. (This can be done with 22 springs.)

5-14 Design a contact network to be applied to a large number of relays (nine or more) which will be closed when the number of operated relays is not divisible by three. Consider that zero is not divisible by three. (This can be done with 26 transfers on 9 relays.)

5-15 Design a contact network to be applied to a large number of relays (nine or more) which will be closed when the number of operated relays is not divisible by three. Consider that zero is divisible by three. (This can be done with 22 transfers on 9 relays.)

5-16 A bridge network may be resolved into its equivalent series-parallel form by any of three methods: (1), tracing all possible paths from input to output terminal; (2), determining all means by which the network may be opened; or (3), employing star-mesh or Y-Delta transformations. Using one of these methods, resolve the bridge network shown below. Check the solution obtained by carrying out resolution by one of the remaining methods.

5-17 Prove the truth of the following equality by the use of theorem (12a):

\[(A + B')(A' + B)(A + C) = (A + B')(A' + B)(B + C)\]

5-18 Prove the truth of the following equality by the use of theorem (12a):

\[(A + B')(B + C')(C + A') = (A' + B)(B' + C)(C' + A)\]
Chapter 6

DESIGN OF COMBINATIONAL RELAY CIRCUITS

6.1 GENERAL DESIGN PROCEDURE

The design procedure for a relay circuit may be divided roughly into three parts:

(1) A determination of the requirements of the circuit in terms of what it must accomplish and what is available for the control of the circuit. This will include a determination of the number of input and output leads, the nature of signals or conditions established on these leads, and the time sequence of the various input and output signals.

(2) An analysis of the requirements, and the development of a scheme or plan of operation. This will include a determination of the number of relays and a plan of time sequence for the action and interaction of these relays.

(3) The detailed design of contact networks and circuit arrangements for receiving the input signals, controlling the relays, and establishing the desired output conditions in accordance with the plan.

During the design of a practical circuit there is often considerable overlapping of the work classified under these three headings. The work under (2) and (3) may indicate that different input conditions are necessary or desirable, while the detailed work under (3) often indicates modifications of the plan developed under (2) which will produce a more satisfactory circuit. Because of the fact that in the design of switching or control circuits there are always a number of alternative ways in which a particular result may be accomplished, experience, good judgment, and ingenuity are necessary factors in the design of such circuits.

In this and the following chapter, design procedures for the basic classes of relay circuits will be outlined in some detail. Emphasis will be placed on the work indicated in (2) and (3) above; the work of part (1) will be assumed completed, since it is often determined by factors over which the designer has little control, or by engineering judgments
based on knowledge of the general requirements of a particular system and capabilities of particular apparatus.

6.2 INPUT AND OUTPUT CONDITIONS

As defined in Chapter 3, the term "relay circuit" is an elastic term used to group together those relays that may be considered as a unit. By its nature, a relay circuit must be controlled and must exercise control over leads connected to other circuits and apparatus. Usually, but not always, the incoming control conditions are carried by leads separate from those carrying the output conditions.

The nature and form of input control conditions which transmit information to a circuit composed of relays or other two-valued devices are necessarily affected by the characteristics of the elemental apparatus. In general, the basic input (and output) signals are two-valued also. Depending upon the type of receptor apparatus, the signals may be presence or absence of ground on a lead, pulse or no pulse, positive or negative voltage or current, etc. Under certain circumstances, the number of signal values on a single input path may be made greater than two. For example, the marginal and sensitive relay arrangement shown in Chapter 3 on Fig. 3-12 can respond to three conditions over a single path. However, this can be considered a special case.

Although these narrow limits to the type of usable input signals may seem unduly restrictive, there are two mitigating aspects to the situation. The first is that a limited number of two-valued signals, taken in combinations or in sequence, can provide a much higher number of distinctive input conditions. The effect of sequence is to increase the number of overall input conditions far beyond the \(2^n\) derivable from \(n\) individual conditions taken in combination. This aspect permits imposing upon a circuit an almost unlimited quantity of input information via a finite number of input paths.

The second important aspect of input control is that if the original information is not in two-valued form, conversion apparatus can almost always be interposed between the sources of raw information and the control or switching circuits in order to change the information into an appropriate form. Many well-known examples of this type of conversion can be quoted. A thermostat converts gradual temperature changes into circuit opens or closures; float and lever systems close or open circuit paths with a change in liquid level; and so on.

With this understood, it will be assumed in this text that control or input signals are always available in proper form to activate the circuits to be designed; that is, the signals will be two-valued. The nature and design of any conversion apparatus that may be required in
actual practice will be considered a separate subject not within the scope of this text.

Output conditions are of the same nature as input conditions with the distinction that the input conditions initiate and control action in the circuit under consideration, while the output conditions either control other circuits or deliver information.

6.3 GENERAL TYPES OF RELAY CIRCUITS

Relay circuits can be divided into two general types, designated "combinational" and "sequential" circuits. In the combinational circuit, a combination of input conditions establishes a definite combination of output conditions independent of the order in which the input conditions are applied. In the sequential circuit, the time sequence in which input conditions are applied affects the output conditions established. A factor of memory is always contained in a sequential circuit.

This chapter will be devoted to the design fundamentals of the first or combinational type of relay circuit.

6.4 CAPABILITIES OF COMBINATIONAL CIRCUITS

In a typical combinational circuit, each of n input leads connects to the winding of a relay of the circuit. The relays are controlled by signals on the leads. Networks of contacts on the relays control the signals (ground or open) on the output leads. Since the input conditions are two-valued, there are $2^n$ possible combinations of input conditions (input combinations), and the n relays can operate in $2^n$ corresponding combinations. The contact networks can establish a combination of conditions on the output leads (output combinations) for each of the $2^n$ input combinations so that the maximum possible number of unique output combinations is also $2^n$. In a combinational circuit, a particular input combination will always produce the same output combination regardless of the sequence in which events occur on the input leads. However, the circuit may be arranged so that several different input combinations produce the same output combination. In many circuits, all the possible input combinations do not occur in normal operation. In any case the number of different output combinations is never greater than the number of input combinations.

The preceding discussion assumed that each input lead connects to a relay which serves to detect signals on this lead. In particular cases, it is frequently possible to eliminate this relay and let the lead connect directly into the contact network. The purpose of the relay is to provide contacts which, in effect, repeat the input signal (and its
negative) in as many different circuit paths as are necessary to achieve the required results. When an input signal condition (e.g., ground) is the same as one or more output conditions, it may be possible to arrange the contact network so that the input signal-detecting relay requires only a single make-contact closing a path to ground. Hence the relay may be omitted, and the signal on the input lead substituted for that produced by the relay contact.

If the circuit is considered from the output end, the number of required output combinations is often a determining factor in the number of relays in the circuit. For example, if twenty output combinations are required, at least five relays, and hence five inputs, will be needed. In another case, if a circuit is required to start from an inactive condition and provide eight active output combinations, three relays with their maximum of eight combinations would not suffice.

Consider now the individual output leads of a combinational circuit. If each lead is grounded for only one input combination, there can be a maximum of \(2^n\) different output leads, each lead corresponding to a particular input combination. However, an output lead may be grounded for any number of the \(2^n\) input combinations. For example, a circuit with two input leads may have \(2^2 = 4\) input combinations. An output lead may be grounded for any combination of these four input combinations. This gives a total of \(2^4 = 16\) networks to which output leads may be connected. Two of these 16 ways — the one which produces a grounded output for all input combinations (a permanently grounded output lead), and the one which produces a grounded output for none of the input combinations (an open output lead) — are trivial and can be omitted from consideration. Thus, in a circuit with \(n\) input signals, there are \((2^2^n - 2)\). Useful ways in which output leads may be connected through contacts of the input relays.

6.5 DESIGN PROCEDURE FOR COMBINATIONAL CIRCUITS

The technique of combinational circuit design entails consideration of each output lead separately, and determination of the input combination which will establish the desired output condition on this lead; that is, the circuit is synthesized from the output end. Usually it is convenient first to develop a separate contact network for each output lead and later to combine the networks into a single multi-terminal network. The circuit for each individual output lead may be designed by inspection or by formal methods.

To develop a circuit by inspection of requirements, the entire group of input and output combinations is examined for coincidences of input and output signals, symmetry of requirements, or other features
of the requirements as a whole which will indicate significant relations between inputs and outputs. The object of this inspection is to obtain directly a statement describing the relations among input signals which must cause a given output signal. These statements are determined for each output and the corresponding circuit paths are combined to form the required network.

In the formal method, circuits are obtained by following routine procedures. To obtain a circuit for a given output lead with this method, every possible input combination is examined to determine which combinations correspond to closed circuit conditions of a particular output lead. For each of the combinations, an individual output circuit path, closed only for that combination, is developed. This path will consist of a series circuit containing one contact on every input relay of the circuit, the contact being a make or a break respectively for relays operated or released in the combination. The individual paths are then combined and simplified to give the circuit for one output lead. This is repeated for each output lead and, finally, the circuits for the several outputs are combined where possible.

It is likely that all the possible input combinations do not occur in normal operation, particularly when there are more than four or five individual input signals. In these circuits there is often considerable latitude in manipulating the contact network, since the designer may elect to let those combinations which are never expected to occur correspond to either open or closed conditions of the network being designed. This often permits considerable simplification of the network, as will be illustrated in the next section. The normally unused combinations, however, represent trouble conditions if they occur during the circuit action. Where closure of output leads during such a trouble condition may cause serious external reactions, a strict requirement may be placed on the circuit that all output leads must be closed only for specifically defined input combinations and must be opened for all others. Another factor that may affect the final form of a combinational circuit is the effect of permitting output leads to be connected together, through contacts of the circuit, during combinations which provide for no output signals on the leads. A circuit which provides complete independence among output leads often requires more contacts than a circuit which does not provide this independence.

6.6 EXAMPLES OF COMBINATIONAL CIRCUIT DESIGN

Combinational circuits are designed to activate a particular output lead for a single input combination or a multiplicity of input combinations, or to interconnect two or more output leads for single or multiple input combinations. The basic principles of design will be
Combinational Relay Circuits

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illustrated by a consideration of the simplest case, a circuit where each output corresponds to a single input combination. Several circuits, based on the same primary requirements, will be designed to indicate the effect of modifying fringe requirements. Following this presentation, more elaborate design examples will be discussed.

A familiar example of a combinational circuit with a single output per input combination is the "transfer tree" which was discussed in Chapter 4 and is reproduced here as Fig. 6-1. Input signals appear on the operating leads for relays (A), (B), (C), and (D). For four relays there are $2^4$ or 16 possible relay combinations, and the 16 individual output combinations corresponding to these relay combinations appear on 16 output leads. No output corresponds to more than one relay combination, and every output path through the network includes a contact on each relay.

The transfer tree is the optimum circuit form when a separate output is required for each of every possible input combination. When
outputs correspond to single input conditions, but not all input conditions are comprehended in the requirements, the circuit form may or may not be a partial tree, depending upon subsidiary requirements. In the example below, assume that the table of requirements must be followed exactly and that output leads must be independent. The requirements are as follows:

<table>
<thead>
<tr>
<th>Input Leads Grounded</th>
<th>Output Leads Grounded</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>s</td>
</tr>
<tr>
<td>c</td>
<td>t</td>
</tr>
<tr>
<td>a,c</td>
<td>u</td>
</tr>
<tr>
<td>a,b,c</td>
<td>v</td>
</tr>
<tr>
<td>b,c</td>
<td>none</td>
</tr>
<tr>
<td>a,b</td>
<td>none</td>
</tr>
<tr>
<td>a</td>
<td>none</td>
</tr>
<tr>
<td>none</td>
<td>none</td>
</tr>
</tbody>
</table>

To illustrate the formal method of circuit design, the algebraic equation for each of the above output conditions is written from a consideration of the requirements, assuming a relay corresponding to each input lead:

\[
\begin{align*}
  f(s) &= A' + B + C' \\
  f(t) &= A' + B' + C \\
  f(u) &= A + B' + C \\
  f(v) &= A + B + C
\end{align*}
\]

These equations*, and their corresponding contact networks, may be combined by inspection to give the circuit of Fig. 6-2. The resultant circuit is a partially developed tree, each output being disjunctive with respect to any other.

If the requirements are such that some input combinations may be assumed never to occur in normal operation (or that the corresponding output conditions are inconsequential if they do occur), a simplification of the basic tree is possible. These combinations, which will be called "invalid" combinations, may be used optionally to eliminate contacts in

* Throughout this text, pertinent equations or groups of equations are identified by hyphenated numbers. The first part of the number indicates the chapter in which the equation is first given, and the second part shows the number within the chapter. The theorems and postulates which were given in Chapter 5, however, are numbered consecutively without reference to the chapter.
the circuit. The tabular notation and method of simplification introduced in Chapter 5 offer a direct approach to the simplification of the network of such circuit. For illustration, let the requirements satisfied by the circuit of Fig. 6-2 be modified by stating that the input combinations \(b,c\), \(a,b\), and \(a\) are invalid. (The condition of "no inputs grounded" will occur as the normal condition when the circuit is idle.) However, it will still be required that the outputs be independent, or disjunctive. The table is then drawn up as follows:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Output Lead</th>
<th>Grounded</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>s</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>t</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>u</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>v</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>Invalid</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Since the invalid combinations represented by lines 5, 6, and 7 of the above table never occur, circuit paths represented by these combinations will never be closed and can be used for contact network simplification in the following manner. Considering the network for output lead \(s\), this network can be closed to ground not only for the active condition represented by line 1, but also may include paths for the invalid conditions of lines 5, 6, and 7. Taking lines 1 and 5 or lines 1 and 6 together, then, it is evident that either \(C'\) or \(A'\) can be eliminated, with the result that \(f(s) = A' + B\), or \(B + C'\). Considering lines 2 and 5, \(B\) can be eliminated from the expression for \(f(t)\). Similarly, by combining lines 3 and 7, and lines 4 and 5 or lines 4 and 6, \(f(u)\) and \(f(v)\) can be simplified, respectively. The algebraic notations for grounding the output leads then become:
The Design of Switching Circuits

\[
f(s) = A' + B; \text{ or } B + C'
\]

\[
f(t) = A' + C
\]

\[
f(u) = A + B'
\]

\[
f(v) = A + B; \text{ or } B + C
\]

Comparison of equations (6-2) with equations (6-1) indicates the utility of the invalid combinations.

Combining paths and grouping contacts with their relay windings gives the circuit of Fig. 6-3A. However, in retrospect it would appear that the network for \( f(t) \) has been simplified to a point where it cannot be easily combined with the other paths. If \( f(t) \) is left unsimplified, \( f(t) = A' + B' + C; \) it can be combined conveniently with \( f(s) = A' + B \) and the further simplification of Fig. 6-3B results. The point at which advantageous simplification becomes over-simplification depends upon each individual case; only experience can teach the designer to recognize when it has been reached.

It should be noted that, in the circuit of Fig. 6-3B, the output paths have been kept disjunctive; that is, no two output paths can be connected together.

---

**Fig. 6-3** A Modified Tree \((\text{Equations 6-2})\)

**Fig. 6-4** Non-Disjunctive Circuits \((\text{Equations 6-2})\)
If, now, the last restriction on the circuit is removed, that is, if the output leads need not be disjunctive, another step of simplification becomes possible. This can be determined by examining equations (6-2) again. It will be noted that the conditions for grounding outputs s and t are

\[
\begin{align*}
    f(s) &= A' + B \\
    f(t) &= A' + C
\end{align*}
\]

which, in this case, permits combining the two \(A'\) factors into one contact since B and C are never validly activated without A. The corresponding circuit is shown on Fig. 6-4. With this circuit, not only is output lead v grounded when relays A, B, and C are operated, but paths s and t are connected together. However, this is permitted by the modified requirements.

One more circuit form to meet the basic requirements is possible. It was stated earlier in this chapter that, if output conditions are of the same nature as input conditions and if a circuit relay can be reduced to a single make contact which repeats the corresponding input condition, that circuit relay may be eliminated. By judicious choice of output expressions from equations (6-2) and by a suitable arrangement of contacts, the circuit configuration of Fig. 6-5A can be attained. This can be changed to the circuit of Fig. 6-5B which employs only two relays.

Four circuits have now been designed to meet the same basic set of requirements. It should be clear that other equivalent circuits are possible. Each of the four designed circuits grounds the required output lead for the appropriate set of input conditions, but the circuits differ in other respects. These differences are tabulated at the top of the next page.
The Design of Switching Circuits

<table>
<thead>
<tr>
<th>Circuit</th>
<th>Apparatus</th>
<th>Circuit Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 6-2</td>
<td>3 relays, 15 springs</td>
<td>No output for invalid input combination. Output paths disjunctive.</td>
</tr>
<tr>
<td>Fig. 6-3B</td>
<td>3 relays, 11 springs</td>
<td>Outputs grounded for invalid input combinations. (Example: ( a, b \rightarrow v )). Output paths disjunctive.</td>
</tr>
<tr>
<td>Fig. 6-4</td>
<td>3 relays, 10 springs</td>
<td>Outputs grounded for invalid input combinations. (Example: ( b, c \rightarrow s, t )). Output paths not disjunctive; ((a, b, c \text{ grounds } v, \text{ connects } s, t \text{ together})).</td>
</tr>
<tr>
<td>Fig. 6-5B</td>
<td>2 relays, 10 springs</td>
<td>Outputs grounded for invalid input combinations. Output paths not disjunctive. One input condition used as an output condition.</td>
</tr>
</tbody>
</table>

This development of a variety of circuits to perform the same basic requirements illustrates a common principle in circuit design: in general, as requirements become more rigid, the circuit becomes more complex and requires more apparatus. The specific requirements and the general situation surrounding a design problem must always be scrutinized carefully to determine in what manner the circuit should be designed and what short-cuts may be taken. Also, a practical consideration that may affect the degree to which combination and simplification can be carried is the electrical load capabilities of the relay contacts.

A frequently used type of combinational circuit is one in which several output leads are grounded in combinations. To obtain a circuit to fit these requirements, a relay is provided, as before, for each input condition. The paths by which each output lead is grounded may be written algebraically, utilizing a contact on every relay. All paths for each output are then placed in parallel and the contact network is combined and simplified. Finally, the composite circuit for all outputs is combined and simplified to whatever extent is possible.

For example, the following is required:

<table>
<thead>
<tr>
<th>Input Leads Grounded</th>
<th>Output Leads Grounded</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. a</td>
<td>w,z</td>
</tr>
<tr>
<td>2. b</td>
<td>w,x</td>
</tr>
<tr>
<td>3. c</td>
<td>w,y</td>
</tr>
<tr>
<td>4. a,b</td>
<td>x,z</td>
</tr>
<tr>
<td>5. a,c</td>
<td>y,z</td>
</tr>
<tr>
<td>6. b,c</td>
<td>x,y</td>
</tr>
<tr>
<td>7. none</td>
<td>none</td>
</tr>
<tr>
<td>8. a,b,c</td>
<td>none</td>
</tr>
</tbody>
</table>
It is a requirement that, although lines 7 and 8 in this table of requirements represent combinations which have no corresponding outputs, these input combinations do occur and hence cannot be used in simplifying the networks for the output conditions.

Let it also be a stated requirement that no output lead is to be connected to another for any output combination except that for which both are grounded. The contact network paths may then be written as below, although the tabular notation would do just as well.

\[
\begin{align*}
  f(w) &= (A + B' + C') (A' + B + C') (A' + B' + C) \\
  f(x) &= (A' + B + C') (A + B + C') (A' + B + C) \\
  f(y) &= (A' + B' + C) (A + B' + C) (A' + B + C) \\
  f(z) &= (A + B' + C') (A + B + C') (A + B' + C)
\end{align*}
\]

(6-3)

Simplifying the expression for each function:

\[
\begin{align*}
  f(w) &= (A + B' + C') [A' + (B + C') (B' + C)] \\
  f(x) &= B + A'C' \\
  f(y) &= C + A'B' \\
  f(z) &= A + B'C'
\end{align*}
\]

(6-4)

The network paths corresponding to the equations (6-4) are illustrated in Fig. 6-6.

Some simplification of the contact network can be obtained by rearranging the expression for \(f(w)\) so that it may be combined with the
The Design of Switching Circuits

paths for other output leads. For example, \( f(w) \) may be manipulated algebraically to contain elements \( B'C' \) and \( A'C' \) for combination with \( f(z) \) and \( f(x) \). Carrying out this manipulation:

\[
f(w) = \left[ (B + C') (B' + C) + A' \right] \left[ C' + B' + A \right]
\]
\[
= (B'C' + BC + A') (A'C' + B' + A)
\]

(6-5)

The resultant network path diagram is shown in Fig. 6-7. Manipulations of this type are recognized only after careful study of all circuit paths, and the combined network must be carefully checked for sneak paths and for disjunctivity when required. Note, in Fig. 6-7, that the presence of \( A + A' \) in the path between \( Z \) and \( W \), and \( B' + B \) in the path between \( W \) and \( X \), insures that the outputs are disjunctive. This final circuit is, of course, not the only one which might be derived, since the algebraic equations might be manipulated in some other manner.

Recalling the theorem of switching algebra stating that \( X + Y = X + X'Y \), there is a temptation to obtain an even more simplified network by expanding the equations (6-4) as follows:

\[
f(x) = A'C' + B = A'B'C' + B
\]
\[
f(y) = A'B' + C = A'B'C' + C
\]
\[
f(z) = B'C' + A = A'B'C' + A
\]

(6-6)

Then, since \( f(x) \), \( f(y) \), and \( f(z) \) all contain \( A'B'C' \), the three are combined and the network paths appear as in Fig. 6-8. Note that the expression for \( f(w) \) represents a symmetric one-out-of-three circuit.

However, inspection of Fig. 6-8 shows that the output paths are not disjunctive as required, since when relays \( (A), (B), \) and \( (C) \) are operated, output leads \( x, y, \) and \( z \) are connected together but not to ground. If the original requirements can be modified to allow this circuit condition to exist, the circuit of Fig. 6-8 represents a saving over
that of Fig. 6-7. Unfortunately, there is no easily applied rule for determining whether or not the network paths are remaining disjunctive during the simplification of a combinational circuit. The final circuit must be inspected for paths not permitted by the original requirements.

As was the case with the first example taken up in this section, a further simplification will usually result if invalid input combinations never occur or, if they do, any output condition is permissible. If the requirements for this problem are amended to allow the input (a,b,c) to establish any output condition when it occurs, the resultant algebraic expression can be written as follows, with the changes underscored:

\[
\begin{align*}
\text{f}(w) &= (A + B' + C') (A' + B + C') (A' + B' + C) \\
\text{f}(x) &= (A' + B + C') (A + B + C') (A' + B + C) (A + B + C) \\
&= B \\
\text{f}(y) &= (A' + B' + C) (A' + B + C) (A + B' + C) (A + B + C) \\
&= C \\
\text{f}(z) &= (A + B' + C') (A + B + C') (A + B' + C) (A + B + C) \\
&= A
\end{align*}
\]

The arrangement of paths is shown in Fig. 6-9. The "no input" condition is reserved for a circuit "normal" condition when outputs must not be grounded.
The design of circuits by inspection of requirements can be illustrated by this problem. The method, as previously stated, is to examine the requirements and determine directly the relations between inputs which must cause each output to be closed. Referring to the table of requirements for the present problem, and assuming a relay operating from each input, it is observed that output lead w must be grounded when exactly one of the relays (A), (B), and (C) is operated. An obvious circuit for \( f(w) \), therefore, is a one-out-of-three symmetric circuit.

The statement for grounding output lead z is, "(A) is operated and, (B) or (C) is released", which may be written:

\[
f(z) = A + B'C'
\]

The part of the statement: "(B) or (C) is released", represented by \( B'C' \), is necessary to insure that z is not grounded when all relays are operated. Similar statements can be made for \( f(x) \) and \( f(y) \), and the resulting circuit is as shown in Fig. 6-6, with the circuit for \( f(w) \) changed to the symmetric form.

A simplified form of the network may be obtained by observing that, although output \( z \) is usually grounded when (A) operates, \( x \) when (B) operates, and \( y \) when (C) operates, none of the three outputs is grounded when all three relays are operated. The network \( A'B'C' \), which is open only when all three of the input relays are operated, can then be made common to the paths for the \( x \), \( y \), and \( z \) outputs to obtain the circuit of Fig. 6-8, where \( f(w) \) is in the symmetric form previously obtained. Inspection of this circuit shows that the outputs are not disjunctive, which may or may not be satisfactory.

If it is assumed that the condition of all inputs grounded simultaneously never occurs (that is, input combination \( a, b, c \) is invalid) then the statement, "\( z \) is grounded when (A) operates" is sufficient for \( f(z) \), and a single contact on (A) will suffice. Similar statements for \( f(w) \) and \( f(x) \) result in the circuit shown in Fig. 6-9.

The choice between the inspection method and the formal method in solving a problem can often be determined by a preliminary analysis of requirements. Inspection often gives a simplified solution directly with less circuit manipulation, but final circuits must be carefully analyzed to see that they conform to requirements, maintain the necessary disjunctivity, and do not contain sneak paths.

The type of combinational circuit requiring the most contacts is one whose output consists of connecting separate leads together, rather than of grounding these leads. Simplification is sometimes possible, but the necessity of keeping paths separate requires many contacts. As an example, take the following requirements:
Combinational Relay Circuits

<table>
<thead>
<tr>
<th>Input Leads Grounded</th>
<th>Output Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>none</td>
<td>connect lead s to t</td>
</tr>
<tr>
<td>a</td>
<td>connect lead u to v</td>
</tr>
<tr>
<td>b</td>
<td>connect lead w to x</td>
</tr>
<tr>
<td>a,b</td>
<td>connect lead y to z</td>
</tr>
</tbody>
</table>

\[
f(s-t) = A' + B' \\
f(u-v) = A + B' \\
f(w-x) = A' + B \\
f(y-z) = A + B
\]

(6-8)

No paths can be combined, and the circuit of Fig. 6-10 is obtained.

Another example is the following:

<table>
<thead>
<tr>
<th>Input Leads Grounded</th>
<th>Output Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Connect lead s to t</td>
</tr>
<tr>
<td>b</td>
<td>Connect lead t to u</td>
</tr>
<tr>
<td>a,b</td>
<td>Connect lead s to u</td>
</tr>
<tr>
<td>none</td>
<td>no connections</td>
</tr>
</tbody>
</table>

The equations for this circuit are:

\[
f(s-t) = A + B' \\
f(t-u) = A' + B \\
f(s-u) = A + B
\]

(6-9)
Here certain inter-relationships among the inputs and outputs indicate that simplification may be possible. For instance, the lead s is associated with A, and the lead u with B. In the noncombined form, the circuit is as shown in Fig. 6-11A; by inspection it can be combined into the form of Fig. 6-11B. Every circuit of this type requires careful analysis to determine what simplifications, if any, may be permitted by the requirements and the configuration of the individual networks.

![Circuit Diagram](image)

Fig. 6-11 Combined Circuit for Connecting Output Paths (Equations 6-9)

When designing circuits using more than four relays, the formal method applied to the problem as a whole often becomes cumbersome and the method of inspection is more satisfactory. For example, take the requirements in the table below:

<table>
<thead>
<tr>
<th>Input Leads Grounded</th>
<th>Output Leads Grounded</th>
</tr>
</thead>
<tbody>
<tr>
<td>v,w</td>
<td>a</td>
</tr>
<tr>
<td>v,x</td>
<td>b</td>
</tr>
<tr>
<td>w,x</td>
<td>a,b</td>
</tr>
<tr>
<td>v,y</td>
<td>c</td>
</tr>
<tr>
<td>w,y</td>
<td>a,c</td>
</tr>
<tr>
<td>x,y</td>
<td>b,c</td>
</tr>
<tr>
<td>v,z</td>
<td>a,b,c</td>
</tr>
<tr>
<td>w,z</td>
<td>d</td>
</tr>
<tr>
<td>x,z</td>
<td>a,d</td>
</tr>
<tr>
<td>y,z</td>
<td>b,d</td>
</tr>
<tr>
<td>none</td>
<td>none</td>
</tr>
</tbody>
</table>

An assumption is that no other input conditions will ever occur.

The correspondence between inputs and outputs can be used to facilitate the design of the contact network as follows:

1. Output lead a is grounded whenever input lead w or z is grounded, except for the input combinations (w,z) and (y,z). This can be stated in terms of relays controlled directly by the input leads
as: output lead a is grounded when relays (W) or (Z) are operated and not when (W) and (Z) are operated and not when (Y) and (Z) are operated. Or algebraically:

\[
f(a) = WZ + (W + Z)' + (Z + Y)'
\]

\[
= WZ + W'Z' + Z'Y'
\]  

(6-10)

(2) Lead b is grounded when (X) or (Z) is operated and not when (X) and (Z) are operated and not when (Z) and (W) are operated; that is:

\[
f(b) = XZ + (X + Z)' + (Z + W)'
\]

\[
= XZ + X'Z' + Z'W'
\]  

(6-11)

(3) Lead c is grounded when (Y) is operated and not when (Y) and (Z) are operated; or lead c is grounded when (V) and (Z) are operated; that is:

\[
f(c) = [Y + (Y + Z)'](V + Z)
\]

\[
= (V + Z)(Y + Z')
\]  

(6-12)

(4) Lead d is grounded when (Z) is operated and not when (V) and (Z) are operated; that is:

\[
f(d) = Z + (V + Z)' = Z + V'
\]  

(6-13)

The combined circuit is shown in Fig. 6-12.

---

**Fig. 6-12** Five-Relay Combinational Circuit  
(Equations 6-10, 6-11, 6-12, and 6-13)
This network could have been obtained by making a table of the thirty-two possible input combinations including the twenty-one invalid combinations and the eleven valid combinations giving particular output conditions, and combining to eliminate contacts. The work involved by this tabular method would be much longer. Circuit design by inspection of requirements is usually the better method when less than half of the possible input combinations occur in normal operation and the circuit action for invalid combinations is not specified.

If the requirements of the above example are made more rigid and state that no output leads are to be grounded should the invalid input combinations occur in this particular case, advantage can be taken of the symmetry of the working input conditions. The inputs are in pairs, and all possible pairs are used. By supplying ground to the contact network developed above through a symmetric circuit closed when two and only two of the five relays are operated, the more rigid requirements are met. This is shown in block form in Fig. 6-13.

![Block Diagram of Circuit for Removing Ground from Unused Output Combinations of Fig. 6-12](image)

A specialized type of combinational circuit is that in which the requirements are stated in terms of the relative position or serial number of the input leads, rather than in terms of correspondence between specific input conditions and outputs. It can easily be seen that the attempt to cope with problems of this nature by the formal method discussed in this chapter will meet with considerable difficulty. However, there are two profitable methods of attack. Since this type of circuit, in general, utilizes a reiterative network, the technique for designing such circuits described in Chapter 4 is applicable. Another attack, in many cases more direct, is essentially the method of inspection as described in this chapter. The requirements are analyzed carefully to discern some one of a variety of equivalent statements of conditions which is most susceptible to expression in circuit form. If such a statement can be found, the circuit is written in algebraic or contact form and manipulated to give the simplest network. Consider,
for example, the following positional circuit requirements: Six control leads, $L_a$, $L_b$, $L_c$, $L_d$, $L_e$, and $L_f$ are grounded at random, singly and in combination. A single output lead, $t$, is to be grounded only when $L_a$ or a number of adjacent control leads starting with lead $L_a$ are grounded.

In the design of a suitable circuit, a relay per control lead is first assumed and the relays are designated (A), (B), (C), (D), (E), and (F) to correspond to the control leads. Given these relays, it should be possible to write, in terms of the relays, the algebraic expression for grounding output $t$. However, the statement "lead $t$ is grounded only when one or a number of adjacent relays are operated starting with relay (A)" does not appear to lend itself directly to translation into algebraic form.

An attempt, then, is made to restate the conditions for network closure in some more suitable form. One such restatement is as follows: "the network should be open when all relays are released or when any relay is released and the succeeding relay is operated; under all other conditions the network should be closed."

The algebraic expression corresponding to these requirements for opening a circuit is the negative of that corresponding to requirements for network closure. Writing the negative expression, then:

$$[f(t)]' = (A' + B' + C' + D' + F')(A' + B)(B' + C)$$
$$\cdot (C' + D)(D' + E)(E' + F).$$

(6-14)

Taking the negative of both sides of this expression, the equation for grounding lead $t$ is obtained.

$$f(t) = [(A' + B' + C' + D' + E' + F')(A' + B)$$
$$\cdot (B' + C)(C' + D)(D' + E)(E' + F)]'$$

$$= ABCDEF + AB' + BC' + CD' + DE' + EF'$$

(6-15)

This equation may be reduced to

$$f(t) = A + AB' + BC' + CD' + DE' + EF'$$

(6-16)

The network represented by this last expression may be drawn by inspection as shown in symbolic form in Fig. 6-14A, on the next page. Placing this network on the relays controlled by the leads "L", a circuit is obtained which fulfills the original requirements. This final circuit is shown in Fig. 6-14B.

6.7 TEMPORARY FALSE OUTPUT CONDITIONS

In obtaining solutions to circuit problems so far, it has been assumed that a relay closed instantly when energized. Actually, of course,
there is always some time delay between the instant of closing the relay's operating path and the instant of the relay closing its make contacts or opening its break contacts. Furthermore, there may be an appreciable stagger time among the operation of several contacts on the same relay. In addition, one relay, due to manufacturing tolerances and variations, may have appreciably different operate and release times from another similar relay. It is probable, then, that in a combinational relay circuit the wrong combination of relays will be operated for a brief time after the proper combination has been energized. Usually this time is so brief that it can be tolerated, but the designer should always be aware that it may exist and make sure that the effects of the "race condition", as this is called, are properly provided for. As an illustration of how a wrong output combination occurs momentarily in a race condition, take the input combination where output lead \( v \) is to be grounded in Fig. 6-4. Suppose that output lead \( u \) is the control lead for a fast-acting relay in another circuit. Input leads a and b are grounded at the same time; relay (A), being lightly loaded, operates in a shorter time than relay (B) and grounds lead u. A short time later relay (B) operates, opening u and grounding lead v; but since u controls a fast-acting relay, there is a distinct possibility that this relay has already operated falsely. Methods of guarding against such a race condition when the output controls a fast-acting relay, are beyond the scope of this chapter, but will be discussed more fully in Chapter 8.

**PROBLEMS FOR CHAPTER 6**

6-1 Draw the fourteen different networks using contacts on one or two relays, as discussed in Section 6.4. Not included are the always open and always closed paths.
6-2 Eight relays are divided into two equal groups, A and B, of four relays each. The relays of group A are designated (A1), (A2), (A3), (A4), while those of group B are designated (B1), (B2), (B3), (B4). Design a contact network to light a lamp when the designation number of an operated relay in group A is greater than the designation number of an operated relay in group B. The lamp should light only when one and only one relay in each group is operated. (This network can be designed with less than 35 contact springs.)

6-3 Eight relays are divided into two equal groups, A and B, of four relays each. Design a contact network to light one of three lamps under each of the following three conditions: (This can be done with 24 transfers).

Lamp (L1): when more A relays are operated than B relays.
Lamp (L2): when an equal number of A and B relays are operated.
Lamp (L3): when more B relays are operated than A relays.

6-4 A circuit for four relays (A), (B), (C), and (D) operates in such a manner that the following chart is satisfied:

<table>
<thead>
<tr>
<th>Relays Operated</th>
<th>Output Leads Grounded</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A),(B); all others released</td>
<td>x</td>
</tr>
<tr>
<td>(A),(C); all others released</td>
<td>z</td>
</tr>
<tr>
<td>(A),(D); all others released</td>
<td>y,z</td>
</tr>
<tr>
<td>(B),(C); all others released</td>
<td>x,z</td>
</tr>
<tr>
<td>(B),(D); all others released</td>
<td>y</td>
</tr>
<tr>
<td>(C),(D); all others released</td>
<td>x,y</td>
</tr>
<tr>
<td>None</td>
<td>None</td>
</tr>
</tbody>
</table>

(a) Design the required circuit assuming that only the combinations above can occur.
(b) Design a second circuit assuming that all possible combinations can occur, but outputs should appear only on the above combinations.

Leads can be connected together while ungrounded.

6-5 A circuit for four relays (A), (B), (C), and (D) operates in such a manner that the following chart is satisfied:

<table>
<thead>
<tr>
<th>Relays Operated</th>
<th>Output Leads Grounded</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A),(B); all others released</td>
<td>x,y</td>
</tr>
<tr>
<td>(A),(C); all others released</td>
<td>x,z</td>
</tr>
<tr>
<td>(A),(D); all others released</td>
<td>y</td>
</tr>
<tr>
<td>(B),(C); all others released</td>
<td>z</td>
</tr>
<tr>
<td>(B),(D); all others released</td>
<td>x</td>
</tr>
<tr>
<td>(B),(C),(D); all others released</td>
<td>y,z</td>
</tr>
</tbody>
</table>

(a) Design the required circuit assuming that only the combinations above can occur.

(13 springs)
(b) Design a second circuit assuming that all possible combinations can occur, but outputs should appear only on the above combinations. (29 springs)

Leads can be connected together while ungrounded.

6-6 Three relays (A), (B), and (C), which may operate in any combination, control a red lamp, R, and a green lamp, G, as follows: when all three relays are operated, the red lamp only is lighted; and when relays (A) and (B) are released and (C) is operated, only the green lamp is lighted. For all other combinations of the relays both lamps are lighted except that no lights are to be operated when all relays are released. Derive the circuit using as few contacts as possible. (9 springs)

6-7 The table below indicates the requirements for a circuit which connects together certain leads depending upon the operated combinations of relays (A), (B), (C), and (D). Design the circuit so that when any relay combination not included in the table occurs, no leads are connected together. (15 springs)

<table>
<thead>
<tr>
<th>Relays Operated</th>
<th>Leads Connected Together</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B)</td>
<td>(s to t), (v to x)</td>
</tr>
<tr>
<td>(C)</td>
<td>(w to x)</td>
</tr>
<tr>
<td>(D)</td>
<td>(u to t)</td>
</tr>
<tr>
<td>(A),(B)</td>
<td>(s to u)</td>
</tr>
<tr>
<td>(A),(D)</td>
<td>(u to t)</td>
</tr>
<tr>
<td>(B),(C)</td>
<td>(v to w), (x to v), (x to w)</td>
</tr>
<tr>
<td>(B),(D)</td>
<td>(s to t), (u to s), (t to u), (v to x)</td>
</tr>
<tr>
<td>(C),(D)</td>
<td>(w to x)</td>
</tr>
<tr>
<td>(A),(B),(C)</td>
<td>(s to u), (v to w)</td>
</tr>
<tr>
<td>(A),(B),(D)</td>
<td>(s to u), (t to u), (s to t)</td>
</tr>
<tr>
<td>(B),(C),(D)</td>
<td>(s to u), (v to x), (v to w), (w to x)</td>
</tr>
<tr>
<td>(A),(B),(C),(D)</td>
<td>(s to u), (v to w)</td>
</tr>
</tbody>
</table>

6-8 In a typical relay storage unit for binary numbers, the binary digit "1" may be indicated by an operated relay and the binary digit "0", by the relay unoperated. Relays (A), (B), (C), and (D) are elements in such a storage unit, where (D) represents 2⁰, (C) represents 2¹, etc. Since there are four relays, the unit may be used to hold the binary equivalent to any decimal number from 0 to 15. For example, the decimal number 13 would be stored by operating relays (A), (B), and (D), since the binary equivalent is 1101.

Design contact networks controlled by relays (A), (B), (C), and (D) to be closed for each of the following conditions:

(a) The stored binary number is equivalent to an odd decimal number.
(b) The stored binary number is equivalent to a decimal number divisible by 4, not including zero.
(c) The stored binary number is equivalent to a decimal number divisible by 3, not including zero.
(d) The stored binary number is equivalent to a decimal number below 5 or above 10.

Combine and simplify each contact network as far as possible. (The four networks can be designed with 39 springs, without combining separate networks).
Chapter 7

TIME CHARTS, SEQUENCE DIAGRAMS, SIMPLIFIED SCHEMATICS, AND GRAPHIC DESCRIPTIONS OF RELAY CIRCUITS

Anyone who has tried to understand from a schematic diagram the functioning of a relay circuit of even moderate size, recognizes that the task is complicated by the difficulty of following circuit paths through a large number of parallel and crossing lines. Furthermore, even if every circuit path is traced through, the dynamic action pattern of a circuit is not shown by the schematic diagram. To assist in understanding relay circuits, schematic diagrams are, therefore, supplemented by sequence diagrams, simplified schematics, and written descriptions. Of these, certain forms of sequence diagram and simplified schematic can be of considerable assistance in the actual designing of circuits. Use has already been made, of course, of simplified schematics and of switching algebra, which can be considered a simplified notation. However, neither of these gives any sense of time relationships, which are of primary importance in the sequential circuits to be discussed in the next chapter. In order to prepare for an understanding of sequential circuits, this chapter will discuss a variety of time charts or sequence diagrams which are useful for circuit design, analysis, and explanation; and, in addition, this chapter will present other forms of simplified circuit notation.

In the design of sequential circuits, the actual time elapsing between successive events is usually less pertinent than the relative sequence in which events take place. In establishing relay control paths in sequential circuits, the chief concern is that a relay act before some and after other relays act. This is also true in the analysis of such switching circuits. Actual times, of course, must conform to requirements and must be consistent with apparatus capabilities. Also, actual time is often a major concern when a circuit is integrated with other circuits into a system.

Time charts and sequence diagrams are a means for indicating graphically the actions taking place in a circuit, and how these actions are related to each other in time. The chief difference between these two general types is that time charts are drawn to an accurate time scale while sequence diagrams are not. Both indicate the sequence in
which events occur. Time charts are used in determining the actual work time of a series of circuit actions. Sequence diagrams are more often used in the design or analysis of switching circuits where a variable time scale permits expansion to show detail in a sequence of rapid actions.

Fig. 7-1 A Sequential Circuit

7.1 SEQUENCE DIAGRAMS AND TABLES

Sequence can be shown in two ways. In one, the "state" type, a continuous record in chart or table form shows all pertinent input, output, and internal relay conditions as present or absent in every interval, and thus shows the combinations of relays operated at all times. The other, the "differential" type, shows only the changes of condition in their proper sequence. To determine, on the differential-type diagram, the state of a circuit in any interval requires a review of the circuit action in the preceding intervals. The state-type diagram, in turn, has the disadvantage that when the number of circuit elements is high, so much of the diagram is taken up by unchanging state indications that it becomes difficult to locate the changes and to follow their sequence. For this reason, the state diagram is usually used with small circuits and the differential diagram with large ones. Indication of circuit control means can be added to either type diagram by additional lines or symbols. Since any additional lines will tend to make the sequence less clear, whether or not they are used will depend on the particular application.

Since no method of denoting sequence has been universally adopted, many variations in both diagrams and symbols are in use. Several representative methods will be described.
State-Type Sequence Diagram. The first diagram to be considered is of the state type. The circuit used for illustration is shown on Fig. 7-1 and the corresponding sequence diagram is on Fig. 7-2. This diagram is divided into spaces of arbitrary length by vertical lines. Each vertical line represents the moment at which some action, such as the operation or release of a relay, takes place, with sequence progressing from left to right. The "state" of the circuit is indicated in the space between vertical lines. (In a relay circuit, the "state" is the combination of relays operated, and the action of a relay in operating or releasing changes the state.)

Relays are assigned horizontal levels and are shown operated (not necessarily energized*) by a solid line on the horizontal level or are shown released by no line on that level†. The solid line runs through all spaces during which the relay is operated. Each vertical space can be numbered for reference. Note that a space must be left between any two actions which do not occur simultaneously. Input conditions, output conditions, keys, and other apparatus can be each assigned to a horizontal level, and their presence or operation indicated in the same manner as for a relay. To bring out the usefulness of such a sequence diagram, first a written description of the circuit in Fig. 7-1 is given as follows. (This circuit is used merely for example and is not shown as performing any useful function.)

Closing key (K) applies ground to the winding of relay (A), operating it. Relay (A) closes the operating path \((A + C')\) which operates relay (B). Relay (B) locks on path \((B + C')\). Releasing key (K) releases relay (A). Now a path \((A' + B)\) operates relay (C) and a second path \((A' + B)\) holds relay (B). Relay (C) locks on path \((B + C)\) and operates relay (A) over path \((C + B)\). Relay (A) opens the locking path of relay (B) and holds relay (C) through path \((A + C)\). Relay (B), releasing, releases relay (A) which in turn releases relay (C).

The sequence of operations described above is more apparent when shown as on Fig. 7-2, although control paths are not indicated on this diagram. Notice that interval (2) represents the operate time of relay (A); interval (3) the operate time of relay (B); interval (5) the

---

* A relay is "energized" when an external source of potential sufficient to cause eventual operation is applied across its winding. Some delay will always exist between this application and the building up of sufficient flux to move the armature. A relay is "operated" when make-contacts are closed and break-contacts open, and "released" when the converse is true. Thus a relay can be energized and still be in a released position immediately prior to operating, or it can be de-energized and in an operated position immediately prior to releasing.

† This is sometimes varied by designating each relay by a horizontal line. When the relay is operated, the line is offset upwards by a slight amount, and is returned to the original level when the relay is released.
release time of relay (A); and so forth. If controlling elements are to be identified, light arrows may be added as in Fig. 7-3. These show, for example, that the operation of (K) causes the operation of (A) which, in turn, causes operation of (B). This type of sequence diagram, without control arrows, will be used extensively in Chapter 8.

Tabular Sequence Chart. A second type of presentation of sequence is one which uses the symbols introduced in Chapter 5 for the tabulation of relay combinations. Since a circuit with relays operating
in sequence is progressing through various combinations of operated and unoperated relays, the sequence can be recorded by tabulation of the combinations in the order in which they occur. In these tables, time will run down the page, and each relay will be given a vertical column. A relay is shown operated by symbol 0 and unoperated by symbol 1. The action of Fig. 7-1 can be expressed by this method as shown in Fig. 7-4. Note that the numbered lines correspond to the numbered intervals of the diagram of Fig. 7-2. The tabular sequence table will also be used in Chapter 8.

**Differential-Type Sequence Diagram.**
The third method of presenting sequence is of the differential type showing only the changes in state. As mentioned earlier, this type of diagram becomes more necessary with large numbers of relays, since each relay is indicated and occupies space on the diagram only at the time it acts. Sequence progresses downward on this diagram, and actions are denoted by symbols. Relay operation is represented by "x" and release by "-", accompanied by the designation of the relay involved. If an action is responsible for a subsequent action, the dependent action is indicated directly below the first and connected to it by a vertical line. When two parallel chains of action are both necessary for one action, that action is placed directly below one chain of action and connected to it by a vertical line. Then a line is drawn diagonally to this vertical line from the bottom of the other chain of action. A comparison of Fig. 7-5 and its sequence diagram of this type, Fig. 7-6, will demonstrate how these symbols are used. The coordinate system of letters and numbers included with this type of sequence diagram is for the purpose of making possible quick references. An application is shown at coordinate point

![Fig. 7-6 Differential-Type Sequence Diagram of Circuit of Fig. 7-5](image)

![Fig. 7-7 State-Type Time Chart for Circuit of Fig. 7-1](image)
M-1 where relay (C) operating is a necessary action for relay (G) to operate at Q-2.

7.2 TIME CHARTS

Very often the length of time that a circuit and certain of its components will take to operate or the time that elapses between two of its output conditions, must be computed in order to determine how it will work in with other circuits and into the system of which it is a part. The values of the acting times of relays can usually be determined and the maximum, minimum, and average times may be computed for the whole or any part of the circuit operation if the corresponding numerical time values of the relays are added properly. If the action is at all complicated, however, it is much more convenient to draw a sequence diagram to a definite time scale. Thus, Fig. 7-2 could be redrawn to scale as in Fig. 7-7 where each vertical space column represents 0.010 second. Any desired time can be easily obtained by measuring it on the diagram. For example, the time between two successive operations of relay (A) is seen to be 0.116 - 0.20 = 0.096 second.

A second method represents the acting time of a relay by segments of horizontal lines which are not assigned to a given horizontal level and which are shown only during the acting time. These line segments are identified by the relay designation, with some mark such as an underline to differentiate between operation and release. Vertical lines denote direct control; slanting lines with arrows, indirect control. A time chart by this method for Fig. 7-5 is shown on Fig. 7-8. The same type of differential representation may also be used with an arbitrary time scale.

Fig. 7-8 Differential-Type Time Chart Showing Control for Circuit of Fig. 7-5
7.3 PARTIAL SCHEMATIC DRAWINGS OR "SHORTS"

In addition to means for presenting a clear picture of sequence of relay operations, there is often a need for showing relay control paths in a manner simpler than that of a complete schematic drawing of the circuit. Usually, when only the contacts necessary for a given operation are shown, what had appeared as an involved path on the complete schematic diagram now appears as an obvious arrangement. This result, as mentioned in the opening paragraph of this chapter, may be achieved to a degree by the algebraic expression; however, where contact sequences, bridge-type circuits, and relay sequences are encountered, algebra does not always produce an adequate representation. Therefore resort is made to simplified diagrams called relay "shorts". The procedure used for preparing a relay "short" is to omit from the diagram all contacts not involved in the operation under consideration, to draw out the relay control paths in order, and to disassociate relay contacts and relay windings, with, perhaps, the exception of locking contacts. Instead of drawing each contact, an "x" may be used to represent a make-contact and "l" to represent a break-contact. Relay spring numbers may be included to identify particular contacts if the diagram is to be used with actual apparatus. Fig. 7-9 shows the operating path of relay (C) of Fig. 7-1 drawn out in this manner.

![Fig. 7-9 Short Schematic for a Relay from Fig. 7-1](image)

7.4 GRAPHIC DESCRIPTION OF RELAY CIRCUITS

A simple but complete representation of the action of a relay circuit can be of great aid to both the designer of the circuit and to others who, for various reasons, need to learn its operation. The graphical sequence diagrams discussed in the first part of this chapter are useful in presenting what circuit actions take place and when they occur, but some abbreviated circuit sketches or relay shorts of the type described in Section 7.3 are needed to round out the presentation by showing how the actions take place. A substantially complete graphical representation of circuit operation by a combined sequence diagram and relay "short" is described in the following paragraphs.

This graphic description, whose symbolic notations are shown on Fig. 7-10, makes use of a sequence diagram of the state type and is similar in this respect to the first diagram described in section 7.1. As in that diagram, relays and other apparatus are assigned horizontal levels, with the state of any particular circuit component denoted by
<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>MEANING</th>
</tr>
</thead>
</table>
| A      | RELAY (A) ENERGIZED  
RELAY (B) NOT ENERGIZED |
| B      | PATH CLOSED |
| C      | PATH OPENED |
| D      | ACTIVE CONTACT |
| E      | PASSIVE CONTACT |
| F      | DIRECT CONTROL PATH CLOSED TO ENERGIZE RELAY (A), THEN OPENED |
| G      | SHUNT CONTROL PATH OPENED TO ENERGIZE RELAY (B), THEN CLOSED |
| H      | TYPICAL CIRCUIT PATH CLOSURE  
PASSIVE MAKE CONTACT ON RELAY (A)  
ACTIVE MAKE CONTACT ON RELAY (B)  
NO CONTACT ON RELAY (C)  
PATH CLOSURE ENERGIZES RELAY (D)  
BREAK CONTACT ON RELAY (E) |
| I      | TYPICAL CIRCUIT PATH OPENING  
RELAYS NOT CONCERNED IN OPENING OF PATH  
OPENING OF PATH DE-ENERGIZES RELAY (D)  
ACTIVE BREAK CONTACT ON RELAY (E) |
| J      | CLOSURE OF LOCKING PATH FOR RELAY (D) BY RELAY (C) |
| K      | RELAY (B) CLOSING ITS OWN LOCKING PATH THROUGH RELAY (A) |
| L      | OPERATION OF SLOW OPERATE RELAY (A) |
| M      | RELEASE OF SLOW RELEASE RELAY (B) |
| N      | ACTIVE PRELIMINARY MAKE CONTACT ON RELAY (A)  
WITH ADDITIONAL ACTIVE BREAK AND MAKE CONTACTS |
| O      | ACTIVE MAKE-BREAK TRANSFER CONTACT ON RELAY (B) WITH AN ADDITIONAL MAKE CONTACT |
| P      | SYMBOL TO REFER TO NOTE 1 |

Fig. 7-10 Symbols for Graphic Circuit Description
the absence or presence of a solid horizontal line at the corresponding level. Sequence progresses from left to right. (The orientation of the whole diagram with respect to horizontal and vertical may, of course, be interchanged.)

To show circuit details, control path indications are included as vertical lines in the proper sequence locations on the diagram, whenever the paths are opened or closed. A solid vertical line is used to denote the closure of a path, and a dashed line to denote the opening of a path. The contacts making up the path are shown by symbols at the appropriate intersections of the horizontal relay levels and the vertical control path lines. The user of the diagram has available, therefore, information concerning both the over-all operation pattern and the particular causes and effects of every individual action.

The number of symbols used has been kept as low as possible to facilitate both the construction and reading of the diagram. Fig. 7-10 lists all of the common symbols. The arrowhead is used to identify the controlled circuit component. It is put on the vertical control path line at the level of the component controlled by that path. Contact symbols differentiate between active and passive contacts. The active contact, whose symbol is a small circle, is the last one to close or the first to open a path, if a number of series contacts are involved in the path. On the other hand, the passive contacts, whose symbols are x's, are those which are necessary to complete the path, but which have been closed prior to the time at which the path is completed. A given contact may be a passive contact when a path is closed and may be the active contact when the path is opened. Whether or not a contact is a make type or break type and whether or not it is opened or closed can be determined by observing the state of its associated relay and the condition of the path of which it is a part. Thus, an active contact of a relay which is shown, after that relay has been released, as closing a control path is evidently a break contact.

Fig. 7-10H shows a typical circuit path being closed by an active contact on relay (B) and passing through passive contacts on relays (A) and (E). Note that the solid horizontal line showing energization of relay (D) starts immediately at the arrowhead. Fig. 7-10I shows the same path opened, but this figure does not have to show passive contacts since such contacts do not contribute to the path-opening operation.

Other symbols for slow relay operation, shunt paths, contact sequence, and the like, are explained in Fig. 7-10. Symbols for the active contacts of a relay are always shown adjacent to each other following the change of state of the relay. No inference is drawn concerning the sequence of action of any of the contacts unless specific symbols denoting contact sequence or relay speed are shown. To keep the number
of symbols small, unusual circuit configurations are not covered symbolically, but in the form of circuit notes.

A difficulty with the state type of sequence diagram, which has already been mentioned is that when the number of circuit elements is large, the diagram is cluttered up with unchanging state indications which conceal the significance of important sequences. To correct this situation, the graphic circuit description brackets those relays which remain relatively inactive for many intervals. The bracketed relays are then carried as a single state line for their inactive period, thus freeing space for the representation of other relay actions. When one or more elements within the bracketed group becomes involved in a circuit control path, it is indicated by the proper symbol on the group state

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Fig. 7-11 A Sequential Circuit

Fig. 7-12 A Graphical Representation of the Circuit of Fig. 7-11
line. External circuits are carried in a similar manner by single state lines. The method can be extended to show the interaction among minor or major components of a system instead of apparatus elements.

To conclude, an example of the graphic circuit description is attached as Fig. 7-12 representing the operation of the circuit of Fig. 7-11. The circuit itself is intended only to illustrate the graphical notation and performs no obvious function.
Chapter 8

DESIGN OF SEQUENTIAL RELAY CIRCUITS

In a combinational circuit, each input combination invariably produces the same output combination every time it occurs. In a sequential circuit, however, the output conditions are determined jointly by the sequence in which input signals occur as well as by the combinations of input signals. The action of a sequential relay circuit depends upon arrangements of relays and circuit paths which recognize certain input combinations, retain a memory of the fact that these combinations have occurred, and use this memory to influence later circuit actions.

Input signals to a sequential circuit take the same forms as those for combinational circuits. In the following discussions it is assumed that there is one input lead for each input signal and that the leads connect directly to relays in the circuit. These relays, which will be called "primary" relays, perform a function similar to that performed by relays in a combinational circuit, in that they detect input signals and may establish a wide variety of circuit paths dependent on the input combinations. A sequential circuit also contains one or more relays called "secondary" relays which provide a record of significant actions the circuit has taken, and thus guide future circuit operations. Secondary relays are controlled within the sequential circuit by local circuit paths which may employ contacts on both the primary and secondary relays. The circuit for each secondary relay is of the locking type where the relay participates in its own control. In many cases, the circuit can be manipulated so that one or more primary relays require but a single make contact and therefore can be eliminated from the circuit. In the extreme situation, no relays identifiable as primary relays appear at all in the circuit.

Sequential circuits can be illustrated by a simple example. Suppose that a signaling system between points A and B has the following requirements.

Grounding a lead a by means of a key at position A is to cause a lamp to light at position B. If a lead b is then grounded by means of a key at B, the lamp is to be extinguished. The lamp must remain extinguished if the key at B is restored to normal before the key at A is restored.

If the input leads control primary relays (A) and (B), the operating sequence can be indicated as in Fig. 8-1. During interval 2, when
the circuit to the lamp must be closed, relay (A) is operated and (B) is released and the circuit path \((A + B′)\) will be closed. However, this same input combination exists in interval 4 when the lamp must be extinguished, and no arrangement of contacts on relays (A) and (B) alone will permit different functions to be performed in intervals 2 and 4. A solution is to provide a secondary relay which will be in an operated state during one of the intervals in question and in a released state during the other. Considering all relays, both primary and secondary, the combinations then will be different during intervals 2 and 4, and a circuit path for the lamp can be developed.

A suitable method of controlling the secondary relay in this example is to let it operate during interval 3 when (B) operates, and lock to a make on (A) so that it will hold through interval 4 and release after (A) releases. This sequence is shown in Fig. 8-2 where the relay is designated (X). The line in the chart for relay (X) indicates the operation of the contacts of this relay. Although (X) starts to operate at the beginning of interval 3 when (B) first operates, there is a short operate time
interval before the contacts act to open or close circuit paths. In a similar manner, the relay contacts remain operated for a short time after interval 4 while the relay is releasing. A circuit to fulfill the requirements can be easily constructed and is shown in Fig. 8-3. A break contact on relay (X) prevents the lamp from lighting during interval 4. Thus, a secondary relay controlled by the primary relays to operate in a prescribed sequence provides control means which cannot be achieved by the primary relays alone.

![Fig. 8-4 Energization of Secondary Relay (X)](image)

### 8.1 CONTROL OF SECONDARY RELAYS

In designing a sequential circuit, the number of secondary relays required must be determined first; then a suitable operating sequence for these relays must be chosen; and finally, control paths to cause the relays to act in this sequence must be developed. Although the design of control paths is usually the last stage in the development of a circuit, it will be discussed first, since a knowledge of control paths is helpful in developing satisfactory operating sequences.

Assume, therefore, that a satisfactory sequence diagram showing the action of all primary and secondary relays has been developed for a circuit problem. By examining the diagram, the combination of relays operated at any given time can be determined. To develop a circuit path which closes when some circuit action occurs, and remains closed until some later action takes place, it is necessary only to observe what relay combinations exist during the time the circuit path is to be closed, and to proceed by combinational methods. For instance, the circuit for controlling a secondary relay can be developed by determining the intervals during which its winding must be energized, and developing a circuit path closed in these intervals. As will be seen, the circuit paths can be developed by methods comparable to the formal approach discussed in Chapter 6, or by the method of inspection.
Consider, as a first example, the control of relay \((X)\) in Fig. 8-2, which shows the sequence of action of contacts on relays \((A); (B);\) and \((X)\). In Fig. 8-4 a dashed line has been added through intervals 3 and 4 to indicate when a circuit path must be closed to energize the winding of relay \((X)\). This does not correspond to the line on the chart showing the action of the contacts of \((X)\), since the acting time of relay \((X)\) subdivides interval 3 into two parts. For the same reason there are two parts to the interval following 4 (in which the circuit returns to the normal condition shown in interval 1). The operate and release times of secondary relays are often of considerable importance, and their existence should always be recognized in the design of sequential circuits.

From the three relay combinations existing during intervals 3 and 4, a circuit for \((X)\) can be developed as follows:

\[
f(X) = (A + B + X')(A + B + X)(A + B' + X)
\]
\[
= (A + B)(A + X)
\]
\[
= A + BX \quad (8-1)
\]

The circuit is shown schematically in Fig. 8-5.

As with combinational circuits, invalid relay combinations may be included to achieve simplification. In this case relay combinations \((A' + B + X')\) and \((A' + B + X)\) never occur in the operating sequence. These combine to give \((A' + B)\) which may be combined with equation (8-1) as follows:

\[
f(X) = (A + B)(A + X)(A' + B)
\]
\[
= B(A + X) \quad (8-3)
\]

This path was shown schematically in Fig. 8-3.
The Design of Switching Circuits

Algebraically, equation (8-3) is not equivalent to equation (8-2), but either contact arrangement will provide the proper control path for relay (X). When such alternative choices occur, the ease of combining with other paths usually determines which will be used. In this example the operating path for the lamp is:

\[ f(L) = A + B' + X' \]

This can be combined with the circuit for (X) in the form (8-2) as follows:

\[ f(X) = A + B' + X' \]

Since relay (A) contains only a single make contact, it is unnecessary to include that relay in the circuit. The resulting simplified circuit is shown in Fig. 8-6.

The method used above, that of employing the combinations in all intervals during which the controlled relay is energized, is general and could be used to design the control path of any secondary relay. However, it becomes unwieldy where many relays and combinations are involved, and a more direct approach is desirable. The circuit of a secondary relay is basically of the locking type shown in Fig. 8-7, which may be written:

\[ f(X) = P(Q + X) \]  \hspace{1cm} (8-4)

The network \( P \) is the "operate path" which must close at the time the relay (X) is required to operate. The network Q, which is connected in series with the locking contact X, is the "hold path" which must close during or before the closure of P and must remain closed until the relay is required to release. The design work may be reduced by developing each of these paths separately. The circuit then can often be
simplified by combining elements in the operate and hold paths. In the previous example, equation (8-1):

\[ f(X) + (A + B)(A + X), \]

the operating path is \((A + B)\) and the holding path in series with the locking contact \(X\), is \(A\). These were combined in equation (8-2):

\[ f(X) = A + BX. \]

If there are no restrictions on the number and type of contacts available on the relays of the circuit, the control path for a secondary relay can always be developed in the general form given in equation (8-4). Cases arise, however, where shunt control or other special circuit arrangements are used because of practical restrictions.

It can be observed that the operate path \(P\) and the hold path \(Q\) in the general form \(f(X) = P(Q + X)\), will contain neither make nor break contacts on relay \((X)\). The circuit combination which causes relay \(X\) to operate must come into existence before \((X)\) operates and will be followed by a combination which is identical except that \((X)\) is operated. Thus the operate path \(P\) obviously does not require contacts on \((X)\). Since the hold path \(Q\) is connected in series with a make on \((X)\), any contacts on \((X)\) contained in \(Q\) can be eliminated by algebraic methods. Since operate and hold networks, \(P\) and \(Q\), do not then contain contacts on the relay being controlled, these networks may be designed by considering the combinations of all relays of the circuit except the one being controlled.

The operate path of a secondary relay must be closed in the interval in which the relay is to move to the operated position. However, this path can often be simplified by permitting it to be closed for other combinations of the circuit relays than the one existing at this time. It may be closed during any of the intervals in which the relay will remain operated, and may also be closed for any of the invalid combinations which do not occur in the normal sequence of actions. It must not be closed for any combination in the sequence in which the relay is expected to remain unoperated. Although the circuit may be designed by symbolic methods, it can often be determined from a direct inspection of the operating sequence. The simplest statement of the position of other relays which identifies the point in the sequence in which the relay acts will indicate the form of the circuit.

The hold path of a secondary relay must be closed in every interval during which the relay is energized. Since it is in series with a make contact on the relay, it may be closed also for any other combination of the circuit relays except the one which occurs at the time the relay must release. The circuit may be designed by formal methods in two ways. An obvious method is to determine those relay combinations
for which the relay must hold operated, and to construct a network closed for these combinations. A second method is to develop a circuit path which opens only at the time the relay must release and is closed at all other times. The combination which exists during the interval in which the relay releases and which must cause the holding path of the relay to open is called the "release combination". A circuit network open only for this combination (the negative of the circuit closed for this combination) can be used as the holding network.

Holding circuits developed by these two methods will not necessarily be the same* although both fulfill essential requirements. The first is closed for the minimum number of combinations required to hold the relay operated, while the second is closed for all except the release combination. The simplest circuit often lies between these two limits. Circuits developed by the first method often can be simplified by including certain of the remaining combinations in the conditions for closing the path. Invalid combinations not in the normal sequence as well as combinations existing while the relay is released may be used.

The determination of a hold path by either of these two methods may not give the simplest path, since if there are many combinations available for simplifying, it is difficult to pick the proper combinations for the simplest result. For this reason, a completely different third approach based on inspection is often taken. After determining the relay's operating path, the sequence diagram is examined for a contact or several contacts in series which will serve to hold the relay from the interval in which the operate path is opened to the interval in which the relay is to release. If none can be found which will serve for the complete holding time, a path may be picked which covers at least a few intervals, at the end of which another path is selected to take over, and so on up to the release interval. Caution must be observed that each hold combination determined in this manner will be open when the relay is to release. The parallel connection of these paths will give the hold path, usually in a simple form.

When a secondary relay operates two or more times in the sequence of circuit actions, the preceding methods must be modified. The operating network must close each time the relay is required to operate. It can be designed by developing a circuit path for each of the several operating intervals, and then placing the combined paths in parallel. The paths should be carefully checked to see that they do not close at times when the relay must remain unoperated. Similarly, holding paths can be developed for each of the periods during which the relay must hold operated, and these paths placed in parallel in the locking

* It can be shown that when all combinations of all circuit relays, both primary and secondary, occur in the circuit action sequence, circuits obtained by both methods are identical.
circuit. Each holding path is required to be open not only at the end of the period during which it serves to hold the relay but also must be open during each of the other intervals in which the relay releases. If a relay is assumed to operate and release three times in a sequence, it will have three operating paths, $P_1$, $P_2$, and $P_3$, and three holding paths, $Q_1$, $Q_2$, and $Q_3$. The combined circuit will be:

$$f(X) = P_1P_2P_3(Q_1Q_2Q_3 + X)$$

The holding path may be developed by a second method. A satisfactory circuit will be one which is closed for all combinations except the ones during which the relay releases. This circuit can be developed by constructing a network for each release condition which is open only when this condition occurs, and connecting the networks for the several release conditions in series. If the release networks for the relay which acts three times are designated $R_1$, $R_2$, and $R_3$ the resulting circuit is:

$$f(X) = P_1P_2P_3(R_1 + R_2 + R_3 + X)$$

From the above discussion it is seen that a circuit to fulfill the control requirements of a secondary relay can be designed by considering only those intervals during which the relay operates or releases. The resulting circuit, however, will not necessarily be the simplest one.

![Diagram](image)

**Fig. 8-8 Sequence with Two operation of a Secondary Relay**

Fig. 8-8 shows a sequence requiring two operations of a secondary relay ($X$), once in interval 2 and again in interval 7. For this sequence:

$$\text{Operate Path} = (A + B' + C')(A' + B + C)$$

The path may be simplified by using the invalid combination $(A + B + C)$ as follows:
Operate Path = \((A + B' + C')(A' + B + C)(A + B + C)\)
\[= (A + B' + C')(B + C)\]

The hold path in intervals 3, 4, and 8 is determined as follows:

\[
\text{Hold Path} = \left[(A' + B' + C')(A' + B + C')\right](A' + B' + C) \\
\text{[first operation]} \quad \text{[second operation]} \\
= A' + B'C'
\]

Then the complete circuit is:

\[
f(X) = (B + C)(A + B' + C')(X + A' + B'C') \\
= (B + C)(A + B' + C')(X + A')
\]

If the hold path is determined by the second, or release combination, method (intervals 5 and 9), then:

\[
\text{Hold Path} = (A + B + C')' + (A + B' + C')' \\
= A'B'C' + A'BC' \\
= A'(B'C + BC') \\
= A'(B' + C')(B + C)
\]

And

\[
f(X) = (A + B' + C')(B + C) [X + A'(B' + C')(B + C)] \\
= (A + B' + C')(B + C) [X + A'(B' + C')] \\
= (B + C)(A + B' + C')(X + A')
\]

Both methods give identical results in this example.

Fig. 8-9 Circuit for Fig. 8-8
The circuit must now be examined for operating hazards due to races or contact stagger sequences. A hazard due to a temporary break in the energization of relay \(X\) may occur on the release of relay \(A\) between intervals 2 and 3 when the path transfers from \((A + B' + C')\) to \((A' + X)\). This can be overcome by using a continuity-transfer for the \(A-A'\) pair. The circuit may also be rearranged so that a break-make transfer is satisfactory by using the following theorem of switching algebra:

\[
(X + Y)(X' + Z) = XZ + X'Y
\]

The control path then is:

\[
f(X) = [AX + A'(B' + C')] (B + C)
\]

The circuit is shown in Fig. 8-9.

Circuits for this example can also be constructed directly from an inspection and analysis of the operating sequence. From the sequence diagram, Fig. 8-8, it is seen that \((X)\) operates when \((A)\) operates in interval 2, but does not operate when \((A)\) re-operates in intervals 5 and 9. The condition of both \((B)\) and \((C)\) released is sufficient to distinguish interval 2 from intervals 5 and 9, since \((B)\) is operated during interval 5 and \((C)\) is operated during interval 9. Therefore \((X)\) must operate when \((A)\) operates and both \((B)\) and \((C)\) are released, i.e. \((A + B' + C')\). Relay \((X)\) must also operate in interval 7, which may be described as the interval of the sequence in which both \((B)\) and \((C)\) are operated, i.e. \((B + C)\). For the holding path, it is seen that the relay could hold to a break on \((A)\) during intervals 3 and 4, and release during interval 5 when \((A)\) operates. However, the hazard of the relay releasing due to a momentary open during the change from interval 2 to interval 3 is immediately recognized. One solution would be to include in the locking path a circuit which is closed continuously during intervals 2 and 3. From the diagram it is seen that the condition "(B) and (C) released" exists continuously during intervals 1, 2, and 3, and will not interfere with the release of the relay during intervals 5 and 9. The path \((B' + C')\) therefore can be included in the locking path. The single break on \((A)\) is satisfactory to hold the relay in interval 8 during its second operation, since this contact is closed in interval 6 before the relay operates and remains closed through intervals 7 and 8. From the above analysis the following circuit can be written:

\[
f(X) = (A + B' + C') [X + A'(B' + C')] (B + C)
\]

This can be simplified as follows:

\[
f(X) = [A + A'(B' + C')] [X + A'(B' + C')] (B + C)
= [AX + A'(B' + C')] (B + C)
\]

This is the same as the circuit in Fig. 8-9 which was obtained by formal methods.
Circuits considered so far have contained only one secondary relay. Design approach is not materially altered when several secondary relays are involved. The control of each relay is considered separately, assuming that other secondary relays operate in the desired sequence even though their control paths may not yet have been designed. With all paths independently completed, the circuit will act in the sequence indicated.

For example, take the sequence of Fig. 8-10. Since the condition of (C) operated and (Y) released in interval 4 is unique, the path (C + Y') can be used for the operation of secondary relay (X). The relay can be locked during intervals 5, 6, and 7 to a break on (C) (C) giving the circuit:

\[ f(X) = (C + Y') (X + C') \]

Rearranging to avoid a continuity-transfer on (C),

\[ f(X) = CX + C'Y' \]

Similarly for the control of relay (Y), the path (B' + X) which is closed during interval 6 may be used for the operating path, and the relay can be held through interval 10 by locking to a make on (A). The circuit is:

\[ f(Y) = (B' + X)(A + Y) \]
The complete circuit for relays (X) and (Y) is shown in Fig. 8-11. Note that relay (A) can be eliminated without affecting the operation of the circuit.

![Circuit Diagram](image)

Fig. 8-11 Circuit for Fig. 8-10.

Much of the combination and simplification of the previous examples could have been done by the tabular method. If the number of relays is not too large, the tabular method of denoting sequence gives a direct approach to the subsequent contact manipulation. Operate, hold, and release combinations can be selected from the table by observing where the secondary relay to be controlled changes its state. The tabular method will be used for a final secondary relay control example in which all the relays are internally controlled and which will run through its sequence cycle continuously as long as power is provided. This particular sequence, shown in the table on the opposite page, uses all combinations of four relays.

The holding paths will be determined from the combinations which cause the relay to release. Relay (D) operates and releases once in the cycle. It is energized in interval 7 by combination \((A + B' + C)\), and released in interval 15 by combination \((A' + B + C')\). The holding path, therefore, is the negative of this latter path.
\[ f(D) = (A + B' + C) \left[ D + (A' + B + C') \right] \]
\[ = (A + B' + C)(D + AB'C) \]

The circuit for relay (A) is determined from intervals 1 and 6 for the operating path, 4 and 11 for the holding path,

\[ f(A) = (B' + C' + D')(B' + C + D')(A + B'C'D + B'C'D') \]
\[ = (B' + D')(A + B'C') \]

The circuit for relay (B) is determined from intervals 2, 9, and 14 for the operating path; 5, 12, and 16 for the holding path,

\[ f(B) = (A + C' + D')(A + C' + D)(A' + C' + D) \]
\[ \cdot (B + AC'D + AC'D' + ACD) \]
\[ = (C' + AD)[B + A(C' + D)] \]
\[ = C'B + AC' + AD \]
\[ = C'(A + B) + AD \]
\[ = (C' + AD)(A + B) \]

(Notice that the holding path for the third operation of relay (B) is eliminated by algebra since the relay releases when its operate path opens.)

The circuit for relay (C) is determined from intervals 3 and 10 for the operating path; and from 8 and 13 for the holding path,
\[ f(C) = (A + B + D')(A + B + D)(C + A'BD' + ABD') \]
\[ = (A + B)(C + BD') \]

The combined circuit for all four relay paths is shown in Fig. 8-12. There are situations in this circuit where a relay acts through the contacts of another relay which it in turn controls. For example, relay (B), which operates through path \((A + C')\) in interval 3, immediately operates relay (C) to open its own operate path. This requires that contact stagger on relay (B) be less than the operate time of relay (C). If this requirement cannot be met, a preliminary make contact must be used for locking (B) and other relays in similar situations.

8.2 EFFECT OF RACE CONDITIONS ON INTERNAL CONTROL

In many sequential circuits, one relay by its operation may affect several relays at the same time. This causes a "race" condition, and the sequence in which the relays act is uncertain. These conditions must be examined to determine whether some relay combination which may exist momentarily may cause adverse reactions. In cases where the exact sequence of action of several relays is unimportant, simultaneous operation may permit simpler relay control paths and may result in the circuit performing its functions in a shorter time.

A sequence diagram with several race conditions is shown in Fig. 8-13. In interval 3, when both \((X)\) and \((Y)\) operate due to the release of \((K)\), there will always be a slight difference between the times when \((X)\) and \((Y)\) reach the operated position. Thus either combination

\[ (K' + W + X + Y' + Z') \]

or  
\[ (K' + W + X' + Y + Z') \]

![Fig. 8-13 Sequence with "Simultaneous" Secondary Relay Actions](image-url)
will exist for a short time in interval 3. The effect of these two combinations must be considered when developing circuit paths, and neither can be treated as an invalid combination. If the sequence were modified to operate relays (X) and (Y) in a prescribed order, one of these two combinations would be specified and the other then could not occur. For example, (Y) could be operated through a make contact on (X) and combination (K' + W + X' + Y + Z') would never occur. The same sequence could be achieved by making relay (Y) slower in operating than relay (X). In interval 5 there is a race of (X) releasing and (Z) operating. Other races occur in intervals 7 and 9.

8.3 PROVIDING OUTPUT CONDITIONS IN SEQUENTIAL CIRCUITS

The output conditions which a sequential circuit establishes may last for one interval only, or extend over several intervals, or appear several times. In each interval the problem of establishing the proper output conditions is of the combinational type. The only difference is that when an output condition extends over several intervals, care must be taken that the condition is maintained continuously. This problem is the same as that of maintaining continuous hold paths for secondary relays, and may require contact manipulation or use of continuity-transfers.

8.4 DEVELOPING AN OPERATING SEQUENCE

From the requirements of practical design problems, it is usually possible to determine the sequence in which input conditions occur and the corresponding sequence in which output conditions must be produced. From these sequences the designer must determine when secondary relays are required, and choose suitable intervals in the sequence for the operation and release of these relays. This work must precede the design of the actual relay control paths and is often the most difficult phase of circuit design. Use of a diagram in developing the sequence will be found helpful. The sequence will start with a "normal" condition and proceed through the sequence until all actions are completed and the circuit has returned again to the normal condition. This is a circuit "cycle". The normal condition is usually the one which exists when the circuit is idle. In most cases this will be the condition in which no input signals are present and all of the circuit relays, including secondary relays which may be added during the design, are unoperated. There are, of course, exceptions.

There are several types of operating sequences. The simplest is where the inputs and outputs always occur in a definite sequence. The examples which have previously been given in this chapter are of this
type. Another type of sequence is where the inputs, starting from a normal condition, may occur in several alternative sequences, and the output is influenced by the particular sequence in effect. For these circuits, the sequence diagram should show all of the possible alternative cycles in some arbitrary order separated by normal intervals. Although the diagram will appear to state a definite order in which alternative events occur, the completed circuit will follow any one of the alternative cycles from the normal state whenever the input conditions are applied in the appropriate sequence. The normal intervals must all be kept alike in both primary and secondary relay combinations, so that no cycle is affected by the previous cycle through which the circuit has passed.

The following example illustrates an alternative sequence:

An intercommunication system is installed between two positions, A and B. Each position is provided with signaling equipment consisting of a key of the mechanically locking type, an answer lamp (ANS-), and a disconnect lamp (DIS-). Operation of the key at one position will cause the (ANS-) lamp at the other position to light. This lamp is extinguished when the key at the called position is operated. If either key is then restored to normal, the (DIS-) lamp at the other position will light and remain lighted until that key is restored to normal. A relay circuit for the control of the signal lamps is required.

Four alternative sequences of input signals are included in these requirements and are shown in Fig. 8-14. In each case the output depends upon the input sequence. Intervals 1, 5, 9, 13, and 17 are all normal with no input leads grounded.

![Sequence with Alternative Cycles](image-url)
In analyzing the requirements of a sequential circuit, consideration may be given to all conceivable sequences. Of these a large number may usually be discarded as being known never to occur. In many cases, however, the statement of requirements may not mention some of the possible cycles. For example, in the previous statement, no mention was made of what action the circuit was to take if a key should be restored to normal without waiting for an answer. In practice, a designer would probably have to verify that a sequence such as this never occurred, or if it did occur, ascertain what action should be taken. In this case, it can be assumed that it may occur, and, if it does, the circuit should restore to normal after the restoring of the key. A circuit to operate in this sequence will be designed later in this chapter.

The number of alternative cycles may easily rise too high to allow the complete representation of each cycle on a practical sequence diagram. For example, in automatic telephone systems all circuits involved in setting up a call from one subscriber to another must be capable at many or all stages of their operation to return to normal should the call be abandoned by the originating subscriber. This sort of general requirement involving a large number of alternative cycles is easily understood and is most usable in its simple verbal form rather than in a long series of repetitious sequence diagrams.

In some situations, signals may occur on a group of similar input leads in a more or less random sequence, and the circuit is required to establish output conditions dependent upon the relative sequence in which the signals occur. An example is where several independent signal sources are associated with a single common device which can perform some function for the individual sources on a one-at-a-time basis. When one source obtains the service of the common device, all others must be made to wait until service to the first is completed. Waiting requests are then served in a prescribed order. It would be impractical in these cases to investigate every possible sequence in which the inputs may occur. Instead it is usually possible to proceed without a sequence diagram by determining a general plan for assigning a preference order to each individual input according to its physical
position relative to others which make simultaneous or prior requests. The circuit design then can usually be carried out by the methods which have been described for positional networks. Lockout circuits, discussed in a later chapter, are an example of circuits of this type.

Another general type of circuit produces a sequence of output signals entirely by internal control means. In these circuits, an input from external sources merely starts the action, and subsequent intervals of the sequence are determined by elements within the circuit and do not correspond to changes of input signals. An example is a tone-signaling system having the input and output sequence requirements shown in Fig. 8-15, where ground applied to an input "ST" lead is to cause two spurs of tone to be applied to an output signal channel. The problem here is to choose relays and to determine an acting sequence in which the operate and release times of the relays will provide the proper time intervals. In some cases other devices, such as electronic tubes, will be used to generate the required time intervals.*

8.5 PLANNING SECONDARY RELAY ACTIONS

After the requirements of a sequential circuit have been determined, the next step in design is to plan the sequence in which secondary relays shall operate. The preliminary diagram of the sequence of primary relay operations and of the output signals serves as a convenient framework to which the sequence of secondary relay operations can be added.

In order to deliver a distinctive output signal during a particular period in an operating sequence, a circuit must provide a combination of operated relays which exists during this period and does not exist when the output signal is undesired. If the primary relays alone provide suitable combinations, secondary relays are not required. However, if there are intervals in the sequence which have identical input combinations but require different output combinations, secondary relays are necessary. These relays must be inserted in the operating cycle so that their combinations, taken in conjunction with the original combinations of the primary relays, provide unique relay combinations which can establish the desired output paths. The need for secondary relays, then, is recognized when an input combination appears more than once in a sequence and is required to produce different outputs on each appearance. As a minimum, the number of secondary relays must be sufficient to provide a distinct combination for each identical input interval which must be recognized. That is, one secondary relay can distinguish

* Specific techniques for developing suitable sequences for a group of relays are discussed in the appendix to this chapter, section 8.7.
between no more than two identical input intervals, two secondary relays between no more than four identical input intervals, and so on.

After recognizing that secondary relays are required, and determining which input intervals are identical and must be identified, other intervals in the cycle must be selected for the operation and release of the secondary relays. These operate and release intervals must have certain characteristics of uniqueness; otherwise it will be impossible to design circuit paths which will control secondary relays to act as desired.

Clues to the selection of appropriate operate and release intervals for a secondary relay are furnished by the methods of designing operate and hold paths, discussed earlier in this chapter. In order to determine the validity of proposed operate and release intervals, it is sufficient to check only the corresponding operate and release combinations established by the states of all relays except the one being controlled. These combinations must meet the following requirements:

1. The operate combination must not exist during any interval in which the relay must be released, but may be repeated during any interval in which the relay is operated.

2. The release combination must not exist during any interval in which the relay is operated, but may be repeated during any interval in which the relay is released.

Sequences which meet these requirements will produce workable circuits. In some cases which appear to require only a single secondary relay, the sequence may not contain suitable intervals for the control of this relay. It then becomes necessary to provide an additional secondary relay in order to establish combinations for the control of the first

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Fig. 8-16 Sequence with Several Combinations Available for Control
relay. Often each secondary relay must participate in the control of the other so that a satisfactory sequence for either relay cannot be developed without considering the presence of both relays. Regardless of the sequence of occurrence or number of repetitions of combinations of input signals, it is always possible to identify a particular appearance of an input combination by providing enough secondary relays.

Some of the problems encountered in selecting secondary relay sequences may be illustrated by the following examples. Fig. 8-16 shows a cycle with two input combinations each of which is repeated: \((A + B' + C')\) in intervals 2 and 8, and \((A + B + C')\) in intervals 3 and 7. An output is to be established only in intervals 7 and 8. One secondary relay, \((X)\), released through intervals 2 and 3 and operated through intervals 7 and 8 will serve to differentiate between the similar input combinations. Several combinations are available for the operation and release of this secondary relay; it may operate in intervals 4, 5, or 6 and release in intervals 9, 10, or 11 (normal). The governing factor in picking the secondary relay control intervals will be the simplicity of the control path. By inspection, if a single make contact on relay \((C)\) is chosen to operate the secondary relay, it will operate the relay in interval 5 and hold it in intervals 6, 9, and 10. To hold it during intervals 7 and 8 requires only a locking circuit through a make contact on relay \((A)\), which is effective through intervals 6, 7, 8, and 9. Since this overlaps with the operating path in intervals 6 and 9, continuity of the circuit is assured. The following simple control path is thus obtained:

\[
f(X) = C(A + X)
\]

The output circuit which is closed during intervals 7 and 8 is:

\[
f(OUT) = A + C' + X
\]

A sequence in which the choice of secondary relay control intervals is restricted is shown in Fig. 8-17. Here, a secondary relay is

---

**Fig. 8-17 Sequence with Two Combinations Available for Control**
needed to differentiate interval 12 from interval 6 in order to establish
the output condition in interval 12. As a first attempt, let the relay be in
the operated state during interval 6 and in the released state during
interval 12. It is evident that if the secondary relay is operated in in-
tervals 2, 3, 4, or 5 and held through 6, it will also operate in one of
intervals 8 through 11 and hold through 12, since the combinations of
inputs from 2 through 6 are repeated in intervals 8 through 12. Simi-
larly, if the relay is to be in the operated state during interval 12 and
in the released state during interval 6, it cannot be operated in any of
intervals 8, 9, 10, or 11. This leaves only the combinations of interval
1 (same as 13) and of interval 7 available as control combinations. As-
suming that it is undesirable to have the secondary relay operated
throughout the normal interval, it must be operated in interval 7 and
released in interval 13 as shown in Fig. 8-18. By inspection, the cir-
cuit is:

\[ f(X) = (A + B + C')(X + AB) \]

The output circuit is:

\[ f(\text{OUT}) = A' + B + C' + X \text{ or } A' + C' + X \]

![Diagram](image)

**Fig. 8-18** Sequence of Fig. 8-17 with Secondary Relay Added

The sequence of Fig. 8-19 offers more difficulty in the search for
intervals containing suitable operate combinations. In order to provide
an output path which is closed in interval 13, but not in interval 7 where
the primary relay combination is the same, first assume that a sec-
ondary relay will be in the operated state during interval 7. It is found
by inspection that the sequence of relay combinations from interval 2
to 7 is the same as the sequence from interval 8 to 13. Therefore if a
secondary relay is controlled by relays (A), (B), and (C) to operate in
one of the intervals 2 to 6 and to hold through interval 7, it will repeat
a similar action in the later part of the cycle and hold through interval
13. Intervals 14 and 15 (or 1, the normal interval) are the only unique intervals in the cycle; and these, obviously, cannot be used to control a relay to be in the operated state in only one of the intervals 7 and 13. It is necessary, therefore, to provide an additional secondary relay which will make one of the intervals between 7 and 13 dissimilar from the corresponding one preceding interval 7. Once this is realized, there is a wide choice of intervals in which to operate and release the two secondary relays. One such choice, not necessarily the simplest, is shown in Fig. 8-20. Designating the secondary relays (X) and (Y) in the order of their operation, the circuit paths are:

\[
\begin{align*}
f(X) &= (A + C)(AB + X) \\
f(Y) &= (X + C')(C + Y) \\
&= CX + C'Y
\end{align*}
\]

The output circuit, closed during interval 13, is:

\[f(OUT) = A + B' + C + Y\]
The requirements of Fig. 8-15 present a different problem in sequence planning. Here, the acting times of secondary relays must generate intervals 3, 4, and 5. No secondary relay combination may be repeated, because then the circuit would continue recycling itself as long as the start lead was grounded. A minimum of two relays is needed in this example. These have four possible combinations, and every one must be used since different combinations must be established in each of the intervals 2, 3, 4, and 5. A sequence with two secondary relays is shown in Fig. 8-21 with a primary relay controlled by the start lead. Some practical difficulty might be experienced, however, in selecting relays which are slow enough in operating to give sufficient duration to the intervals. A sequence such as that of Fig. 8-22 might prove more

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**Fig. 8-21** One Sequence of Secondary Relays for Fig. 8-15

**Fig. 8-22** Another Sequence of Secondary Relays for Fig. 8-15
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desirable, where interval 2 is formed by three relay operating times added together, and intervals 3 and 4 are both generated by relays releasing.

The choice of an operating sequence of secondary relays, when several sequences are satisfactory, may affect the complexity of required control paths. The complexity of these paths can usually be judged by inspection, then making a tentative design of the circuits for several optional sequences and observing how the modification of a given sequence will alter the corresponding circuit. Simple paths for output circuits will usually result if the operated intervals of secondary relays correspond to closures of output circuits. It may be desirable in some cases to arrange the sequence to give simple output paths at the expense of added complexity of secondary relay control circuits.

In developing an operating sequence, it is often possible to use a secondary relay to perform several functions by reoperating it at several points in the sequence. This practice will reduce the number of relays required, but it has a tendency to complicate the circuit since all conditions for operating and holding the relay must be represented in the control paths. An overall improvement is often obtained in these cases by using a larger number of secondary relays controlled by simple contact networks.

In the design of combinational circuits, it is frequently found possible to reduce the required contacts of certain input relays to a single make-contact and thereby eliminate corresponding relays. The same situation exists with the primary relays in sequential circuits. If a sequential circuit has been designed on the basis of a primary relay per input lead, the attempt should always be made to select secondary relay operate and release intervals which will permit elimination of one or more primary relays in this manner.

A more fruitful approach in many cases, perhaps, is to start the circuit design with the idea that no primary relays are provided. The operating sequence diagram is laid out on the basis of single lead inputs and the attempt is made from the start to plan secondary relays that will function directly from input control or from the actions of other secondary relays. Primary relays are added only when necessary. Either of these methods should yield the same or equally satisfactory circuits. The second method merely approaches the desired result of minimum apparatus more directly.

8.6 ILLUSTRATIVE EXAMPLES OF SEQUENTIAL CIRCUIT DESIGN

The design of a sequential circuit to meet stated requirements will be illustrated first by the intercommunication signaling problem which was stated in Section 8.4. This problem involves answer and
disconnect signals to be given at positions "A" and "B" served by the intercommunication system. The circuit will require two primary relays (A) and (B) which operate when keys are operated at the corresponding positions. There are four alternative sequences of events which were illustrated in the diagram of Fig. 8-14. Two conditions occur where the same input combinations must produce different output conditions depending on the sequence of input events: the condition of (A) operated and (B) released must sometimes light the ANS-B lamp and at other times light the DIS-A lamp; likewise, (A) released and (B) operated sometimes lights ANS-A and at other times lights DIS-B.

From a study of the sequences it is seen that the ANS-lamp signals always occur before an interval in which both (A) and (B) are operated and DIS-lamp signals always occur after such an interval. Therefore a single secondary relay which distinguishes between ANS- and DIS-conditions should be sufficient, since the combinations of input signals can indicate the position in which one or the other of the lamps is to light. A secondary relay (X) can be inserted in the sequence as shown in Fig. 8-23. Note that this relay must return to normal in each of the normal intervals in order that one of the alternative sequences will not influence the sequence which follows. From inspection, relay (X) must operate when both (A) and (B) are operated, and hold as long as either (A) or (B) is operated. The circuit is:

\[ f(X) = (A + B)(X + AB) \]

From the combinations existing during intervals 2 and 6 of the sequence diagram, Fig. 8-23, the circuit for the ANS-B lamp is:

\[ f(\text{ANS-B}) = A + B' + X' \]

This circuit will light the ANS-B lamp when (A) operates and extinguish it when (B) operates. The circuit may be simplified by reconsidering the original requirements. It will be entirely satisfactory from an operating standpoint if the ANS-B lamp is not extinguished until the secondary relay (X) operates. Thus the ANS-B lamp can light at any time when (A) is operated and (X) is not operated, or:

\[ f(\text{ANS-B}) = A + X' \]

This circuit does not strictly follow the sequence diagram of Fig. 8-23, since the closure of the ANS-B circuit path in interval 2 will extend into interval 3 to the point where (X) operates. The simplified circuit might have been obtained by including the combination existing in the first part of interval 3 in the symbolic expression as follows:

\[ f(\text{ANS-B}) = (A + B' + X')(A + B + X') = A + X' \]

From an examination of the sequence diagram it will be seen that the circuit path (A + X') will be closed (or the condition of (A) operated and (X) released will exist) momentarily at times other than those
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| SECONDARY RELAY X |    |   |   |   |    |   |   |   |    |    |    |    |      |    |    |    |     |

| ANS - A          |    |   |   |   |    |   |   |   |    |    |    |    |      |    |    |    |     |
| ANS - B          |    |   |   |   |    |   |   |   |    |    |    |    |      |    |    |    |     |

| OUTPUTS |    |   |   |   |    |   |   |   |    |    |    |    |      |    |    |    |     |
| DIS - A |    |   |   |   |    |   |   |   |    |    |    |    |      |    |    |    |     |
| DIS - B |    |   |   |   |    |   |   |   |    |    |    |    |      |    |    |    |     |

* NORMAL INTERVALS

Fig. 8-23 Sequence for Signaling Circuit of Fig. 8-14 with Secondary Relay Shown

specified for lighting the lamp, for example in the first part of interval 11 after (A) operates and before (X) operates. However, if (X) is a fast relay, its operating time will not be sufficient to permit the lamp to light to the point of visibility before the circuit path is opened. Accepting this as satisfactory, similar circuit paths can be developed for the remaining lamps:

\[
\begin{align*}
    f(\text{ANS-A}) &= B + X' \\
    f(\text{DIS-A}) &= B' + X \\
    f(\text{DIS-B}) &= A' + X
\end{align*}
\]

The various paths may be combined in several ways, one of which is shown in the schematic diagram of Fig. 8-24.

A second example will illustrate some of the hazards to the proper action of secondary relays which are sometimes encountered in circuit design. During the development of a complex relay circuit, it may not be worthwhile to consider completely the effects of every temporary path closure that may possibly occur while relays are in the process of operating or releasing. However, an analysis of these temporary closures must be made at the conclusion of design to determine whether or not they will be hazardous to proper circuit operation.

A pulse-frequency divider circuit is required which will receive a series of ground pulses on a single lead and deliver output pulses at half the rate of input pulses. This circuit has many practical applications. For example, it can indicate whether the number of pulses in a series is odd or even, and it may be used as a binary counter. In solving this problem it is necessary first, to determine the number of
secondary relays required; next, select a suitable operating sequence; and then, design the circuits for controlling these relays. Additional circuit paths to perform required output functions can finally be developed by combinational methods.

The requirements of this problem are indicated in the sequence diagram of Fig. 8-25, in which a lead O is shown grounded during and after the odd pulses and a lead E during and after the even pulses. It will be satisfactory for the normal condition before the first pulse to give an even indication. At least two secondary relays are required. Although one relay operating through intervals 2 and 3 will provide the proper output, no control combinations for such a relay are available, since the relay must be operated and released by the same input combination. Designating the two secondary relays (W) and (Z), the sequence

![Sequence Diagram](image_url)

Fig. 8-24 Intercommunication Signaling Circuit
of Fig. 8-26 is the only suitable sequence with the trivial exceptions of changing the starting point of the cycle or interchanging relay designations. From the sequence diagram, control paths for (W) and (Z) can be developed as follows:

Operate Path for (W) = \( P + Z' \)
Hold Path for (W) = \( P' \)
\[ f(W) = (P + Z')(W + P') \]
\[ = PW + P'Z' \]

Operate Path for (Z) = \( P' + W \)
Hold Path for (Z) = \( P \)
\[ f(Z) = (P' + W)(Z + P) \]
\[ = PW + P'Z \]

The term \( PW \) may be made common to both circuits as shown in Fig. 8-27A. This can be further simplified by modifying \( f(W) \) and \( f(Z) \) to
\[ f(W) = PW + (P' + Z)Z' \]
\[ f(Z) = PW + (P' + Z')Z \]
and making \( P' \) a bridging path between \( Z \) and \( W \) as shown in Fig. 8-27B.
In this form, when relay (Z) operates, the path of relay (W) is shifted from \((PW + Z')\) to \((PW + P' + Z)\), so that a continuity transfer is required on Z. Similarly, the shift from the operate to the hold path of relay (Z) on its operation requires the continuity transfer. This configuration is shown in Fig. 8-28.

The hazards of this circuit are due primarily to contact stagger on the (P) relay resulting in time differences in the action of the make and the break contacts of this relay. If the contact adjustment of relay (P) is such that the make P closes before the break P' opens, a hazard will occur when the first pulse operates relay (P) in interval 2. A path will be closed momentarily through \(P + Z' + P'\) which will cause (Z) to start to operate falsely. If the stagger time of the P and P' contacts is sufficient, relay (Z) may succeed in closing its locking contact and operate fully. A second hazard occurs at the beginning of interval 4 when (P) operates on the second pulse. If P' opens before P closes (opposite of first hazard) and if (W) is very fast in releasing, the holding path for relay (Z) through contact W may open before the path through P closes, thus permitting (Z) to release sufficiently to open its locking contact and thereby fully release. This hazard seems safe unless the contact stagger on (P) is excessive. A third hazard condition occurs on the release of the (P) relay following the second pulse when the circuit restores to normal. At this time, if P' closes before P opens (same as first hazard condition), a momentary operating path through \(P + Z + P'\) is closed to the winding of relay (W), which may operate falsely and lock.
Since either sequence of the operation of the P and P' contacts produces a hazard, any solution will be a compromise. The best arrangement in this case is to insure by adjustment or construction of the contact assembly of relay (P) that the break contact P' opens before the make contact closes when the relay operates. This will eliminate the first and last hazards. The second hazard can be overcome by insuring that the release time of relay (W) is sufficient to cover the time between the opening of P' and the closure of P which occurs when relay (P) releases.

![Diagram](image)

**Fig. 8-29 Two-Relay Pulse-Frequency Divider**

A circuit without operating hazards, which is often used as a pulse-frequency divider, is shown in Fig. 8-29. This circuit uses shunt control paths and operates directly from ground pulses without a pulse repeating relay. The acting sequence is the same as in Fig. 8-26, and the operation is as follows: When the first pulse is applied, relay (W) operates through its own break contact and immediately transfers through the make-before-break spring arrangement to a direct locking ground. During the first pulse, the relay (Z) is prevented from operating by the pulse ground applied through its break contact Z' as a shunt. When the first pulse terminates, relay (Z) operates in parallel with (W) which continues to hold to direct ground. When the second pulse starts, ground from the pulse lead through a make on (Z) shunts (W), causing it to release. Relay (W), in releasing, transfers the holding path for (Z) back to the pulse lead. When the second pulse ends, relay (Z) releases, returning the circuit to normal.

In this circuit the hazards are overcome by using, in effect, a single make contact corresponding to the input pulse, and hence difficulties due to contact stagger on the pulse-repeating relay (P) are avoided. Since the opening of the pulse ground must cause (Z) to operate and the closure of this ground cause (W) to release, the use of shunt control paths is indicated. The control paths for (W) and (Z) in Fig.
8-29 may be expressed as follows, where \( f_d \) represents direct control paths and \( f_s \) represents shunt control paths:

\[
\begin{align*}
  f_d (W) &= (P + W') W \\
  f_s (W) &= P + Z \\
  f_d (Z) &= (P + W') W \\
  f_s (Z) &= P + Z'
\end{align*}
\]

The intervals during which these paths are closed are shown in the sequence diagram of Fig. 8-30. The circuit might be developed by recognizing the need for shunt control, and developing the sequence diagram for direct and shunt control paths. The paths can then be developed individually and made properly disjunctive so that only a single contact, \( P \), is required. This example illustrates that considerable ingenuity is often needed in the solution of circuit problems, and that it may be necessary to try several approaches to a given problem before a suitable circuit is found.

8.7 APPENDIX: DEVELOPMENT OF AN OPERATING SEQUENCE FOR A GROUP OF RELAYS

A systematic method of developing suitable operating sequences for a group of relays can be of considerable help to the circuit designer. The criterion for a suitable sequence will be that the change in combinational state at each successive interval of the sequence will be produced by the operation or release of but one element of the group. Hereafter, such sequences will be known as legitimate sequences. If
there is a choice of sequences, the optimum sequence in a particular case is determined by circuit considerations.

The possible states which a single relay can occupy can be represented as in Fig. 8-31. The two ends of the line represent the operated and released condition of the relay, and a change in state is specified by a shift from one end of the line to the other. The concept of sequence has little meaning in this case.

The four possible states of two relays can be represented by the corners of a square as shown in Fig. 8-32. Change from one state to another can take place only along the sides of the square; and, since actual sequence is not affected by interchange of designations, there is only one possible legitimate sequence, shown on Fig. 8-34. The four states of the two relays can also be represented by the line array of points also shown on Fig. 8-32, and the sequence can be determined by motion along the line. The line can be considered as folded back on itself so that a step from the B state to the null state, or vice versa, is permissible.
The eight states of three relays are represented by the corners of a cube, as shown on Fig. 8-33. Motion is restricted to the edges of the cube, and two complete legitimate sequences, indicated on Fig. 8-35, can be developed. The simpler representation of a planar array of points is also shown on Fig. 8-33. In this array, the state of a point is determined by the co-ordinate values. For example, the lower right point represents BC. Again, the array can be considered as folded back on itself so that a step from B to null, or BC to C, or vice versa is allowable.

The geometrical representation of the sixteen states of four relays is a four-dimensional cube whose portrayal will not be attempted here. However, the planar array of points corresponding to this cube is drawn as on Fig. 8-36, and from this any desired sequence can be obtained. The same rules as previously described apply to motion through the array, with the addition that the top and bottom as well as the sides can be considered as folded back for the purpose of indicating allowable steps. This permits motion as shown by the dashed lines of Fig. 8-36, for example. One of the several different legitimate sequences possible with four relays is shown on Fig. 8-37. This was obtained by motion through the array as indicated by the numbered arrows on the figure.
The discussion up till now has been based upon the development of complete cycles. However, the same arrays can obviously be used for obtaining any desired partial cycle, or for developing sequences in which the same state is repeated one or more times.

The expansion from the array for four relays to that for five is illustrated in Fig. 8-38, representative of a five-dimensional cube. The two arrays of Fig. 8-38 must be visualized as stacked one above the other with point E connected to the null point by a line, point EA connected to point A by a line, etc. This is indicated by the dashed lines of the figure. Rules for developing sequences are a natural extension of the previous methods.

A 64-point array showing the states of six relays consists of four 16-point arrays stacked one above the other. The first two are the same as the arrays of Fig. 8-38, and, if the second one is designated the E array, the last two are the EF array and the F array, in that order.

Although large numbers of different sequences for particular purposes can be developed for four, five, six, and more relays, there is a general-purpose sequence which may comprise any number of relays, in which each additional relay can be entered in a fixed, repetitive pattern. This is shown for three relays at the top of Fig. 8-35, and for four relays on Fig. 8-37. The method of extension to additional relays can easily be seen.

**PROBLEMS FOR CHAPTER 8**

8-1  (a) Analyze the sequence chart at the top of page 176 to determine the minimum number of relays which must be added to distinguish between each appearance of input K. Relay (A), (B), and (C) are externally controlled.

(b) Show a circuit to control any added secondary relays using contacts on relays (A), (B), and (C).
8-2 Design a circuit to ground a lead a during interval 7 and a lead b during interval 2, as shown in the sequence diagram below. Relays (A) and (B) are externally-controlled primary relays; a minimum number of single-winding secondary relays may be added if necessary. Avoid continuity-transfers.

8-3 Four relays act in a cycle of twelve (12) intervals as shown in the adjacent table. The unused combinations are assumed invalid and will never occur. Develop circuits on the contacts of these relays as follows:

(a) A circuit closed during intervals 2, 3, 6, and 7. (Eight springs)
(b) A circuit closed during intervals 4, 5, 6, 7, 8, 9, and 10. (Six springs)
(c) The circuit for controlling relay (B) as an internally controlled relay; relays (A), (C), and (D) being externally controlled. (Nine springs)
(d) The circuit for controlling relay (C) as an internally controlled relay; with relays (A), (B), and (D) being externally controlled. (Eleven springs)

8-4 Design a circuit which responds to sequential input ground conditions on leads A, B, and C in the diagram below by grounding lead X or Y as indicated by the diagram. The four cycles shown are alternative. (This can be done with either four or five relays, the four-relay solution being quite difficult.)
8-5 A model railroad track layout consists of three loops A, B, and C and four switches W, X, Y, and Z arranged as shown below. The switches are controlled by electromagnetic solenoids arranged so that the track is switched to the curved spur as long as the solenoids are energized, and restored by a spring to the straight section when the solenoids are de-energized. Three track sections a, b, and c, are each provided with a single make contact which closes while the train passes over it, and which may be used to control relays for actuating the switches. The switch solenoids have no contacts.

Arrange a relay circuit so that a train starting at point p and moving in the direction of the arrow will first make a complete circuit of loop A, switch to loop C, then to loop B, back to loop C again, and then to the starting point, and thereafter repeat this action indefinitely. (This can be done with two relays.)

8-6 A relay circuit is controlled by two keys (A) and (B), each with a single make-contact, one spring of which is grounded. This circuit grounds a lead s under the following conditions: When either key alone is operated, ground is placed on lead s. Upon the release of the key, the ground is removed from s. However, if after one key is operated and s is grounded, if the second key is operated before the first is released, the release of either key will remove ground from the lead s. Design such a circuit with the additional requirement that no relays shall remain operated when both keys are in an unoperated condition. Avoid shunt-down conditions if possible. (This can be done with three relays.)

8-7 In order to protect the power tubes of a particular radio transmitter, the plate voltages must not be applied to the tubes until the cathodes are sufficiently heated. A relay circuit is to be devised which will supply current to all filaments when the transmitter switch is turned on, will supply plate voltage to the driver stage not less than 30 seconds (and not more than 45 seconds) after the filaments are energized, and will apply plate voltage to the power amplifier not less than 45 seconds (and not more than 1 minute) after the filaments are energized. Times need be accurate to no more than one second.

To obtain the necessary timing intervals, a 115-volt a-c synchronous motor timer is supplied which switches a transfer-contact from its back to its front contact for one second every 15 seconds. This timer should be energized only during the sequence described above, and may rest on either the front or back contact when the switch is turned on. Design the required relay circuit. (This can be done with three relays.)

8-8 A circuit controlled by two single-make non-locking keys, A and B, has as outputs a red lamp and a green lamp. The circuit always assumes a normal condition with
no relays operated and no lamps lighted when the two keys are released. Starting from a normal condition, the first key operated, either A or B, lights lamp G. With the first key still operated, operating the second key, either B or A, will light lamp R. Thereafter, as long as either or both keys remain operated, the lamp first associated with a particular key will follow the condition of that key; i.e., it will light when the key is operated and go out when the key is released. (This can be done with two relays.)

A rotating shaft carries a single grounded brush which makes contact with three stationary commutator segments arranged symmetrically around the shaft. A relay circuit is required which will indicate the direction of shaft rotation by lighting one of two lamps, designated CW and CCW, for clockwise and counterclockwise respectively. The shaft may reverse its direction at any time. Assume that the shaft is driven so that a brush contact closure is 0.25 second, and that the open time between the brush leaving one segment and reaching another is 0.25 second. When the shaft changes direction, the output indication must change as quickly as possible, at most within 2 seconds. (This can be done with four relays; to indicate change within one revolution may take six relays.)
Chapter 9

ELEMENTARY SWITCH CONTROL CIRCUITS

Some of the functions which switching devices may be called upon to perform can be efficiently implemented by the aid of rotary and relay-type switches. This is true, for example, of the task of establishing connecting linkages through telephone switching systems - a major application for the crossbar switch, since the ratio of connectable contacts to electromagnetic control parts for this device is high. Similarly, the mechanical construction of a rotary selector switch makes its use as a sequence controller highly attractive where one circuit is to be connected, in turn, to each of a number of other circuits. Switches find many other applications throughout the field of automatic control systems.

As is found in the case of relays, certain basic switch circuits repeatedly make their appearance in the switching art, the particular configuration of circuit depending upon the type of switch employed and the function it is to perform. Switches occur in a wide variety of kinds, each of which demands particular consideration as to the best method of control.

In this chapter only the construction, operation, and control of the so-called rotary selectors and relay-type switches will be discussed. Although many other types of switch mechanisms are extant, the operation and control of these mechanisms should present no difficulties if the rotary selector and relay-type switches and associated control circuits are thoroughly understood.

9.1 ROTARY SELECTOR SWITCHES: MECHANICAL CONSIDERATIONS

Rotary-type selector switches consist, primarily, of arcs of terminals over which associated wipers pass. An electromagnet mounted on the switch assembly provides power to move the wipers from one terminal position to the next, each separate energization and de-energization cycle of the magnet causing the wipers to move one position. There are two basic types of rotary switches: forward-acting or direct driven switches, which step from one terminal to the next terminal on the energization of the magnet; and back-acting or spring driven
switches, that step on the de-energization of the magnet. The control magnet of either type of switch is known as the "step magnet".

When the step magnet of a forward-acting switch is energized, a pawl coupled to the magnet armature is forced against the teeth of a ratchet wheel on the shaft supporting the wipers, causing the shaft to rotate through a small angle, thus moving the wipers from one terminal to the next. A detent engaging the ratchet wheel insures that the wipers remain on the terminal just reached when the magnet is de-energized. In the case of a back-acting switch, energization of the step magnet pulls a pawl away from the ratchet wheel on the wiper shaft, against the force of a spring attached to the frame of the switch. When the magnet is de-energized, the pawl is pulled back by the spring, engaging a tooth on the ratchet wheel and advancing the wipers a single step.

Some rotary switches may be caused to step continuously in the same rotary direction over the same set of terminals, whereas others, after stepping their wipers over the associated arcs, must be returned to a normal position before the wipers can again be moved over the arc terminals. These two types of switches are designated "non-homing" and "homing", respectively. Switches of the homing type are normally equipped with a second magnet, a release magnet, which removes a restraining detent from the ratchet wheel and allows a spring to restore the wipers to the starting position.

Wipers may be either of two types: bridging, in which adjacent arc terminals are short-circuited by the wiper as it steps from one to
the other; and non-bridging, in which the wiper leaves one terminal before it makes contact with the next.

Contact springs are often provided on the step magnet to indicate that the magnet is fully energized. Also, for those switches which must be returned to a normal position, off-normal contacts are employed to indicate when the switch is not in the starting position. These contacts are in their normal or unoperated state only when the switch wiper assembly is in a so-called "home" position.

Illustrated in Fig. 9-1 is a back-acting rotary switch consisting of six arcs of 22 terminals each. The wipers are double-ended so that, when one wiper end has passed over the half-circle of 22 terminals, the other end is in position to start stepping over the same 22 terminals. Occasionally, single-ended wipers mounted in pairs staggered 180 degrees apart are utilized on a similar switch so that two adjacent arcs of 22 terminals may be employed as a continuous bank of 44 terminals. The stepping magnet, on the right-hand side of the picture, is equipped with a break contact. The switch may be driven by external circuit pulses at a rate of up to 25 or 30 steps per second, and a rate of 50 or 60 steps per second may be realized if the switch runs under self-interrupted control as discussed later in the chapter. The resistance of the step magnet winding is normally low (200 ohms or less) in order to meet mechanical power and speed requirements.

Circuit drawing conventions for such a switch are pictured in Fig. 9-2. The bridging and non-bridging wipers are indicated by BDG and NBG respectively, and the numbers in the center of the wiper circles are the arc numbers. It is seen that, in actuality, there is a twenty-third terminal on the 22-position arc which is permanently connected to the associated wiper. This terminal merely serves as a means for joining external wiring to the wiper.

![Fig. 9-2 Drawing Conventions for 22-Point Rotary Selector Switch](image)

A forward-acting ten-terminal two-arc switch is illustrated in Fig. 9-3. In addition to the stepping magnet, this switch is furnished with a release magnet, shown in the lower left corner, and off-normal springs, in the upper right corner. In the normal position, the wipers stand in the position just preceding the first terminal. The switch may be driven at speeds up to 25 steps per second. As in the previously
described switch, the winding resistance of each control magnet is generally low.

As a wiper passes over its arc, a second wiper connected electrically and mechanically to the first moves along a copper strip placed beneath the arc. This copper strip provides an electrical path to the first wiper and, for this purpose, is equipped with a wiring lug. Fig. 9-4 shows drawing conventions for this switch.

In the text, both types of switches will be shown by a somewhat simpler schematic symbol, illustrated by Fig. 9-9. Unless a homing switch is specified, it should be understood that the wiper can step directly from one end terminal of the arc to the other.
9.2 CONTROL OF ROTARY SELECTOR SWITCHES

As discussed above, rotary selector switches are advanced one step for each cycle of energization and de-energization of a stepping magnet. The circuit which insures that the switch takes one and only one step per external control action is termed a "stepping" circuit.

Switching mechanisms are frequently required to step continuously, repeatedly recycling, when only a single external condition is present. This type of switch operation is known as "running" and the controlling circuit is then a running circuit. If a separate circuit action is employed to stop the running process at a specific switch position, the circuit providing the means for stopping the switch is referred to as a "halting" circuit.

Finally, it is often necessary, after a certain sequence of switch operations, to return the wipers of that switch to a starting or normal position. This is accomplished by a "restore-to-normal" circuit which may either step the switch to the required position or operate a release magnet provided on the switch.

Any one of these control circuits is composed of one or more of the following elements: relays or other control devices external to the switch; contacts activated by the stepping magnets of the switch; and terminals of the switch arc and the associated wiper. The sections to follow indicate a number of methods by which rotary switches may be controlled in the stepping, running, halting, and restoring functions.

9.4 SWITCH STEPPING CIRCUITS

The most convenient method of stepping a rotary switch from one terminal to the next is that of directly energizing the step magnet until the pawl has reached its fully operated position and then de-energizing the magnet, allowing the pawl to return to rest. This may be accomplished by such a circuit as that shown in Fig. 9-5, a direct stepping process in which the step magnet is energized by each closure of the make-contact on relay \( P \) in the external circuit. If the switch is of the forward-acting type, it will step at the beginning of the ground pulse; whereas a back-acting switch will step after the step magnet is de-energized.
It may occur that the ground pulse applied by the external circuit to the switch is not of sufficient duration to operate the step magnet fully, and hence, might fail to step the switch. In this case it is necessary to lengthen the energizing pulse. This is effected by such an arrangement as the circuit of Fig. 9-6. The input ground pulse operates relay (A) which locks to the break-contact on the step magnet. Ground is applied to the step magnet until relay (A) has released. The adjustment of the contact on the step magnet influences the duration of the magnet energization in that the contact does not open until the pawl has been moved the required distance.

Conversely, the actuating ground pulse may be applied by the external circuit to the step magnet for a considerable time. As mentioned above, switch magnets are in general of low resistance to facilitate rapid stepping, and therefore should not be energized for a time greater than is necessary to permit the switch to step. One method by which ground may be removed after the switch has stepped is illustrated in Fig. 9-7. Here, ground is applied to the switch only during the operate time of the slow-operating (A) relay. Note that the switch steps either when relay (P) operates or when relay (A) operates, depending upon whether the switch is forward- or back-acting. Relay (A) may be replaced by any timing device which performs an identical function.
A forward-acting switch may be made to advance in a manner similar to a back-acting switch, the wipers remaining fixed in position until the termination of the input pulse. A circuit making this possible is shown in Fig. 9-8. The pulsing relay (P) of the external circuit operates slow-release relay (A). When (P) releases, (A) remains operated for an appreciable time because of its release characteristics, closing the path to the step magnet. The forward-acting switch steps before (A) removes ground from the magnet.
In a given application, it may be necessary to step the wipers of a rotary switch from a particular position to the next. This is accomplished by applying ground to an arc terminal in that position, and connecting the step magnet to the wiper of the associated arc. The actual circuit used depends upon the type of switch employed; Figs. 9-9A and 9-9B illustrate arrangements for forward- and back-acting switches. A break-contact is utilized on the step magnet for the back-acting switch to allow the switch to step while relay (P) is still operated. If the switch is in proper adjustment, this break does not open until the magnet is sufficiently energized to operate the pawl satisfactorily. It is necessary in the forward-acting arrangement that the detent pass over the ratchet tooth before the wiper moves completely off the arc terminal which establishes the stepping magnet operate path. Switches are designed to meet these requirements.

![Diagram](image_url)

**Fig. 9-10 Direct Running, Self-Interrupted Circuit**

### 9.5 SWITCH RUNNING CIRCUITS

In running, it is necessary for the switch step magnet to be alternately energized and de-energized for as long as an external ground signal is applied to the input of the circuit or until the switch is halted. The most frequently used running arrangement is the "self-interrupted" type of circuit in which contacts on the stepping magnet structure are employed in the operating path. These contacts, which close or open on each activation of the magnet, are known as "interrupter" contacts since, in a self-interrupter circuit, they are used to interrupt the flow of current to the magnet. It is normally assumed that these contacts will act only when the pawl and detent are in the proper mechanical relation to the ratchet wheel to insure complete operation of the stepping mechanism. However, such factors as inertia of the armature assembly may permit the contact to be effective in a self-interrupted arrangement even though the mechanical relationship is not perfect.

The basic self-interrupted running circuit, illustrated in Fig. 9-10, uses a break contact in series with the magnet winding. When the magnet has been energized to an extent sufficient to actuate the switch, the energizing path is opened and the pawl assembly returns to normal,
completing the cycle by reclosing the interrupter contact. This arrangement is applicable to both back- and forward-acting switches. In such a direct running circuit, some switches may attain a speed of 50 or 60 steps per second since full energization and de-energization of the magnet for each step is not necessary.

Such rapid running may be undesirable, particularly if the switch is to perform functions at certain positions during the running process. If an additional relay is included in the circuit, the acting time of this relay can be added to that of the step magnet for each stepping cycle. To illustrate, two circuits are shown in Fig. 9-11, one employing a make interrupter contact and the second, a break. In Fig. 9-11A, the step magnet operating path is not opened immediately after the magnet has been energized, but instead, remains closed during the operating time of relay (A). After the operating path is opened, it is not again closed until (A) has released. If the spring load of relay (A) is light, its release time will ordinarily be greater than its operate time; and as a result the switch, either back- or forward-acting, is de-energized for a greater duration than it is energized.

Fig. 9-11 Two Examples of Slow-Running Circuits
If only a break interrupter contact is available, an arrangement similar to that of Fig. 9-11B is employed. Here, the period of magnet energization is longer than the period of de-energization. This may be a disadvantage in the case of a back-acting switch since, with such a type of switch, arc-terminal conditions are tested during the de-energization period. If, for example, a condition on a particular terminal is to inform the back-acting switch to cease running when that terminal is reached, information that the operating path is to be opened must be transmitted, and the opening of the path must occur, all during the de-energization period which follows after the switch has reached that particular terminal. To increase this de-energization period, the relay (B) in Fig. 9-11B can be chosen to be somewhat slow in operating; the circuit in Fig. 9-11A can also be made slower running by selecting a slow-release relay for (A). It is normally of advantage, however, to permit the switch to run at as great a speed as is consistent with the requirements of the circuit.

An interrupter contact is not always available on the stepping magnet for running purposes, precluding the use of a self-interrupting circuit. In such instances, an interrupter circuit or pulse generator must be employed, as in Fig. 9-12A. The pulse generator may be a

Fig. 9-12 Running with Pulse Generator Control
motor-driven interrupter, a gas-tube pulsing circuit, or any other convenient pulsing arrangement. A switch controlled by a relay pulse generator is illustrated in Fig. 9-12B. The slow-acting characteristics of the relays (A) and (B) limit the speed at which the switch runs. As in the circuits of Figs. 9-11A and 9-11B, the release of relay (R) will immediately remove ground from the step magnet, preventing further running. Such direct control is desirable in that the progression of the wipers can be halted in the shortest possible time.

![Diagram](image_url)

**Fig. 9-13 Direct Halting by Absence of Ground**

### 9.5 SWITCH HALTING CIRCUITS

The presence of a running circuit in the control of a rotary switch ordinarily implies that at some point the switch must cease to run. If there is included a method of halting the switch other than merely removing the input condition causing running, this arrangement assumes the form of a halting circuit.

In halting circuits, means are provided for stopping the progression of the switch in some position as indicated by the presence or absence of a potential (battery or ground) condition on an arc terminal. In all the examples to follow, the switches run to a terminal marked by the presence or absence of direct ground, it being understood that battery or some marginal condition may be substituted for the ground condition by appropriate circuit design.

One of the most useful of halting arrangements consists of a self-interrupted running circuit with the running ground passing through an arc and wiper of the switch under control. In the position or positions at which the switch is to be halted, the arc terminal is not connected to the external circuit ground. This basic circuit is illustrated in Fig. 9-13. The arc terminals are protected by the use of a bridging-type wiper. In this figure, as well as in some of the others of this chapter, the switch arc is shown connected to ground and the wiper to the step-magnet circuit. However, if it seems desirable, these connections may be reversed. The current through the switch arc can be reduced by the
addition of a repeating relay placed between the wiper and the interrupter contact, with ground being applied to the step-magnet circuit through a make-contact on this added relay. Used with a bridging wiper, this relay remains operated until the desired position is reached. The relay must be fast in releasing when the ungrounded terminal is found to prevent the step magnet from remaining energized for an additional step.

With the circuit of Fig. 9-13 it is necessary to employ an additional switch arc and wiper to detect that the switch has been halted. This may be undesirable if all arcs are otherwise used to capacity; in such a case it would be convenient to provide means in the halting circuit itself to indicate switch stoppage.

A halt-indicating circuit for use with a forward-acting switch is shown in Fig. 9-14A. Relay (S) is marginal in that it will release when connected in series with relay (H). The (S) relay is a part of a running circuit similar to that of Fig. 9-11B. Relay (H) is locked down until the control wiper reaches the ungrounded terminal, at which time it operates, removing the running ground from (S) and placing ground on lead a as an indication that the switch is halted. If a non-bridging wiper is employed, the dashed wiring should be incorporated to insure that (H)
will remain unoperated as the wiper passes between grounded terminals. A back-acting switch with bridging wiper may be substituted in Fig. 9-14A. The dashed wiring is of no assistance in this case.

In all the running circuits mentioned so far, the operation of a relay (R), external to the running circuit, has provided the running ground. In the circuit of Fig. 9-14B, relay (R) must be operated and released before running can commence. Upon operation of (R), relay (A) operates and locks to the break interrupter contact of the switch. When (R) is released, ground is applied to the step magnet which runs in self-interrupted fashion until relay (A) releases. Relay (A) can release only after the step magnet has been energized, the interrupter contact has opened, and the wiper has reached an ungrounded terminal. Thus the forward-acting switch will halt on the ungrounded terminal, whereas the back-acting switch will step to the next adjacent terminal. In the circuit, the wiper need not be of the bridging type since the locking path of relay (A) through the interrupter contact covers the interval during the wiper step movement. Relay (A), however, must be fast in releasing when the ungrounded terminal is reached since the interrupter contact in a rapidly self-interrupted circuit does not long remain open.

![Fig. 9-15 Basic Halting by Presence of Ground](image)

It is often more appropriate to halt a running switch on a grounded terminal since the running circuit can then be opened by the operation of a control relay rather than by its release. This is desirable since the control relay with its small contact-spring load is usually more rapid in operation than in release. Fig. 9-15 shows a self-interrupted running circuit controlled by fast-operating relay (A). This relay acts when the marked terminal is reached, halting the switch and placing an indicating ground on lead a. This is a widely-used circuit, both as shown and also in a form where the two connections to the arc and wiper of the switch in Fig. 9-15 are reversed.
Fig. 9-16 Halting by Presence of Ground: Shunt Operation

Fig. 9-17 Basic Restoring Circuit: Homing-Type Switch

Fig. 9-18 Restoring Circuit: Homing-Type Switch

Fig. 9-19 Arc-Controlled Homing Switch

Fig. 9-20 Off-Normal Contact Controlled Restoring Circuit: Non-Homing Type Switch
The control relay is not vital to a ground-halted circuit, as is illustrated by Fig. 9-16. Here, the step magnet is shunted down when a grounded terminal is reached, preventing further stepping. This circuit gives no output indication of switch stoppage; moreover, considerable current is drawn through the arc and the protective resistance.

9.7 SWITCH RESTORING CIRCUITS

When it is desired to restore a homing switch, a path, usually through an off-normal contact on the switch, is closed to its release magnet. The release magnet is thus energized and held energized until the switch has been restored to normal. A circuit effecting this is shown in Fig. 9-17. Since the restoring ground is supplied through the off-normal contact, the release magnet draws current only until the switch is restored.

If the input ground from the external circuit is of insufficient duration to insure that the switch is fully restored, a circuit similar to that of Fig. 9-18 must be employed. Otherwise the released detent may damage the ratchet wheel during the restoring movement of the switch mechanism. Care must also be taken that the step magnet is not energized during switch restoration.

In Fig. 9-19, the switch is returned to normal automatically when the wiper has reached its final position on the arc. Means can be provided by the circuit shown in dotted lines for restoring the switch to its home position at any time desired.

Rotary switches which do not have a mechanical normal position frequently require, for circuit reasons, the designation of a particular terminal as "home". To reach this terminal, the switch is run through its arc and halted at the home position by some method similar to those already discussed. The circuit of Fig. 9-13 is usually preferable as a result of its speed of action.

If a non-homing switch is equipped with off-normal springs, the off-normal contact may be used in restoring the circuit to normal. As in Fig. 9-20, the switch runs in a self-interrupted manner until the off-normal contact opens.

Occasionally, a set of "off limits" contacts are provided to indicate that the switch has reached the limiting position on its arc and must be returned to normal before the circuit can again function. When these contacts are actuated, the switch may be restored in a circuit similar to that of Fig. 9-19, in which the wiper and grounded arc terminal are replaced by an off-limits make-contact.
9.7 CROSSBAR SWITCHES: MECHANICAL CONSIDERATIONS

The crossbar switch, shown in front view in Fig. 9-21, comprises a welded rectangular frame on which are mounted a number of vertical relay-like units and five horizontal bars turned by so-called selecting magnets. Each vertical unit has a "holding" magnet and ten sets of contacts in a vertical row. The holding magnet, or, briefly, hold magnet, actuates a vertical armature which is adjacent to the ten vertical sets of contacts. Each contact set, known as a crosspoint, consists of from three to six make-contacts, all sets of a given unit usually containing the same number. The internal structure of a vertical unit connects the fixed spring of each make-contact to the corresponding springs of the other sets on the same vertical unit, thus producing a permanent vertical multiple. The movable springs are brought out individually, but may be wired externally to form a horizontal multiple. Thus, a fully multipled switch would comprise a group of vertical units each connectable to any one of ten horizontal levels. A co-ordinate representation of this is shown on Fig. 9-22.

![Crossbar Switch, Front View](image)

![Crossbar Switch, Co-Ordinate Representation](image)
The horizontal select bars are rotatable in either of two directions by two select magnets associated with each. At each vertical unit location there is, on the horizontal bar, a flexible selecting finger which, when the bar is rotated by the energization of one of its two select magnets, interposes itself between the armature of the vertical unit and the associated set of contacts. This is shown on Fig. 9-23. If a hold magnet operates after the select finger is in this position, the set of contacts at the crosspoint so selected is closed. Thus, these cross-point contacts can be operated independently of each other by a coordinate action of the horizontal bars and vertical armatures. The select magnet, when de-energized, restores the horizontal bar to its neutral position, but the operated crosspoint holds its associated selecting finger. If a select magnet is energized after a hold magnet is operated, the contacts at the selected crosspoint will not close since prior operation of a hold magnet prevents the inter-position of a select finger into any crosspoint in the associated vertical unit. There are, in general, two sizes of crossbar switches; one with 10 hold magnets and 100 crosspoints, the second with 20 hold magnets and 200 crosspoints. Other variations with respect to the number of holding magnets are also in use.

![Diagram of Crossbar Switch](image)

**Fig. 9-23 Crossbar Switch: Crosspoint Detail**

On inspection of Fig. 9-23, the details of a crosspoint become more evident. The movable contact springs are bifurcated and the contacts themselves are of precious metal, thus assuring good contact characteristics through the switch. The number of make-contacts at each crosspoint is frequently used in describing a switch. For example, a switch with three make-contacts per crosspoint is referred to as a three-wire switch.
The various select and hold magnets themselves may be equipped with off-normal spring combinations. These are operated directly by the associated magnet irrespective of which crosspoint is closed. Wiring lugs for the movable springs of the crosspoints are arranged for individual wiring, as illustrated in Fig. 9-24 which shows the wiring terminals for two crosspoints. The terminals are provided with staggered notched projections to facilitate horizontal strapping of corresponding terminals of horizontal groups of crosspoints. Fig. 9-25 shows the circuit diagram conventions used for the crossbar switch.

9.8 CONTROL OF THE CROSSBAR SWITCH

The control of the crossbar switch is very similar to that of ordinary relays except that mechanical considerations dictate certain rules which must be followed in controlling the switch. Because of the construction of the switch, once a crosspoint has been closed and is held by its associated hold magnet, no other crosspoints in the same vertical unit can be actuated. More than one crosspoint in the same vertical unit may be closed only if the corresponding select magnets are operated before energizing the hold magnet. In this connection, it must be remembered that there are two select magnets for each horizontal bar and that only one of these should be energized at a time.

The operation or release of the horizontal bars results in transient vibration of the select fingers. For this reason, time must be allowed in the circuit operation for the fingers to become reasonably stationary before attempting to operate a hold magnet, in order to insure against either missing a finger or "snagging" an extra finger when
the hold magnet operates. Since each finger of a select bar, held by a vertical armature, exerts further tension on the bar, a limit of nine select fingers held on any horizontal level may be prescribed.

The hold magnets of a crossbar switch are usually energized for a considerably greater time than the select magnets, since they hold the established connection. For this reason, their windings are ordinarily of high resistance to conserve power. The operating time for a crosspoint, measured from the instant the circuit to the hold magnet is closed until the last contact of the crosspoint operates, varies from about 0.025 second to 0.060 second, depending upon the number of contacts at the crosspoint and upon the off-normal spring combination.

Select magnets, to be rapid in operation, are ordinarily provided with 600-ohm coils, though smaller values may be used. They are generally energized only for short periods of time. Time from the closure of the select magnet circuit until the hold magnet may be safely energized ranges from about 0.010 to 0.040 second.

To provide the necessary delay between the operation of a select magnet and that of the hold magnet, any convenient timing arrangement may be used. Select magnet off-normal contacts may also be employed in the control of the hold magnet. Very often, in large circuits using crossbar switches, it is possible to utilize the inherent time delays resulting from the performance of other functions to guarantee the necessary time interval, rather than providing specific relays for the purpose.

One of the principal advantages of the crossbar switch is that it is inherently more rapid in operation than the rotary-type switch. To disconnect an input from one output terminal and reconnect it to another,
the rotary-type switch must move in a progressive manner over all intervening terminals. In accomplishing an identical task, the relay circuit associated with the crossbar switch can control the disconnection and re-connection, one immediately following the other.

Another factor contributing to the speed of crossbar switch circuits is that no connection through the switch exists until a hold magnet is operated. This allows the select magnets to be operated by the controlling circuits at some time before the connection is to be made. The select magnets of the switch may follow the actions of control circuits with no effect on their associated crosspoints until such time as a hold magnet is energized.

Although, generally, only one select magnet on a switch is operated to prepare for a connection, in certain applications two crosspoints must be closed to provide a path through the switch, thus usually necessitating the energization of two select magnets. An example of such a case is discussed in the following paragraph.

Fig. 9-26 Splitting Crossbar Switch to Increase the Number of Outputs
In normal operation the vertical unit of a crossbar switch is equipped with a single input with access to any one of ten outputs. By holding the number of leads in each output to three or less, the number of outputs may be increased. This can be done, for example, with a six-wire switch. For this purpose the vertical unit of six vertical multiple straps is divided into two groups of three straps each, as illustrated in Fig. 9-26, thus effectively providing the unit with twenty crosspoints, each consisting of three make-contacts. The vertical unit is equipped with one three-wire input which is connected to the "0" horizontal level that is associated with the first group of vertical multiples, and is in addition connected to the "1" level associated with the second group of vertical multiples. The crosspoints of the "1" horizontal level of the first group and the "0" level of the second group are left vacant. The remaining sixteen crosspoints of the unit each provide an output connection as shown in the figure. The outputs can be multiplied across all vertical units. By this device, each unit has been altered to give sixteen outputs rather than the standard ten at the expense of reducing to three the number of leads to be connected. For any connection, two select magnets must be operated as well as a hold magnet. To illustrate, if input leads "0" are to be connected to output leads "12", select magnets (1) and (6), and hold magnet (0), are energized. The particular hold magnet to be operated depends on the input leads, and the select magnets depend on the output desired. It should be remembered that the crossbar switch has no directional sense, so that the terms input and output can be interchanged in the above example.

Since the design of a crossbar switch control circuit depends primarily upon the function which the switch is to perform and its relation to other associated circuits, detailed discussion of specific control arrangements is not included at this juncture. However, no great difficulty in the understanding of such circuits should be experienced because of the previously mentioned similarity of the crossbar switch to a relay arrangement.

PROBLEMS FOR CHAPTER 9

9-1 A circuit is required to count up to 100 pulses from a key K with a single make-contact, giving a continuous output indication by grounding one of 100 leads. The circuit is to employ a 22-point, back-acting, non-homing rotary switch and need not be recycling. The circuit should be active only when an ON-OFF switch is turned ON, and should restore to normal at any time when the switch is turned OFF. (This can be done with four relays in addition to the switch.)

9-2 A 10-point forward-acting homing-type rotary switch is to be used in an application in which it is necessary that the wipers normally rest on the No. 5 terminal. When a non-locking key K is operated momentarily, the wipers run through terminals 6,
The Design of Switching Circuits

7, 8, 9, 10, 1, 2, 3, and 4, in order, and halt at terminal 5. On alternate rounds, the output leads connected to terminals 6, 7, 8, 9, and 10 are to be grounded in reverse order: that is, for odd rounds the order of grounding output leads is 6, 7, 8, 9, 10, 1, 2, 3, 4 and for even rounds, the order is 10, 9, 8, 7, 6, 1, 2, 3, 4.

The switch is furnished with a step magnet and a release magnet, neither equipped with contacts. However, switch off-normal springs may be specified. Two terminal arcs are available on the switch.

Design a suitable circuit. (This can be done with five relays in addition to the switch.

9-3 (a) An 11-point three-arc back-acting rotary switch is used as a translating device. A ten-button keyset grounds two leads out of five designated 0, 1, 2, 4, 7, in an additive two-out-of-five code as shown below. A sixth lead from the keyset is grounded to indicate that a key is depressed. One of the switch positions is designated as the home or normal position. There is no release magnet; the switch is of the non-homing type. Design the switch circuit to light one of ten (10) lamps indicative of the key which is depressed. Arrange the circuit to return the switch to the normal position after any depressed key is released. Allow only the desired lamp to light during the circuit operation. (This can be done with three relays.)


<table>
<thead>
<tr>
<th>Key Operated</th>
<th>Leads Grounded</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0, 1</td>
</tr>
<tr>
<td>2</td>
<td>0, 2</td>
</tr>
<tr>
<td>3</td>
<td>1, 2</td>
</tr>
<tr>
<td>4</td>
<td>0, 4</td>
</tr>
<tr>
<td>5</td>
<td>1, 4</td>
</tr>
<tr>
<td>6</td>
<td>2, 4</td>
</tr>
<tr>
<td>7</td>
<td>0, 7</td>
</tr>
<tr>
<td>8</td>
<td>1, 7</td>
</tr>
<tr>
<td>9</td>
<td>2, 7</td>
</tr>
<tr>
<td>10</td>
<td>4, 7</td>
</tr>
</tbody>
</table>

(b) Using a six-arc switch in place of the three-arc switch, arrange the circuit of part (a) to check for exactly two-out-of-five leads grounded without employing a 2/5 or 3/5 symmetric contact configuration on any of the relays. An incorrect indication should cause an alarm signal to lock in and prevent any numerical lamp indication. (This can be done with one additional relay.)

9-4 When an ON-OFF switch is turned ON, a back-acting 22-point non-homing rotary switch is to follow the operation of a key K, equipped with a single make-contact. The switch is to step on each depression of the key and again on each release of the key. If the key is held operated for an interval longer than approximately 1/4 second, the switch is to restore to its normal position and to ignore the subsequent release of the key. After the switch has restored to normal, and after this final release of the key, the switch must again be free to act as specified above.

Design the required circuit. (This can be done with two relays.)

9-5 A radio receiving system is to be installed. There are to be ten loudspeakers distributed throughout the premises, but only one of these is to be connected to the central receiving unit at any one time. A 22-point rotary switch is to be used as the connecting device, cutting through the two output leads from the receiver to the desired speaker.

Characteristics of Apparatus:

- Speaker Impedance: 500 ohms
- Receiver Output Impedance: 500 ohms
- Rotary Switch: 22-point, back-acting, non-homing.
  - Three terminal arcs.
  - One break-contact on step magnet.
- Selector Keys: Non-locking, two make-contacts per key.
The particular speaker to be driven is selected by operating one of ten non-locking keys. The connection thus established to this speaker is maintained until some other key is operated. During the acting time of the switch no momentary connection should be made to any speaker. However, the load impedance presented to the receiver during this acting time should be the same as that provided when a connection is established. The switch and its control relays should draw no current except when establishing a new connection.

(This problem can be done with two relays.)

9-6 Design a control arrangement for a select and hold magnet of a crossbar switch to meet the following requirements:

(a) Action is initiated by closure of a single make contact which remains closed only long enough for the select magnet to operate fully.

(b) The circuit shall make sure that the select magnet has operated before starting timing for hold magnet operation.

(c) The operate time of three relays shall intervene between select and hold magnet operation.

(d) The circuit shall make sure that the hold magnet has locked operated to a separate ground before releasing the select magnet and restoring to normal.

(e) The hold and select magnets are each furnished with a single off-normal make-contact.

(This problem can be done with three relays.)
Chapter 10

ELEMENTARY ELECTRONIC SWITCHING CIRCUITS

Electronic tubes and the semi-conductor elements, varistors and transistors, can be used as two-valued devices and hence are suitable for performing switching functions. The various electronic tubes, both vacuum and gas-filled, are essentially voltage-sensitive elements capable of being driven to a conducting or non-conducting state by control signals at a low energy level. Varistors and transistors, on the other hand, are primarily current-sensitive elements again functioning at low energy levels. Varistors offer either a low or a high resistance to current flow, depending upon voltage polarity, while transistors incorporate current amplification. Comparatively speaking, vacuum and gas tubes operate at higher voltage levels, and the semi-conductors at lower voltage levels.

The chief advantages of electronic switching devices are high speed and sensitivity. Offsetting these factors, at least at the present time, are lower life expectancy and reliability, and probably less flexibility as circuit elements, as compared to relays. Also important in a negative sense is the relatively heavy power drain required by hot-cathode tubes.

The design of electronic switching circuits is a young and dynamic art, subject to almost daily change and development, and advancing on many fronts. For this reason, no comprehensive treatment of the subject will be attempted in this volume. Rather, this chapter will discuss the basic characteristics of electronic apparatus that make it suitable for switching, together with some of the fundamental switching circuit arrangements. Examples of typical functional electronic circuits will be presented in later chapters. The theory of gas discharge, electron flow, and other physical phenomena related to the science of electron tubes and semi-conductors has been omitted since it is assumed that the reader is familiar with such concepts.

It will be seen that electronic circuit design follows two main trends. One is to take advantage of specific characteristics of apparatus to perform particular switching functions. For example, gas tubes with multiple elements arranged in a certain geometrical relationship have been designed to count pulses. The other trend is to utilize electronic
elements in the same sense that relays are used, that is, to interconnect the elements in patterns that satisfy the "and" - "or" logical requirements of the problem. Both these methods have validity and enhance the flexibility of tubes and semi-conductors as switching devices.

10.1 CHARACTERISTICS OF ELECTRON TUBES AND SEMI-CONDUCTORS AS SWITCHING ELEMENTS

Hot-Cathode Vacuum Tubes. The construction and characteristics of the many hot-cathode vacuum tubes in general use are well known. The output of such tubes as used in switching applications usually consists of a voltage condition on either anode or cathode, or of a current flow in the cathode-anode circuit, with the tube in either case fully conducting or completely cutoff. This output depends upon potentials on one or more control grids which provide the means for setting up a particular output condition as a result of one or more input conditions. Two separate output leads, one from the anode and one from the cathode, can be furnished to external circuits; and, when the tube is non-conducting, the two output leads are isolated. The vacuum tube may be used for impedance transformation, as in the cathode follower circuit for which the input impedance is high and the output impedance is low.

Vacuum tubes are extremely sensitive and very rapid in response to control signals, their speed being dependent upon the small inter-electrode capacitances and the electron transit time from cathode to anode. Unfortunately for some possible switching applications, it is difficult to "lock operated" a single vacuum tube in the manner of a relay*. In general, the control grid potentials must be maintained even after the tube has been operated. Also, the vacuum tube has the disadvantage of requiring either standby filament current or sufficient time to heat the cathode adequately before the tube is called upon to operate.

Hot-Cathode Gas Tubes. There are three types of hot-cathode gas tubes available for general use: the diode, the three-element thyratron, and the screen-grid (four-element) thyratron. The diode, normally employed as a non-linear unit capable of conduction in only one direction, rests in a non-conducting, or de-ionized, state until the anode-to-cathode voltage is raised to approximately +15 volts. At this breakdown potential, the tube ionizes and remains ionized until the anode-to-cathode voltage is reduced below +15 volts. Thus, the sustaining voltage for the diode is approximately equal to the breakdown potential. In the ionized state, the voltage drop from anode to cathode of the tube is substantially constant over a wide range of current values.

* A non-conducting tube may be referred to as unoperated, and a tube passing current as operated.
The three-element thyatron may be considered as a gas-tube diode with an added control grid. When the voltage on this control grid is sufficiently low with respect to the cathode, negligible current conduction occurs from anode to cathode. When, however, the grid voltage is raised to a point more positive than the so-called cutoff value, the gas ionizes and conduction commences. After tube ionization, the grid loses control over the discharge, and the anode-to-cathode voltage drop is maintained at a sustaining value of about +15 volts. The tube is extinguished by reducing the anode-to-cathode voltage to zero for sufficient time to de-ionize the gas. The grid cutoff value (the value of grid voltage just negative of that at which the tube is permitted to fire) depends upon the potential of the anode. An increase in anode voltage increases in a negative direction the voltage necessary to keep the tube cut off.

Two varieties of three-element hot-cathode gas tube are available: the negative-grid and the positive-grid thyatrons. These designations refer to the cutoff characteristics of the tubes; the grid cutoff voltage for the negative-grid tube is of the order of two to ten volts negative, and the cutoff voltage for the positive-grid tube is about nine volts positive in a typical tube.

The four-element thyatron is similar to the triode with the addition of a screen grid. This screen grid tends to increase the control grid impedance by reducing the current drawn by the control grid when that grid is near the cutoff point. In addition, the screen grid provides a means of control of the ionizing potential of the tube.

In accordance with the characteristics of gaseous discharge, large currents may be passed through the tube with low voltage drop. In this connection it should be noted that the life of a gas tube is a function of the time during which it conducts as well as of the magnitude of the conducted current. The useful life, then, may be increased by allowing only low values of current to be passed through the tube; or, if high current is necessary, by permitting it to flow only momentarily. The hot-cathode gas tube, like the vacuum tube, requires standby filament current.

The speed of response of the tube is contingent primarily on the ionization and de-ionization times of the tube. Depending upon the gas, the ionization time ranges from a fraction of a microsecond to several microseconds; the de-ionization time is ordinarily of the order of a hundred to a thousand microseconds, though lower values have been achieved. The tube, then, can respond very rapidly to input signals applied to operate the tube, but considerably more time must be allowed for extinguishing the tube.
Cold-Cathode Gas Tubes. At present, the cold-cathode tube is the most attractive type of electron tube from the point of view of switching applications where extreme speed is not necessary. Although two-element and three-element cold-cathode tubes are most common, many other types with a greater number of elements have been constructed for special purposes. Certain of these more intricate tubes are illustrated in schematic form, together with the cold-cathode triode, in Fig. 10-1. Those tubes with multiple main anodes provide for several outputs which are mutually isolated until the tubes fire; those with multiple starter anodes allow the tubes to be ionized by any one of several isolated input conditions.

Although the theory of gaseous discharge in the cold-cathode tube is fundamentally different from that in the hot-cathode gas tube, the two types of tube are somewhat similar in operating characteristics. As in the hot-cathode tube, sufficient potential difference between two electrodes of the cold-cathode tube causes the tube to ionize and the anode-to-cathode gap to conduct. A tube remains ionized until the potential from anode to cathode is reduced below the sustaining value.

The firing and sustaining voltage values for the cold-cathode tubes differ from those encountered in the hot-cathode gas tubes. A typical cold-cathode diode ionizes at an anode-to-cathode potential of about +70 volts, and remains ionized until this potential is reduced below +60 volts. The three-element cold-cathode tube employs a starter...
anode (rather than a control grid as in the corresponding hot-cathode tube) which must be raised to a potential of the order of about +70 volts to fire the tube. In firing this tube, the starter-anode-to-cathode gap is ionized, and the discharge is transferred to the main-anode-to-cathode gap. To effect this transfer, the main anode potential must be above the sustaining value of the main gap but not above the self-ionizing value. The sustaining potential of the main gap is normally of the order of 75 volts, and the self-ionizing potential varies from about 150 to 250 volts. After the main-anode-to-cathode gap is ionized, the starter anode has no further effect on conduction until the tube is extinguished.

Such properties as high sensitivity, low conduction impedance, multiple outputs, and multi-control pertain to the cold-cathode tube. Moreover, since the cold-cathode tube has no filament, no standby current is consumed. The speed of response, though somewhat less than that of the hot-cathode gas tube, is sufficient for most applications. The ionization time depends upon the time necessary to transfer the discharge from the starter gap to the main gap, and it is generally less than a hundred microseconds. Main gap de-ionization times are of the order of one to ten milliseconds. Because of its suitability to switching circuits, the electron tube circuit examples contained in the remainder of the chapter are, in the majority of cases, based on the cold cathode-tube.

Semi-Conductors. Although the advent of semi-conductors into the field of switching circuits is relatively recent, two types of semi-conductors — the non-symmetrical germanium or silicon varistor, and the germanium transistor — appear to be suitable as switching devices in numerous applications. As with the cold-cathode gas tube, no standby filament power is expended and no warm-up time is required before operation. Upper-frequency limits of operation lie above the megacycle mark; the germanium varistor retains its non-linear characteristics up to a frequency of the order of 100 to 1000 megacycles, and the maximum useful frequency for the transistor appears, at present writing, to be well above 10 megacycles.

The non-symmetrical varistor is a non-linear element with a high ratio of non-conduction ("reverse") resistance to conduction ("forward") resistance, the actual values of these resistances depending upon the voltages applied to the element. A typical varistor may have a reverse resistance of 50,000 ohms at 50 volts across the unit, and a forward resistance of 300 ohms at 1 volt.

The germanium transistor is an amplifier element equipped with three electrodes — the emitter, the collector, and the base — as shown in Fig. 10-2. In normal operation, the emitter voltage $V_e$ and current $I_e$ are positive, and the collector voltage $V_c$ and current $I_c$ are negative.
As a result of these polarities, the emitter-to-base circuit acts as a semi-conductor rectifier conducting in the forward (low-resistance) direction, and the collector-to-base circuit acts as a rectifier conducting in the reverse (high-resistance) direction. However, there is a large degree of interaction between these two circuits: a change in the magnitude of \( V_e \) or \( I_e \) causes a corresponding change in the magnitudes of \( V_c \) and \( I_c \). Ordinarily, the emitter is used as the control electrode, the input signal to the transistor varying the emitter voltage or current. Where the transistor is used primarily as an amplifier, the voltage or current output is obtained from the collector electrode, although for certain applications the output may be derived from the emitter or base terminals.

The transistor may be employed to take advantage of its nonlinear properties in addition to its use as an amplifying device. Unlike the vacuum tube, the transistor cannot be placed in a cutoff state by a large negative potential on its control electrode. However, when such a potential is applied, the amplification factor of the transistor is a small fraction of unity; for a less negative control condition, the voltage gain is appreciable*. Also in contrast to the vacuum tube, the input (emitter-to-base) impedance of the transistor (as normally used) is low, and the output (collector-to-base) impedance is high; typical values of input impedance and output impedance are 200 ohms and 10,000 ohms, respectively.

10.2 BASIC TUBE AND SEMI-CONDUCTOR CIRCUITS FOR SWITCHING APPLICATIONS

Lock-In or Memory Circuits. In most switching circuits, certain relays are required to operate in response to some particular condition and to remain operated, or locked-in, until a second particular condition obtains. As discussed in previous chapters, such a requirement is met by a relay furnished with a holding path, the relay operating when its operating path is closed, and locking operated until the holding path

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* In actuality, the transistor is a current amplifier with a current gain. Specific values are not quoted here since the device is still in the comparatively early developmental stage.
is opened. The relay, by virtue of its associated control networks, has two stable states: when it is unoperated, it remains unoperated until the operate condition is applied; and when it is operated, it remains operated until the release condition is applied.

Elements with a similar characteristic are necessary in many electronic switching circuits. Hot-cathode or cold-cathode gas tubes may be used directly as lock-in elements since, after being ionized,
they remain in a conducting state until the potential between conducting electrodes is reduced below the sustaining value. Inherently, then, gas tubes are devices with two stable states.

As an example of an elementary gas-tube lock-in circuit, consider the cold-cathode diode circuit shown in Fig. 10-3A. This circuit consists of the tube in series with a load resistance in the cathode circuit and a variable potential \( E_p \) applied to the anode. As the anode potential is increased from zero in a positive direction, the voltage drop \( E_k \) across the load resistance remains at zero as long as \( E_p \) remains below the firing potential of the tube, about 70 volts for a representative cold-cathode tube. When \( E_p \) reaches this ionizing value, the tube conducts; and, as the characteristic curve in Fig. 10-3B shows, the voltage drop across the tube is reduced to 60 volts. Since at this moment \( E_p \) equals +70 volts, \( E_k \) is +10 volts. Thereafter, as \( E_p \) is increased from +70 volts, \( E_k \) increases by an like amount, the voltage drop across the tube remaining almost constant.

Note that the characteristic curve of Fig. 10-3B consists, in effect, of two positive dynamic resistance regions — one of very low current (very high resistance) and the other of moderately high current (low resistance) — separated by a region of negative dynamic resistance. The tube, in firing, passes through this negative resistance region between non-conduction and conduction.

When \( E_p \) is reduced, in the circuit of Fig. 10-3A; the current through the tube diminishes until, at a very low value of current, the knee ("a" in the diagram of Fig. 10-3B) of the characteristic curve is reached, the voltage across the tube begins to decrease rapidly, and the tube de-ionizes. This occurs at a value of \( E_p \) of slightly less than +60 volts. As a lock-in element, then, the anode may be biased at a point between the sustaining and firing voltages. The tube is operated by application of a pulse sufficient to raise the anode-to-cathode potential to the ionizing value, and is released by reduction of the anode-to-cathode potential to a point below the sustaining value. Release can also, of course, be obtained by opening the conduction path with a switch or its equivalent. Since the margin between ionizing and sustaining values may be small (in this case, 10 volts), a diode is not practical as a lock-in element in some applications.

The gas-tube diode as a lock-in device also has the disadvantage that the input and output leads are not isolated during the interval of tube conduction. If isolation at all times is necessary, the gas-tube triode may be employed, as shown in Fig. 10-4. In this circuit, the main anode is connected to a positive voltage source, and the cathode is connected to ground through a load resistance and a contact or some other means capable of opening the main-anode-to-cathode conducting
path. The main-anode voltage supply is of a value lower than that necessary for ionizing the main conducting gap but greater than the main-gap sustaining potential.

With the circuit at rest, the starter anode is at ground potential. When a sufficiently positive voltage $E_{st}$ is applied to the starter anode, in this instance by the operation of the (ST) key, the starter-anode-to-cathode gap is ionized, the discharge is rapidly transferred to the main anode, and the main conducting gap is ionized. After the main gap is fired, the starter anode exercises no further control until the tube is de-ionized. Main-gap de-ionization is accomplished by reducing the main-anode-to-cathode potential below the sustaining value. This is done in the circuit of Fig. 10-4 by opening the conduction path through the cathode circuit.

The operating input condition for the cold-cathode tube circuit consists of a positive voltage signal, and the corresponding output consists either of a current flow through the load resistance or a potential $E_k$ measured across this resistance*. Since the starter gap requires only a few micro-amperes for ionization, the source of the input signal may be of high impedance.

The transistor as a lock-in element is somewhat similar to the cold-cathode diode. This similarity is clearly indicated, for example, by the curve of emitter current as a function of emitter voltage shown in Fig. 10-5. This characteristic curve applies to the circuit of the same figure where the collector voltage is held constant and the resistance $R_b$ in series with the base is large. In this figure, the arrows associated with $I_e$, $I_c$, and $V_e$ indicate the sense of positive current flow and positive potential. Like the curve representing the characteristics of the cold-cathode diode, the curve of Fig. 10-5 indicates two regions of positive dynamic resistance separated by a region of negative resistance. Therefore, the transistor connected as shown in the

* In some gas-tube circuits, where the starter-anode-to-main-anode potential is sufficiently high, ionization between these two electrodes may occur and the resulting current may be used as an output indication.
The figure has two stable states, one characterized by low negative values of emitter current $I_e$, and the other characterized by high positive values of $I_e$. By selecting appropriate values of emitter bias $E$ and the series resistance $R_e$, the circuit can be pulsed from one stable state to the other, thus fulfilling the general requirements of a lock-in device.

The collector-to-ground characteristic of the transistor with constant emitter potential and a high resistance in the base circuit also indicates the presence of two stable states. Here, however, the collector current $I_c$ is negative in both states, the difference between states being one of current magnitude.

![Fig. 10-5 The Transistor Used as a Lock-In Device](image)

Lock-in circuits may also be composed of electronic elements which do not, in themselves, exhibit lock-in characteristics. Consider, for example, the vacuum triode. Two triodes may be interconnected in such a manner that when one is conducting, the other is cut off. With appropriate values of components such an arrangement has two stable states, the circuit remaining in one until forced into the other by external means.

One triode circuit of this type is the Eccles-Jordan circuit, shown in somewhat modified form in Fig. 10-6. Assuming that, with the circuit at rest, tube $T_1$ is conducting and tube $T_2$ is non-conducting, a negative pulse applied to lead 1 decreases the grid potential of $T_1$. The plate potential of $T_1$ rises sharply, carrying with it the grid potential of $T_2$, thus allowing tube $T_2$ to conduct. The accompanying decrease in plate potential of $T_2$ forces the grid potential of tube $T_1$ below cut off. Tube $T_2$ remains in a conducting state and $T_1$ remains in a non-conducting state, until a negative pulse applied to lead 2 returns the circuit to its original state. Thus, the presence of a negative pulse on lead 1 is the operating condition. An indication of the state of the circuit may be obtained by examining the plate voltage or plate current of either
tube. In Fig. 10-6, the plate current of tube $T_2$ gives an output voltage indication. In applications of this circuit, it is necessary to insure that the desired tube is conducting (in this case $T_1$) before the circuit is required to function.

By suitable changes in the arrangement of Fig. 10-6, the circuit may be made to operate and release on consecutive re-appearances of the same input condition; i.e., the negative pulse. A circuit operating in this manner is evidently a pulse-frequency divider, since it provides a given output indication for every alternate input pulse, and corresponds in action to the relay pulse-frequency divider presented in Chapter 8.

![Fig. 10-6 Eccles-Jordan Circuit as a Lock-In Element](image)

**Lockout Characteristics.** A characteristic of gas tubes which is of great importance to one functional circuit in particular, the lockout, which is discussed in a later chapter, concerns the operation of tubes connected in parallel.

It has been experimentally determined that if two or more gas tubes are provided with a common load resistance, as in Fig. 10-7, either in the anode or in the cathode circuit, only one tube, except in rare cases, will ionize and remain ionized even if firing potentials are applied to the several tubes either simultaneously or in sequence. This phenomenon is due to the region of negative resistance in the characteristics of the gas tube through which the tube passes in the range
between breakdown and the stable sustaining voltage. In this region, as the current through the tube increases, the voltage across the tube decreases, tending to prevent other tubes with the common load resistance from firing. However, if two tubes reach the breakdown potential simultaneously and then travel through this unstable region exactly together, it is possible for the two tubes to remain ionized, since in the stable condition of normal conduction the characteristics exhibit a positive resistance.

To reduce this slight possibility of dual operation, the common load resistor is replaced by an impedance including an inductive element. The effect of the inductance is to increase the time interval necessary for the tubes to traverse the unstable negative resistance region, decreasing the likelihood of the two tubes passing through this
region exactly together. A possible second modification is that of employing multi-element tubes which ionize in successive stages, corresponding stages having common inductive impedances.

One such arrangement is shown in Fig. 10-8. The tubes in this circuit have a separate starter cathode, with the result that the starter-gap firing circuit is isolated from the main anode-cathode gap. The tubes are fired in two stages: first, the starter gap is ionized, and then the discharge transfers to the main gap. In the circuit of Fig. 10-8, the two tubes can be fired coincidently only if both pass together through the unstable region for each ionization: that of the starter gap, and that of the main gap. If the probability of two tubes traversing one such region is very small, the probability of covering all such regions together is infinitesimal.

![Diagram of gate circuit](image)

**Fig. 10-9 Elementary Diode Gate Circuit**

**Gate Circuits.** A basic element which is necessary in many switching circuits is one which is capable of opening and closing a signaling or control path between other elements. Such an element is, in effect, a switch which is either open or closed to signals passing along a lead, depending upon the conditions controlling the switch. In usual terminology, this electronic switch element is called a "gate".

A gate circuit may operate in one of a number of different ways. It may act to alter the impedance of a path carrying signals through the gate, or it may act as an amplifier which is characterized by an approximately zero gain when the gate is closed and appreciable gain when the gate is open. In most cases, a gate circuit is designed to operate on a path carrying signals comprising positive or negative pulses rather than on a path carrying a-c signals. Many gates have non-linear characteristics which make them unsuitable for a-c transmission.

An elementary gate circuit employing a vacuum diode is illustrated in Fig. 10-9. In this circuit, the bias voltage $E$ is maintained on the output lead as long as the anode-to-cathode potential is negative. The magnitudes of the signal pulse $E_s$ and the gate-open voltage condition $E_g$ are such that the anode-to-ground potential remains less than
the cathode-to-ground potential unless the signal pulse and the gate-open voltage condition are present simultaneously. When both signal pulse and gate-open voltages are applied to the anode, the tube conducts, raising the cathode or output voltage. The wave shape of the output pulse is similar to that portion of the signal pulse which occurs during the period of tube conduction. There is an inherent loss in signal amplitude between input and output due to the voltage-dividing action of the circuit resistances.

A varistor rectifier may replace the diode in the circuit of Fig. 10-9 if the specific requirements for the circuit permit. Comparison of the characteristic curves for the vacuum diode and the varistor, shown in Fig. 10-10, indicates that the forward resistance of the varistor is less than the conducting resistance of the diode, and the reverse resistance is less than the diode non-conducting resistance. Therefore, the output voltage level may be affected by signal pulses with the gate closed if a varistor is substituted for the diode, although the conducting loss through the varistor when the gate is open will be less.

Where gates are to be constructed using varistors, slightly more complex circuit arrangements are often employed to increase the gating efficiency. Consider, for example, the gate of Fig. 10-11. The operation of this circuit in either the gate-open or gate-closed condition is to some extent equivalent to that of a transmission filter T-section. When the gate voltage is negative, series varistors \( V_1 \) and \( V_2 \) are biased in the reverse direction, and the shunt varistor \( V_3 \) is biased in the forward direction; when the gate voltage is positive, the directions of the varistor biases are reversed. Inspection of the varistor characteristic curve of Fig. 10-10 shows that the small-signal (small in comparison to the magnitude of d-c voltage across the varistor) a-c resistance is

\[
\begin{align*}
\text{VOLTAGE-CURRENT CHARACTERISTICS OF VACUUM DIODE} \\
E_p & \quad +I_p \\
-E_p & \quad -I_p \\
\text{VOLTAGE-CURRENT CHARACTERISTICS OF NON-SYMMETRICAL VARISTOR} \\
E_v & \quad +I_v \\
-E_v & \quad -I_v
\end{align*}
\]

Fig. 10-10 Characteristics of Vacuum Diode and Varistor Rectifiers
high when d-c current flows through the varistor in the reverse direction, and low for d-c current in the forward direction*. Therefore, for a negative gate voltage the series resistance is high to the signal, and the shunt resistance is low. For a positive gate voltage, the converse is true: the series resistance is low and the shunt resistance is high. The resistor R is placed across varistor V3 to increase the forward current through V1 and V2, thus effecting a greater reduction in series resistance when the gate is open.

![Diagram](Fig. 10-11 Varistor Gate Circuit)

A simple gate circuit based on d-c control of small-signal a-c varistor resistance is shown in Fig. 10-12. As in the circuit just discussed, a negative gate voltage increases the series resistance to the signal voltage while the series resistance is reduced by application of a positive gate voltage. No shunt varistor appears in the arrangement of Fig. 10-12, although more complicated varistor networks making use of variable resistance shunt elements may be constructed.

If signal gain is desired when the gate is open, a gate employing an amplifying tube may be used. One such gate circuit is shown in Fig. 10-13. In this arrangement, the grid of the triode is held below cutoff except during the interval when the gate is to be open. During this interval, the gate voltage biases the grid approximately to the cutoff point. Thus, positive signal pulses placed on the grid have no effect on the output voltage except while the gate is open. The magnitude of the signal voltage must be limited to a value insufficient to raise the grid above cutoff when the gate-open voltage is not present.

A transistor may also be used for gating purposes where signal gain is desirable. The operation of a transistor in a gate circuit, as for example in the circuit of Fig. 10-14, is very similar to that of a vacuum triode. With the gate voltage sufficiently negative, the input signal

* Note that the small-signal a-c resistance of the varistor at a given d-c current level is given by the inverse slope of the characteristic curve at that current: $R_{a-c} = d(E_v)/d(I_v)$; while the d-c resistance at the same current is the ratio of values of $E_v$ and $I_v$ coordinates: $R_{d-c} = E_v/I_v$. 
causes only a very small change in collector current; with the gate voltage at ground potential, the input signal is amplified by the transistor action and appears on the output lead.

"And" - "Or" Circuits. In most relay circuits, the state, at a given instant, of a particular relay (operated or unoperated) or lead (activated or not-activated) depends upon the combination of states of other relays or leads at that moment. This is also true of electronic control circuits in that the state of some one element depends upon the state of a number of other elements. The design of an arrangement which will exhibit this required dependency among elements involves the use of circuits which associate the independent elements with the dependent elements. These circuits correspond, to some extent, to the contact networks constructed on a group of relays, allowing these relays to control the action of certain other relays.

There are two basic forms of these electronic control circuits: the "and" circuit and the "or" circuit. In both forms, a number of input
terminals and a single output terminal are provided. For the "and" circuit, the output terminal is activated only if all input terminals are activated; for the "or" circuit the output terminal is activated if any input terminal is activated. Therefore, the "and" circuit may be considered as corresponding to a relay circuit in which each input lead is connected to a relay winding, and the output lead is controlled by a series chain of make-contacts placed on the relays. Similarly, the "or" circuit corresponds to a relay circuit in which the output lead is controlled by make-contacts placed in parallel.

An example of an "and" circuit employing either vacuum diodes or non-symmetrical varistors is shown in Fig. 10-15A. In this circuit, the rectifier units are poled in such a manner that the output lead can
never assume a positive potential appreciably above that of the least positive input lead. Assuming that the input leads are at zero volts when unactivated, and that the resistance $R$ indicated in Fig. 10-15A is large in comparison to the forward resistance of the rectifier units, the potential of the output lead remains close to zero volts as long as any of the inputs are unactivated. The input leads, in this case, are activated by signals of $+E$ volts, and when all these leads are thus activated, the output lead is allowed to rise to a point somewhat less than $+E$ volts. The exact positive potential which the output lead may reach depends upon the comparative values of resistance $R$, the external resistance faced by the output lead, and the back resistance of the rectifiers.

In the circuit of Fig. 10-15A, power to the output circuit is supplied through resistor $R$ by the bias battery $E$. If it is permissible to draw power from one of the circuits controlling an input lead, the circuit of Fig. 10-15B may be employed. Inspection shows the operation of this circuit to be similar to that of Fig. 10-15A with the distinction that the bias source of constant potential $+E$ is replaced by an input signal source which has a potential of either zero volts or $+E$ volts. Here, the resistance of $R$ must be considerably less than the reverse resistance of a rectifier unit in order to maintain the potential of the output lead near zero volts when input lead 3, in the figure, is at zero volts, and leads 1 and 2 are at $+E$ volts. Conversely, $R$ must be substantially greater than the forward resistance of the rectifiers to reduce the voltage rise on the output lead when terminal 3 is at $+E$ volts and terminals 1 and 2 are at zero volts.

The corresponding "or" circuit, as shown in Fig. 10-16, is equally simple. As a result of the polarity of the rectifier elements, the potential of the output lead is close to $+E$ volts when a potential of $+E$ volts is applied to one or more of the input leads. Again, the actual voltage reached by the activated output lead depends upon the resistance $R$ and also upon the forward resistance of the rectifiers. In this circuit, power to the output circuit is supplied by whichever inputs are activated. The principal effect of the rectifiers is to isolate the input leads.

![Fig. 10-16 An "Or" Circuit](image-url)
As an elementary example of the utility of these circuits, consider the circuit of Fig. 10-17 in which the cold-cathode gas tube D is to be fired when all three control tubes A, B, and C are in a state of conduction. The cathodes of the control tubes are connected to an "and" circuit similar to that illustrated in Fig. 10-15; the output of this "and" circuit — point P in the diagram of Fig. 10-17 — is joined to the starter anode of tube D. As a result of the characteristics of the "and" circuit, point P maintains a potential nearly equal to that of the least positive cathode, assuming the cathode resistances to ground to be small. This potential is insufficient to ionize the starter gap of tube D when any of the tubes A, B, and C are non-conducting.

As each control tube fires, its cathode voltage rises; and when the last of the three tubes fires, point P and the starter anode of tube D take on a potential approximately equal to the cathode voltage of the ionized tubes. Thus, the starter gap of tube D ionizes and the tube fires. All four tubes remain ionized until the connection between the main anodes and the voltage supply is opened. Note the equivalence in action of this circuit and the relay circuit also shown in Fig. 10-17. Similar circuits can easily be visualized in which a varistor "or" circuit permits a tube D to ionize if any of tubes A, B, or C are conducting, or prevents a tube D from ionizing if any of tubes A, B, or C are conducting, etc.
The switching algebra, introduced in a preceding chapter, may be used as an aid to the combination of "and" and "or" circuits to fulfill stated requirements for activating a particular lead. Here, the algebraic expression which is equivalent to a statement of requirements is written in terms of leads activated and unactivated rather than in terms of relays operated and unoperated. For example, if lead D is to be activated when lead B and either lead A or lead C are activated, the corresponding algebraic expression is: \( f(D) = B + AC \). Except for this difference in reference, the algebra as applied to the activation of leads

Fig. 10-18 Two Combinations of "And" and "Or" Circuits Corresponding to the Expression: \( f(D) = B + AC \)
is identical to that used in the design of relay circuits, and may be similarly employed in the manipulation and simplification of circuit configurations.

The "and" and "or" circuit combination which corresponds to an algebraic expression may be derived by inspection. Any such expression consists of variables which are associated by addition ("and") and multiplication ("or"). Thus, all that is necessary in obtaining a circuit equivalent to the algebraic expression is to separate the expression into "and" and "or" associated terms, and then to draw the corresponding circuit, representing the "and" associations by "and" rectifier circuits and the "or" associations by "or" rectifier circuits.

As an illustration of this method, consider the example mentioned above: \( f(D) = B + AC \). Here, the expression indicates that leads A and C should be associated by an "or" circuit, and the result (the output lead of the "or" circuit) should be associated with lead B by an "and" circuit. This arrangement is shown in Fig. 10-18A. Study of the figure indicates that the original requirements have been fulfilled; lead D is driven positive for a positive pulse on lead B occurring simultaneously with a positive pulse on lead A or lead C.

However, it is clear from an inspection of Fig. 10-18A that the values of resistances \( R_1 \) and \( R_2 \) are somewhat critical. As stated in connection with the discussion of the basic "and" and "or" circuits, resistances \( R_1 \) and \( R_2 \) should be greater than the forward resistance of a rectifier element, and \( R_2 \) must be less than the reverse resistance of a rectifier element and also less than the output resistance faced by lead D. Moreover, \( R_2 \) must be greater than \( R_1 \) in order to minimize the potential of lead D when lead B is activated and leads A and C are at zero potential. Also, when "and" and "or" circuits are combined in this manner, care must be exercised to maintain low resistances at the sources of the input signals. If the source resistance at terminal B, for example, is high, positive voltages on terminals A and C may cause an appreciable positive potential on output terminal D, giving a false output indication.

Analysis of this kind must be employed in the determination of the values to be assigned to the resistances employed in combinations of "and" and "or" circuits. Evidently, as the number of circuits which are to be combined increases, it becomes more difficult to obtain sufficient margins for practical application. In some cases it may be necessary to regenerate the combination of activated and unactivated conditions on an intermediate output lead by a gas-tube or vacuum-tube circuit before allowing these conditions to affect further circuit action.

In the circuit of Fig. 10-18A, power is supplied by the source connected to lead B to the external circuit connected to lead D. Under
certain conditions it may be desirable to distribute the power load over
the input sources in some other manner. To effect this distribution, the
circuit may be modified by such methods as interchanging the resistor
and rectifiers in the legs of the "and" circuits — in this case inter-
changing rectifier $V_3$ and resistor $R_2$. Another method is to manipulate
the original algebraic expression from which the circuit configuration
was derived. As an example, the original expression $f(D) = B + AC$
is equal to: $f(D) = (B + A)(B + C)$. This latter form suggests a combina-
tion of two "and" circuits associating leads $B$ and $A$ and leads $B$ and $C$,
respectively, with their outputs associated in an "or" circuit, as shown
in Fig. 10-18B. In this arrangement, power is supplied by leads $B$ and
$C$, a result which could not have been obtained as easily by direct ma-
ipulation of the circuit of Fig. 10-18A.

A convenient system of symbolizing these "and" and "or" cir-
figs is indicated in Fig. 10-19. Here, the basic "and" or "or" element
is provided with multiple input leads and one output lead. The charac-
teristic behavior of the element is specified by the polarity signs adja-
cent to the input and output leads. For example, the symbol of Fig.
10-19A indicates that when $A$ is positive (+) and $B$ is positive (+), $C$
is positive (+); the symbol of Fig. 10-19B indicates that when $A$ is positive
or $B$ is positive, $C$ is positive. Thus, these symbols correspond to the
circuit forms shown in Figs. 10-15 and 10-16.

The actions of these two symbolic circuit elements may be ex-
pressed in another way. In the first, that of Fig. 10-19A, $C$ is negative
(-), with respect to its activated condition, when either $A$ or $B$ is nega-
tive. For the element of Fig. 10-19B, $C$ is negative when both $A$ and $B$
are negative. Now if, in an algebraic expression, the prime (') is used
to indicate the negative (-) condition and the absence of the prime indi-
cates the positive (+) condition, the behavior of the two elements may be
expressed in either of two forms:

For Fig. 10-19A, $A + B = C$ or $A'B' = C'$
For Fig. 10-19B, $A B = C$ or $A' + B' = C'$

Algebraically, the two forms characterizing the action for each element
are equivalent. It is interesting to note that the algebraic expression

\[
\begin{align*}
A + B &= C \\
(A'B') &= C'
\end{align*}
\]

\[
\begin{align*}
A B &= C \\
A' + B' &= C'
\end{align*}
\]
The Design of Switching Circuits

Fig. 10-20 Additional "And" and "Or" Elements

indicates that the "and" element of Fig. 10-19A can be considered an "or" circuit for negative activation of its leads. Similarly, the "or" element of Fig. 10-19B is an "and" circuit for negative activation.

In addition to these two symbolic elements, four other elements shown in Fig. 10-20, are logically possible. Each of these is shown with its characteristic algebraic expressions. The physical realization of these four elements necessitates some means for signal inversion through the use of vacuum tubes or transformers. As an illustration, a circuit corresponding to the symbolic element \( A + B = C' \) (or \( A'B' = C \)) is indicated in Fig. 10-21A. In this circuit, only when both grids A and B are raised in a positive (+) sense above cutoff does the anode C become negative (−) with respect to the anode supply voltage. Similarly, in a circuit for \( A B = C' \) (or \( A' + B' = C \)) in Fig. 10-21B, the anode C falls below the supply potential when either A or B becomes positive with respect to cutoff. The remainder of the symbolic elements may be realized with somewhat similar circuit arrangements. In fact any given gate, say the pentode of Fig. 10-21A, can be used to realize any of the six circuits by inserting signal-inverting elements (triode amplifiers or transformers) in the appropriate input and output leads.

Fig. 10-21 Typical Elementary "And" and "Or" Circuits Providing Signal Inversion
This symbolism may be an aid to the design of electronic switching circuits in that a circuit composed of symbolic elements may immediately be drawn from an inspection of the algebraic statement of the circuit requirements. By manipulation of the algebraic expression several different, but equivalent, circuit forms may be derived. This is illustrated by the example of Fig. 10-22. The process of translating the
A second method of de-ionizing one tube when a second is operated is illustrated by the circuit of Fig. 10-25. Tube $T_1$ is ionized by a positive input pulse as in the last example. Values of the common anode and the individual cathode resistances are such that after $C_1$ has become charged, the cathode potential is about +45 volts. When the second tube $T_2$ is fired, the initial drop across the anode resistance $R$ is $135 - 75 = 60$ volts, since $C_2$ is uncharged at the instant of firing. Thus, with its anode at $135 - 60 = +75$ volts, and its cathode held at +45 volts by $C_1$, tube $T_1$ is de-ionized. The cathode potential of $T_1$ then decreases to zero volts as $C_1$ discharges through $R_1$, and that of $T_2$ rises to +45 volts. When steady-state conditions are reached, the next operation of tube $T_1$ will extinguish $T_2$. Again, the time duration necessary between activating pulses depends upon the circuit time constants, in this case those regulating the charge and discharge rates of the two cathode capacitors.

Another fundamental circuit action is that of preventing one tube from firing until a second has been fired. One method of accomplishing this is by causing the latter tube to control the bias on the starter anode of the former, as in the arrangement of Fig. 10-26A. In this circuit, the bias on tube $T_2$ is controlled directly from the cathode of $T_1$. The value of this bias is either +50 volts or 0 volts, depending upon whether $T_1$ is conducting or extinguished. The input positive pulses have an amplitude of the order of 40 volts, with the result that $T_2$ can be ionized only if $T_1$ has already been fired.

By a minor modification this circuit may be made to operate from a single input pulse lead, tube $T_1$ firing on the first pulse and $T_2$ firing on the second. In this modified circuit, shown in Fig. 10-26B, a
capacitor is added across the cathode resistor of tube $T_1$. As a consequence of this added capacitor, the bias on the starter anode of $T_2$ rises exponentially toward +50 volts after $T_1$ has been ionized. The capacitance of $C$ is adequately large to prevent the bias from increasing during the application of the pulse to such a value that this first pulse can fire tube $T_2$. By the time the second input pulse occurs, the bias on the starter anode of $T_2$ is sufficient to allow this pulse to ionize $T_2$. The circuit can be expanded to any number of tubes by connecting the starter anode of each tube to a tap on the cathode resistor of the next preceding tube, as $T_2$ is connected to $T_1$, and adding a capacitor from the cathode of each tube to ground.

Fig. 10-26 The Second Tube Cannot Be Fired unless the First Tube is Ionized
The arrangements of Figs. 10-24 and 10-26A may be combined to form a circuit in which the second tube cannot be fired until the first has been ignited; and when the second does fire, the first is extinguished. This circuit, shown in Fig. 10-27, is somewhat similar in operation to the relay pulse-frequency divider mentioned in an earlier chapter of this text. The first positive input pulse is applied to the starter anodes of both tubes; but, since only tube $T_1$ has adequate starter-anode bias, only tube $T_1$ fires. After $T_1$ ionizes, its cathode rises to +60 volts, the starter-anode bias of $T_2$ increases to +50 volts, and the cathode of $T_2$, coupled to the cathode of $T_1$ through capacitor $C$, rises to +60 volts. Because this rise in cathode potential for tube $T_2$ accompanies the increase in starter-anode bias, the first pulse does not fire $T_2$ after the ionization of tube $T_1$. As in the previously discussed circuit of Fig. 10-24, the capacitor $C$ charges before the second pulse, allowing the cathode potential of $T_2$ to fall to zero. The second pulse fires $T_2$ and, through capacitor $C$, extinguishes $T_1$. The third
pulse fires $T_1$ and extinguishes $T_2$, and so on. Tube $T_1$ is ionized by the odd-numbered pulses, and $T_2$ by the even-numbered pulses.

Control of Relays by Electron Tubes. It is sometimes necessary in electronic switching applications to operate a relay under the control of a tube. Since a relay is inherently a current-operated device and is often of relatively low impedance, the tube controlling a relay must be capable of passing a moderately large operating current to that relay. For this reason, gas tubes are more generally applicable to the control of general-purpose relays than are vacuum tubes.

The basic relay-control gas-tube circuit is shown in Fig. 10-28. When the starter anode is supplied with a potential exceeding the breakdown of the starter gap, the tube fires and the relay (K) operates. To release the relay, key (L) must be operated, interrupting the flow of current through tube and relay. A better arrangement employing a cold-cathode tube, illustrated in Fig. 10-29A, provides means for extinguishing the tube after the relay (K) has operated. In this figure, operation of relay (K) places a comparatively low resistance in parallel with

![Diagram A](image1)

![Diagram B](image2)

Fig. 10-29 Circuits Illustrating Gas-Tube Control of Relays
the tube and in series with (K), reducing the voltage across the tube to below the sustaining value. The disadvantage of this circuit is that the relay is held operated by a current considerably larger than is necessary. Also, unless the capacitor and resistor network shown in dotted lines is added, there is the probability that the tube will refire when (L) is opened to release the relay, as a result of the accompanying transient negative voltage kick at the cathode.

Another circuit, shown in Fig. 10-29B, employs a continuity-transfer contact on relay (K). Operation of relay (K) closes a holding path through key (L) to ground, and opens the conducting path of the tube. Either a resistance-capacitance network CP is used to avoid re-operating the tube on the release of relay (K), as shown, or a secondary winding for lock-up may be used.

Vacuum tubes may be utilized for operating relays from high-impedance inputs, though the current requirements for operation of a particular relay may make the selection of an appropriate tube difficult. On the other hand, the amplification characteristics of a vacuum tube make possible relay operation from a signal of small voltage amplitude. Ordinarily, only the more sensitive types of relays are used directly in conjunction with vacuum tubes. It is often convenient to couple the tube-amplifier circuit to a general-purpose relay through a gas tube, thus providing the relay with sufficient operate current.

10.3 ELECTROSTATIC AND MAGNETIC EFFECTS IN ELECTRONIC SWITCHING CIRCUITS

Although it is their characteristic sensitivity and speed of operation that make electron tubes of considerable importance to the switching art, these same characteristics add to the difficulty of designing reliable electronic circuits. Transient conditions of such short duration and such low energy level that their effects on relay circuits are negligible may cause an unprotected tube circuit to operate falsely or to fail to operate at all. In the design of tube circuits, the designer must anticipate these possible transient conditions and provide the proper protection where necessary. Skillful arrangement of the circuit components, physically as well as electrically, also minimizes the effects of transients.

Trapped Charges. When the lead to an electrode of a gas tube is opened, to extinguish the tube for example, the electrode tends to become charged as a result of electrostatic coupling and leakage between the electrode lead and other circuit elements. If the electrode is electrically floating with respect to ground and the rest of the circuit, the charge will be trapped on the electrode. Depending upon the magnitude
and polarity of the charge, the tube may fire falsely or refuse to fire on the next usage of the circuit.

The simplest solution to the problem is to drain trapped charges from the cathode of the tube by a very high resistance (of the order of 10 megohms) connected between the cathode of the tube and ground. With the cathode at ground potential under standby conditions due to the presence of the resistance, a trapped positive charge of sufficient potential on the starter anode will ionize the starter gap, and the charge will fall below the sustaining voltage value for the starter gap. Conduction through the main gap cannot be sustained with the gap in series with 10 megohms resistance.

Surge Conditions. High-speed transient surges of potential which may affect electron tube circuits may appear on battery and ground supply leads as a consequence of coupling between these leads and those of other circuits. Also, magnetic coupling between relays in the electron tube circuits and external closely adjacent relays may result in transient conditions on those tube electrodes which are connected to relay windings.

Transients appearing on battery and ground supply leads may be by-passed around the tube circuit by connecting a capacitor between these leads and ground as electrically close to the circuit to be protected as possible. Surge conditions resulting from transformer action between relays in the circuit and those external to the circuit may be minimized by proper physical placement of the circuit relays. In practical circuits, it is also well to allow ample operating voltage margins. Voltage surges produced by opening relay contacts in series with relay windings may be reduced by placing resistance-capacitance networks across such contacts, as in the circuits of Fig. 10-29.

PROBLEMS FOR CHAPTER 10

Unless otherwise specified, assume in all problems that grounded battery supplies of +135 volts and ±48 volts are available. Also assume the following characteristics for all cold-cathode gas tubes and varistors:

Cold-Cathode Tubes:
- Starter-gap breakdown voltage = 70 volts
- Starter-gap sustaining voltage = 60 volts
- Main-gap sustaining voltage = 75 volts at 20 milliamperes
- Maximum main-gap current = 20 milliamperes
Varistors:
- Forward resistance (d-c) at 2 volts = 100 ohms
- Reverse resistance (d-c) at 25 volts = 1.0 megohm
- Forward resistance (d-c) at 0.5 volts = 300 ohms
- Reverse resistance (d-c) at 48 volts = 500,000 ohms
- Maximum reverse voltage = 115 volts
- Maximum forward current = 20 ma.

Assume a linear variation of reverse resistance from 25 to 48 volts and of forward resistance from 2.0 to 0.5 volts and from 0.5 to 0 volts.

10-1 Design varistor networks (in the form illustrated in Fig. 10-18) satisfying the following algebraic expressions:
   (a) \((A + B)(C + D)(B + D)(C + A)\)
   (b) \(AB + BC + CD\)
   (c) \(E[(A + C)BD + C]\)

Determine suitable values for the component resistors in each network. Assume that the input leads are activated with +48 volts, and design for at least a 4:1 ratio between output and no output.

10-2 A cold-cathode gas tube \(E\) is to be controlled by four cold-cathode tubes, \(A, B, C,\) and \(D\). The tube \(E\) is to fire when \(A\) or \(B\) is fired, if, at that instant, \(C\) or \(D\) is extinguished. Tubes \(A, B, C,\) and \(D\) are three-element tubes, and tube \(E\) is equipped with a starter anode and a starter cathode. The circuit is to be restored to normal by momentarily opening appropriate leads. Design a suitable circuit, including values of component resistors.

10-3 Derive networks for the following algebraic expressions using the "and" and "or" symbolic elements of Figs. 10-19 and 10-20. Simplify each network where possible.
   (a) \(D(AB' + C')(A' + CE)\)
   (b) \((A + B' + D)(A' B + C)\)
   (c) \((A + B' C)(B + C + D')\)
   (d) \((A + B + C')(A + B' + C)(A' + B + C)(A' + B' + C')\)

10-4 The cold-cathode tube circuit of Fig. 10-27 is to be driven by pulses with an effective duration of three milliseconds at a rate of forty pulses per second. Determine appropriate values for the component capacitor and resistors.
The preceding chapters have presented the design fundamentals of circuits comprising two-valued elements, with little regard for any specific applications for these circuits. In this chapter, and in those to follow, the design techniques of circuits which must fulfill certain often-encountered functional requirements are considered. These circuits, each of which is developed to perform a single more or less distinct function, are called "unifunctional circuits".

The various unifunctional circuits serve as building blocks available to the circuit designer in the synthesis of a switching system. Given the requirements which the system must satisfy, the designer can, by analysis, transform them into a set of basic and individual functions which can be interrelated and integrated to fulfill the stated requirements. He can then select, or design, appropriate unifunctional circuits to correspond to these individual functions, interconnecting them to obtain the desired switching system. The determination of circuit arrangements which perform these basic functions is a major part of the design of a switching system.

The first of the unifunctional circuits to be presented herein is the "counting" circuit. Stated in general terms, it is the primary function of a counting circuit to receive information in the form of repeated pulses, the number of repetitions imparting the desired information. These pulses, consisting of voltage or current conditions on one or a number of leads, may be generated by some other circuit within the same switching system or by an external control source.

In its operation, a counting circuit must recognize each appearance of the information-bearing input condition and it must establish a unique state for each successive appearance of the input condition. The number of appearances can be determined at any time by an examination of the state of the elements in the circuit.

The nature of the elements of the counting circuit and the character of the input condition which actuates the circuit are interdependent. That is, if the counting arrangement is to be based on relays, it is convenient to detect and count ground pulses.* The pulses and the intervals between pulses must be of sufficient duration to allow one or

* A "ground pulse" is here defined as the appearance and disappearance of ground on a lead.
more relays to act during and between pulses. For electron tube circuits, input pulses must be of great enough potential and duration to actuate a tube. The minimum time which must be allowed between pulses depends upon the particular circuit arrangement used. The validity of these requirements will become more evident as counting circuits are discussed in detail.

In circuits designed to count the number of pulses in a pulse train, the combination in which the elements (whether relays or tubes) are operated after the train is completed indicates the number of pulses in the train. Such circuits must be capable of passing stage-by-stage, under control of the input pulses, through combinations representing all the numbers through the maximum to be counted. Circuits external to the counting circuit may make use of the count information before the pulse train has been completely received. In such cases, while the first pulse of a train is in progress, an output signal may have to be produced; during the second pulse a second output signal may have to be produced; and so on. The output signals are often derived from a translating circuit, associated with the counting elements, which converts from the system of enumeration employed by the counting circuit to some other desired enumeration system. Circuits for translating functions are discussed in the next chapter.

At all events, the counting circuit must at some time control external circuits. Either the information as to the number of pulses received may be used by some other circuit directly, or that information may be placed in an external storing device for later use, leaving the counting circuit free to accept a second pulse train. Discussion of the storage of count-information is deferred until later in this volume; only the fundamentals of the actual counting process are considered in this chapter.

11.1 SYSTEMS OF ENUMERATION

In order to give more significance to the counting schemes which are employed in switching systems, it is of value to review, in a very general manner, the arithmetic systems of counting and notation. Just as certain systems of notation are more adaptable to numerical calculation, for example, the Arabic notation as compared to the Roman notation, so certain systems are more adaptable than others to counting with relays or electron tubes.

In physically enumerating a series of objects, a collection of arbitrary symbols arranged in a definite sequence is employed. Each of these symbols is placed, in the defined order, in correspondence with one of the objects to be counted. Thus, the total number of objects is
given by the symbol corresponding to the final object, and each inter-
mediate object is uniquely characterized by its corresponding symbol.

The choice of symbols for enumeration permits a large degree of
freedom. For example, a different symbol might be used for each digit
in the counting sequence so that if, say, twenty-three objects were to
be counted, twenty-three different symbols would be required. Such a
system has obvious disadvantages. In order to reduce the total number
different symbols, combinations of a limited number of symbols can
be employed. Numerically, four symbols in all possible combinations
serve to designate and count up to fifteen objects, one combination being
reserved to correspond to the absence of all objects. For counting
purposes these combinations should, of course, follow a definite though
arbitrary sequence. It is apparent that this combinational system might
become highly complex if large numbers of objects were to be counted.

A preferable system of enumeration is that of using a compara-
tively small number of symbols cyclically and assigning a "position
value" which is some multiple of the number of symbols utilized. This
is the procedure in the ordinary decimal, or base-10, system. Here,
ten digits: 0, 1, 2, . . . 9, are used cyclically in each position, or place,
with the result that, together, the symbol and its position indicate the
corresponding numerical value. To illustrate, the number 347 is equal
to \((7 \times 10^0) + (4 \times 10^1) + (3 \times 10^2)\). Each symbol, in effect, is multiplied
by the counting base (10 in this instance) raised to a power, the partic-
ular power being determined by the position of the symbol.

There is no fundamental reason for using the decimal or 10-
valued system of enumeration other than, perhaps, the physiological
fact that man is equipped with ten fingers, which probably composed
the first counting device. Another counting device might require, or at
least suggest, a system constructed on some base other than ten. This
concept is pertinent to the design of relay and tube counting circuits
since these elements are of a two-valued nature.

With relays or tubes as elements of a counting circuit, the two-
valued or binary system of enumeration might be adopted as logical
choice. The count from 0 to 7 (decimal notation) in a binary system is:
0, 1, 10, 11, 100, 101, 110, 111. The next figure, that corresponding to
8, is 1000. If the missing 0's in places ahead of the first significant
figure are supplied as shown in Table 11-1, the binary count from 000
to 111 contains all the eight possible combinations of 0's and 1's in three
places. With 0 and 1 represented by a relay released or operated, re-
spectively, Table 11-1 may be regarded as a table of combinations for
three relays.* As will be seen, a counting arrangement fulfilling this

* It is important that these 0's and 1's be differentiated from the same symbols used in
the tables of combinations, describing the operation of combinational and sequential
circuits, in Chapters 5, 6, and 8.
table is easily designed. The table indicates that three counters are needed to count from 0 to 7 (000 to 111 in binary form) and four counters from 0 to 15 (0000 to 1111).

Regardless of what counting system is used, some symbol must correspond to zero. Consideration of the cyclical system just described shows that the symbol for zero, or its equivalent, may indicate either of two conditions: (1), the enumeration has not yet commenced and the counting device is in its normal or "null" position; or (2), the count has reached a point one unit beyond the final symbol in a particular place. This second condition appears in decimal notation for 10, 100, 1000, and so on.

Zero may be represented in a cyclical counting system in either one of two ways. A separate apparatus unit (a relay or tube) may be provided to correspond to zero, just as an apparatus unit is provided for each of the other counting symbols. On the other hand, zero may be represented, as in the binary system just described by counting elements unoperated. In decimal notation, for example, the first method would require 10 elements (one assigned to each digit from 0 to 9) per cyclical position, and the second method would require 9 (one for each digit from 1 to 9, with 0 being represented by all elements released).

In many switching applications it is desirable to convert the decimal notation to some other notation or vice versa. Since a discussion of this problem appears elsewhere in this volume, it is sufficient at this point merely to note that this conversion may be simple or complex depending upon the systems of enumeration used. For example, inspection of Table 11-1 indicates that no simple correspondence exists between the binary and decimal systems of notation. As a result, conversion from one system to the other tends to be complicated.

A more easily converted notation involves a mixed base. One such mixed base system is the biquinary notation where two digits are used to represent each decimal digit, the first assuming the value 0 or 1 and the second assuming one of five values: 0, 1, 2, 3, or 4. Thus, the decimal number 47 would appear in the biquinary notation as 04-12. Although this system requires more counters to represent symbols than does the binary notation, the ease of translation is far greater. To convert from biquinary to decimal notation, the first figure of each biquinary pair is examined: if this figure is zero the decimal value of the pair is that of the second figure; while if the first figure is one, five is added to the second figure to obtain the value of the pair.
There is an unlimited number of mixed-base systems of notation. However, the general method of notation for any particular system is the same: the number of unique symbols in each place is equal to the base used for that place and, as the counting proceeds, a complete cycle of symbols in any place corresponds to a single change in symbol in the place immediately to its left.

11.2 BASIS OF RELAY COUNTING-CIRCUIT DESIGN

A counting circuit, in response to a set of identical and successive pulses over a single lead, must perform two principal functions in relation to each individual pulse of the set:

1. Establish a unique combination of elements corresponding to the count of each pulse.

2. Establish a unique condition whereby the next following pulse can activate its corresponding combination.

Analysis of these two functions indicates that the second must not take place until after the pulse has terminated. Otherwise the circuit will respond to an input pulse by racing through as many successive combinations as the duration of the pulse permits, and the output indication will be completely indeterminate.

There are two methods of taking care of this situation. One is to introduce a time interval, longer than the pulse duration, which delays the setting up of the condition permitting the next combination to be activated. This is applicable to tube counting circuits which react to very short pulses and can incorporate simple delay networks. In general, it is impractical or uneconomical to apply this method to relay counting circuits because of the timing variability inherent in general-purpose relays.

The alternate method of deferring the second function is to make it contingent upon an action taking place at the termination of a pulse. That is, the circuit will be required to assume two unique combinations per input pulse, one during the pulse and the other after the pulse. The second combination will establish the condition for control by the following pulse. This method is eminently suitable for application to relay counting circuits and will be taken hereafter as the basis for design of such circuits.

11.3 SINGLE-PULSE COUNTERS

A circuit designed to count a single pulse, although elementary, provides a natural introduction to the design methods applicable to more
extensive relay counting circuits. In order to make the design procedure consistent, the previously implied requirement that the pulse count is not complete until the termination of the pulse has been recorded will be imposed.

Under these circumstances, a circuit to count a single pulse requires at least two relay actions, one when the pulse is applied, to record the start of the pulse, and a second when the pulse is removed, to indicate that the pulse is completed. Three distinct circuit conditions, therefore, must be established: one before the pulse appears, a second during the pulse, and a third after the pulse is completed. This requires a minimum of two relays, for which an operating sequence must be devised. One obvious sequence is shown in Fig. 11-1A, in which one relay, (X), operates on the application of the pulse and the second relay, (Y), operates on the termination of the pulse, both relays then remaining operated. Considering this sequence from a circuit standpoint, it appears that there is no particular difficulty in causing (X) to operate and lock on the application of the pulse, but that the means for

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**Fig. 11-1** Counting a Single Pulse
operating the second relay (Y) when the pulse is terminated are less straightforward.

There are two general methods for obtaining the desired operation of relay (Y). The first of these, shown in Fig. 11-1B, employs a pulse-repeating relay (P) to provide a break contact for the control of relay (Y). However, as an answer to the original problem, this arrangement has the disadvantages of requiring an additional relay and of requiring two relays to act in sequence during and after the pulse. The latter disadvantage becomes important for short pulses.

The second method involves a shunting arrangement where the operation of (X) connects ground to the winding of (Y), but a shunt on this winding is maintained by the pulse to prevent the operation of (Y) until the pulse is removed. Two schemes for accomplishing this are shown in Figs. 11-1C and 11-1D. The basic arrangement of Fig. 11-1D is frequently used and is commonly called a "prime pair" or "prime counter", since in many existing applications the two relays are given the same letter designation; the shunted relay, (Y) in this case, is designated the prime (') relay.

The three circuits in Fig. 11-1 satisfy the original requirements for a single-pulse counter that three distinct combinations before, during, and after the pulse, be established, but the latter two do not satisfy two additional requirements that are frequently imposed on these circuits. These are the so-called "non-interfering" requirements which may be stated as follows:

(a) The circuit must not ground or otherwise interfere with the control lead.

(b) The circuit must ignore subsequent pulses or grounds on the control lead.

Circuits which satisfy these conditions find much broader application, since the control lead may be re-used after a single pulse for subsequent control functions without interference with or by the counting device. The requirements can be met for the circuits of Figs. 11-1C and 11-1D by passing the control lead through a break-contact on the relay which operates at the end of the pulse. In Fig. 11-1C the break-contact on (Y) may be placed either in series with the incoming pulse lead or only in the extension of this lead to the bottom winding-terminal of (Y), since the winding of (X) is isolated from the lead by virtue of the continuity-transfer. This second arrangement is shown in Fig. 11-2A, while the arrangement for the series relays is shown in Fig. 11-2B. Both these circuits completely free the control lead after the passage of the original pulse.
Fig. 11-2 Non-Interfering Single-Pulse Counters

An alternative sequence to that of Fig. 11-1A, shown in Fig. 11-3, is also possible; a non-shunting two-relay circuit based on this sequence can be designed. The sequence requires the action of two relays one after the other during the pulse, tending to produce a slow circuit. However, this plan has the advantage that the (X) relay operates at the beginning and releases at the end of the pulse, a situation which is adaptable to performing certain control functions. If the non-interfering requirements are dropped, a circuit can be easily realized by allowing (X) to operate directly from the pulse lead, and by operating relay (Y) from a make-contact on (X), locking (Y) to ground from some other source. Non-interfering circuit arrangements of a more complex form may also be designed.

Fig. 11-3 Alternative Sequence for Counting a Single Pulse

11.4 TWO-PULSE COUNTERS, OR PULSE-FREQUENCY DIVIDERS

A highly useful circuit in many switching applications is a two-pulse counter which recycles after the second pulse. The cycle for such a circuit consists of four intervals as shown in Fig. 11-4A, where relay (X) releases at the start of the second pulse and relay (Y) releases at its end. The circuit, then, counts two pulses. Circuits which recycle after the second pulse are frequently called "pulse-frequency dividers" since, when the input consists of a continuous train of pulses, the relays act at half the repetition rate of the input pulses.

A circuit fulfilling these recycling requirements was discussed in Chapter 8. This circuit, shown in Fig. 11-4B, is very similar in form to
the circuit of Fig. 11-2A from which it may be derived. Briefly reviewing the operation of the circuit, the first ground pulse on the lead p is applied to the windings of both relays (W) and (Z) through the break of the continuity-transfer on relay (W). Only (W) operates, however, since relay (Z) is locked down through its own back-contact. After the first pulse is removed, (W) remains operated as a result of its locking path to ground, and relay (Z) operates from the locking ground of (W) since (Z) is no longer shunted down by the pulse ground.

The second pulse passing through the make-contact of (Z) shunts relay (W), releasing it. Relay (Z) remains operated since its operating path is transferred to the pulse lead by the action of the continuity-transfer on relay (W). When the second pulse is terminated, relay (Z) is released; and the circuit is then in the normal state. Further pulses actuate the circuit as just described, the circuit returning to normal after every even-numbered pulse. Inspection of the sequence chart for the circuit shows that information as to whether the last pulse received was even or odd in number can be obtained from a simple contact network placed on relays (W) and (Z).

This pulse-frequency divider does not entirely meet the non-interfering requirements. That is, when relays (W) and (Z) are both operated, a potential between battery and ground is connected to the pulse lead through the winding of relay (W) and one of the resistors. The circuit has the further disadvantage that it is inclined to be slow, both in the release of the (W) relay as a result of the shunt across its winding and in the operation of the relays through series resistances.

![Pulse Sequence Chart](image1)

![Circuit Diagram](image2)

Fig. 11-4 A Two-Relay Pulse-Frequency Divider
It is also possible to modify the prime pair of Fig. 11-2B to produce a second type of pulse-frequency divider. The application of a second pulse to this latter circuit finds both relays (X) and (X') operated and held in series by the make-contact on (X). This second pulse must release (X) and hold (X'), a requirement which suggests the use of a shunting path to effect the release of relay (X). However, this introduces design difficulties which impose rather rigid requirements upon the relays. A more practical approach is to add a third relay (Y) to the arrangement for the purpose of opening the operating and holding path of relay (X) when the second pulse appears. The operating sequence of such a circuit is as shown in Fig. 11-5A, and a circuit designed for this sequence is shown in Fig. 11-5B. Minor rearrangements of the circuit are possible. For example, relays (X') and (Y) might be held by a parallel rather than a series circuit. Also, the pulse through the break-contact of (X') might be connected directly to the winding of (X) rather than through the back contact of (Y), although the arrangement as shown affords a positive check that (Y) is released before (X) is called upon to re-operate.

This three-relay pulse divider is somewhat faster than the shunting two-relay pulse divider of Fig. 11-4 since the operation of relay (Y) and the release of relay (X) in sequence can usually be accomplished more rapidly than a single relay can be shunted down. If circuit margins are too close, the momentary decrease in the current through (X), occurring when the winding of relay (X') is inserted in the circuit at the end of the first pulse, tends to release relay (X), restoring the circuit to normal. This tendency is also present in the original prime pair circuit.
A cold-cathode gas-tube circuit which is similar in operation to the relay pulse-frequency dividers has been discussed in Chapter 10. This counter, as shown in Fig. 11-6, is actuated by positive pulses of an amplitude of about 30 volts. The first pulse applied to lead p ionizes tube (T1); the second ionizes tube (T2) which, in turn, extinguishes (T1). Each succeeding pulse then fires the tube previously extinguished. Thus, tube (T1) is ionized by the odd-numbered pulses and (T2) by the even-numbered pulses. Note that the state of the gas tubes is the same during

![Diagram of a cold-cathode gas-tube circuit](image)

**Fig. 11-6 Cold-Cathode Tube Pulse Divider**
The Eccles-Jordan circuit, described in Chapter 10, may also be arranged as a pulse-dividing counter as shown in Fig. 11-7. By virtue of the polarities of the varistors V1 and V2 which serve to couple the pulse lead to the anodes of the two tubes T1 and T2, a negative pulse appearing on the pulse lead affects only the anode of the cut-off tube. This follows since with one tube cutoff and the other conducting, the varistor connected to the anode of the conducting tube is biased in the reverse or non-conducting direction. Therefore, if tube T2 is conducting and tube T1 is cut-off, for example, a negative pulse drives the anode of T1 and, in addition, the grid of T2, in a negative direction, causing T2 to be cut-off and T1 to conduct. A second pulse, in a similar manner, restores the circuit to its original condition of tube T2 conducting and T1 cutoff.

Some means, not shown in Fig. 11-7, must be provided so that the same tube always conducts in the null condition. One method by which
this may be accomplished is to hold the anode circuit of one tube open until counting is to commence; another method is to drive the desired tube to the conducting state in preparation for counting.

The basic advantage of these electron tube circuits is their speed of operation. The cold-cathode tube circuit is easily capable of counting pulses at a rate of 60 per second, and, if interelectrode and wiring capacitances are kept low, the vacuum tube counters can be operated at pulse rates of the order of several million pulses per second.

11.5 COUNTING A TRAIN OF PULSES

Consideration of the general problem of counting a train of pulses shows that a particular counting circuit should be designed to be capable of counting some maximum number of pulses, this maximum number controlling the circuit arrangement and the amount of apparatus used. Such a circuit will also count a train of any number of pulses less than the maximum for which it was designed. This follows since the circuit must hold its relays operated under its own local control during the open intervals between pulses, and if no further pulse appears to terminate an inter-pulse interval, the relays will hold indefinitely in some combination corresponding to the number of pulses received. A counting circuit, then, is based on an operating sequence containing the maximum number of pulses which must be counted. A counting circuit, having received its maximum number of pulses, may (1), lock up during the last pulse and ignore the termination of this last pulse and any succeeding pulses; (2), lock up following the termination of the pulse and ignore succeeding pulses; or (3), return some or all relays to normal on the termination of the last pulse. If all relays are restored to normal, a re-entrant cycle is established and the circuit is permitted to recycle, starting the counting process anew on the arrival of a succeeding pulse. As will be evident later, cases (1) and (2) are more common, although many circuits recycle part of their relays. Circuits for cases (1) and (2) must be released to their normal condition by some control other than the input pulses.

Three general types of counting circuits are discussed below. The first includes schemes based on the use of a minimum number of relays. The next type comprises arrangements of one-pulse and two-pulse counter circuits, described in the preceding section, usually connected in cascade. Finally, there is the class of counting circuits which employ one or more common relays to detect the pulses, and, sequentially, activate the relays recording the count. This last type is, in many cases, the most satisfactory. As indicated by the preceding paragraph, counting circuits may also be classified as to whether they are non-recycling, recycling, or partially recycling. In partially re-cycling
circuitsthat some but not all of the component relays of the circuit recycle under the control of the pulse-detecting relays.

As already stated, a relay counting circuit will establish a different combination of operated relays for each pulse, and paths through contacts on the relays can produce signals indicating which pulse is in progress, how far the pulsing has progressed at a particular time, or when a particular number of pulses has been received. The discussion will concern only methods of controlling the relays, and not the translating circuits which interpret the relay combinations in terms of the pulse count.

Counting With a Minimum Number of Relays. The basic problem in designing a relay counting circuit is to establish some operating sequence of relays, allowing for the acting time of these relays, which will set up a unique combination for each pulse and each interval between pulses. Three relays, for example, can be operated in eight combinations. They can therefore count only four pulses, and then only if the circuit locks up during, or recycles after, the last pulse. Four of the eight combinations may be allotted to the four pulse intervals, one to the normal or null interval before pulsing starts, and the remaining three to the three intervals separating the four pulses. The circuit must lock up during the last pulse and ignore its termination, or return to normal after the last pulse. If the circuit is to recognize the termination of the last pulse and then lock up, a ninth combination is required which must be different from the normal condition preceding pulsing. In this latter case, a fourth relay is necessary.

If a circuit is to count a number of pulses, m, a number of relays, n, is required, such that $2^n$ is equal to or greater than $2m$ or $2m + 1$, depending upon the action of the circuit after the last pulse. For example, if ten pulses must be counted, either twenty or twenty-one combinations must be established, and at least five relays are required since four will produce only sixteen combinations.

Circuits based on a minimum number of relays can be designed, but usually become fairly complicated when more than four or five pulses are involved. Circuits in which the relays are used in most of their possible combinations are often characterized by large relay spring loads and complex contact networks, especially if the number of component relays exceeds three or four. Also, as mentioned earlier in the chapter, the translation to decimal notation from counting information based on many or all combinations of counting symbols becomes very involved.

In designing a minimum-relay counting circuit, the first step is to develop an operating sequence. It is evident from a consideration of the necessary circuit action during and after the pulses that either a
shunting scheme or the sequential operation of two or more relays must be used at one or more stages of the sequence. In the 10-pulse counter example above, the sequence must establish a different combination of relays operated for each interval of the original twenty or twenty-one interval sequence. No combination of the relays taken together with the pulse must be repeated even during the additional intervals (at least twenty more) created by the acting times of the relays. After a satisfactory operating sequence plan has been devised, the paths for operating and holding the various relays can then be developed by the methods presented earlier in the text.

Counting With Pulse Dividers. The method of counting a number of pulses discussed in this section employs more relays than the theoretical minimum, but allows a more orderly and less complicated sequence of relay operations as pulses are received and detected. As in most circuits, the simplest arrangement usually results from a scheme where each relay is controlled by only a few other relays or where the group of relays is controlled by some form of reiterative network. Also, if certain circuit events always follow certain other events and the action is repeated a number of times, a simple circuit normally results.

The following discussion deals with two-relay or three-relay circuit units rather than with single relays, the circuit units being the previously discussed pulse-frequency dividers. A pulse-frequency divider counts and recycles on two pulses. If a train of pulses is applied to the circuit, such as the arrangement of Fig. 11-4, the (W) and the (Z) relays will act once for each two input pulses. If one combination of these relays controls a second circuit identical to the first, and if this in turn controls a third, a pulse-counting scheme capable of counting a number of pulses results. The sequence of operation is shown in Fig. 11-8A and the circuit in Fig. 11-8B. Obviously the scheme can be extended by the employment of additional divider units, and any other type of pulse-frequency dividing circuit might be used. The circuit can also be varied by different interstage control networks.

Note that one divider unit counts two pulses and recycles, two divider units count four pulses and recycle, while three count eight pulses. All relays of three stages return to normal after the eighth pulse as in Fig. 11-8. By extension, n pulse dividers will count up to \(2^n\) pulses, and the total number of relays will be \(n\) times the number of relays per pulse divider unit. Since a circuit using \(n\) pulse dividers will return to normal and start the count over after the \(2^n\)th pulse, if the termination of the \(2^n\)th pulse is to be marked by a unique combination, at least one more relay must be added. Where the circuit is to count ten pulses, four divider units could be employed, to provide a circuit with a capacity of sixteen pulses. However, by adding to the three divider
units of Fig. 11-8B a single relay, which operates and locks during the eighth pulse, the count can be extended to a maximum of fifteen.

The counting arrangement of Fig. 11-8B corresponds exactly to the binary counting system illustrated in Table 11-1. Considering only the (W-) relays and assigning to them the value 0 when released and 1 when operated, the operation of (W1) corresponds to the farthest column to the right, that of (W2) to the center column, and that of (W3) to the farthest column to the left. The relays progress as in the binary count, and the conditions of the three relays give the number of the pulse in binary notation. The analogy also indicates why the three divider units of Fig. 11-8 are not sufficient to count eight pulses. Eight in binary notation is a four-place number, 1000, and three divider units do not provide a fourth place, but allow the count to return to 000 after the eighth pulse.

Fig. 11-8  Cascaded Relay Pulse-Frequency Dividers
A circuit made up of cascaded pulse-frequency dividers can be easily arranged to isolate itself from the pulse lead, freeing this lead and ignoring further pulses after a given number of pulses have been received, if this number is less than the $2^n$ maximum which the circuit may count. Consider the combination existing after all relays have completed their action following the end of the last desired pulse. This combination is unique and does not exist at any previous stage of the sequence. A circuit path can be developed which is open only during this interval. If this path is connected in series with the input pulsing lead, it will open the pulse lead at the proper stage and prevent further action of the circuit relays.

Counting schemes of the type illustrated in Fig. 11-8 have the disadvantage of being slow, since at a number of stages several relays must act in sequence. However, only the first pair of relays, (W1) and (Z1), are controlled by the pulse and, therefore only these relays must complete their action during the pulse or during the interval between pulses. The sequential action of subsequent relays may lag over into the following interval of pulsing without adverse effect, since the control pulse to each succeeding pulse divider is obtained from a preceding divider stage and not from the input pulse lead. The only disadvantage is that sufficient time must elapse after a given pulse for all the relays to complete their action before information on the count can be obtained from the circuit.

Another disadvantage of this type of circuit, particularly when the number of pulses to be counted is large, is that a number of relays must be examined to determine what the count is at any point in the counting sequence. It is not always necessary to examine all relays, although at least one relay in each pulse divider must be inspected. Thus, in order to close a circuit path during any one interval, a number of contacts are required, and in large circuits the total may be excessive.

Stages of electron tube two-pulse counters may also be connected in cascade, as shown in Fig. 11-9. The input of the second counter is coupled through a suitable coupling element to the output of the first, and so on. The operation of the coupling circuit used in the illustration deserves some attention. The starter anode of tube (Cl) receives sufficient voltage to ionize the tube when the capacitor coupling the cathodes of tubes (T1) - (T2) is charged and tube (T1) breaks down. This occurs on all odd input pulses except the first. Initially, the full supply voltage is applied to the main anode of (C1) but, as capacitor C becomes charged, this voltage decreases. The series combination of resistors R1 and R2 is of such a value as to cause (C1) to de-ionize as soon as the starter anode potential falls below the critical point. Thus, a moderately sharp pulse appears on the cathode of the tube (C1) for
each ionization of (T1), and the input to the second counting stage consists of positive pulses at a repetition rate of half that of the input to the first stage. The over-all operation of the circuit is indicated by the associated sequence diagram. This cascaded counting circuit suffers the same limitations as the corresponding relay circuit, although the reduction in the inherent speed may be of no practical significance.

![Diagram of cascaded cold-cathode tube binary counters](image)

**Fig. 11-9** Cascaded Cold-Cathode Tube Binary Counters

Counting With One Set of Relays Per Pulse. This section discusses circuits for pulse counting where one set of relays is assigned to each pulse in sequence and acts only when this pulse appears. The relays of each set not only detect the presence of a particular pulse, but
record its position in the sequence as it passes. This arrangement corresponds to a system of enumeration in which a unique symbol is assigned to each digit in the counting sequence.

The non-interfering single pulse counters discussed in Section 11.2 are easily adapted to counting a series of pulses. Consider first the prime counter of Fig. 11-2B. This is a very adaptable arrangement since relay \((X')\) acts after the pulse terminates, and any function it may perform commences after the termination of the pulse and is completed under local control. It may thus switch the pulse lead without interfering with its own action and without interrupting or clipping a control pulse. The break-contact on relay \((X')\) opens after the completion of the control pulse, disconnecting \((X)\) and \((X')\) from the control lead. The closure of the make-contact on \((X')\) can pass the control lead through to another similar circuit. Thus, a series of these circuits may be controlled in cascade as shown in Fig. 11-10 where the relays are designated numerically to correspond to the pulses to be counted. The action of the circuit is indicated by the sequence diagram.

This basic circuit of Fig. 11-10 can be modified in a number of ways. The control lead may enter the circuit at one of the intermediate
counters rather than at the first one. For example, if the pulse lead is connected to the armature spring of relay \( (2') \), as shown by the X wiring, the Nth counter will be actuated when the pulse numbered N-1 appears. This is a convenient arrangement when some function must be performed after a given number of pulses and this number of pulses is variable but known in advance. The last counter in this case carries contacts to perform the desired function, and some external circuit connects the pulse lead to the armature spring of the correct prime relay so that the last relay will always be operated after the desired number of pulses. In such cases the relays are usually designated numerically in the reverse order, \( (N) \) being \( (1) \). This system is known as "counting down", since with the new designations the counters act in descending order; for example, if the pulse lead is connected to counter number 6, it will "count down" to the number 1 counter on the sixth pulse.

![Diagram](Fig. 11-11 Prime Counter Circuit with Reversed Transfer Chain)

Inspection of the circuit of Fig. 11-10 shows that, as a result of the coupling transfer chain, the requirements for the operation of any particular counter is that all preceding counters in the chain be operated. This requirement might be replaced by one stating that, to operate a counter, all succeeding counters must be released. This may be satisfied by reversing the coupling transfer chain as shown in Fig. 11-11. This circuit acts in the same sequence as that shown in Fig. 11-10, but has the advantage that the relays preceding the last-operated pair may be released before the count is concluded.

In Fig. 11-10 and 11-11, each pair of relays is held operated by ground through a make-contact on the nonprime relay. In order to release the relays, this ground must be removed by control means not shown. The circuit of Fig. 11-11 may be modified so that each pair of relays is released by the action of some higher numbered relay. An example is shown in Fig. 11-12, in which only one pair of relays remains operated after any given pulse as indicated by the sequence diagram. The circuit makes no provision in itself for releasing the last pair. However, if the make of the transfer contact on relay \( (N') \) is connected to the winding of \( (1) \) and the locking ground for the Nth prime
counter is connected through a break on the \((1')\) relay, the circuit will be completely recycling.

Other modifications also are possible. For example, if the ground for each pair is obtained through a break-contact on either relay of the second higher numbered pair, two adjacent pairs will remain operated after each pulse other than the first. By other similar techniques, the action of any pair may be made to release different combinations of the lower numbered pairs which are in the operated condition at that stage.

A variation of the counting circuit based on prime counters is that in which all relays are normally operated; in the counting process the relays are successively released. In this arrangement, each pair of relays is operated from external sources and locked under control of the prime relay before pulsing starts. The appearance of a pulse shunts

![Diagram of the counting circuit](image)

Fig. 11-12 Prime Counters Released in Sequence
down the prime relay of the corresponding stage, the nonprime relay holding until the pulse terminates, as shown in Fig. 11-13. In this circuit, drawn to show the relays in their operated position, the relays must be pre-operated by a ground on their operating leads. This ground must be removed from each stage before pulsing proceeds to that stage. This may be accomplished by grounding the individual leads momentarily before pulsing starts or by operating each pair from ground through a make-contact on the next lower numbered prime relay and providing external means for operating the first pair. The circuit may be rearranged in various ways, many of which are analogous to the modifications discussed above for the original circuit of Fig. 11-10.

Basic circuits other than the prime counter, such as the circuit of Fig. 11-2A, may be similarly adapted to counting a train of pulses. Also, the principle of the circuit of Fig. 11-1A, which employs a pulse-repeating relay, may be extended to count multiple pulses. The relays that record the count are divided into two sets, one responding to front-contact closures of pulse repeating relay (P) and the other to back-contact closures. Transfer chains steer the control paths to the appropriate relays of each set. A circuit is shown in Fig. 11-14 in which the counting relays controlled by the front-contact on (P) are designated
(-F) and those controlled by the back-contact are designated (-B). The acting sequence is similar to that of Fig. 11-10, the (-F) relays corresponding to the nonprime relays and operating during the pulses, and the (-B) relays corresponding to the prime relays and operating between pulses. If the control contacts are to be completely freed after the last relay (NB) operates, a continuity-transfer on (NB) is required as shown; otherwise it may be omitted.

The arrangement of this circuit may be varied along lines similar to those indicated for the prime counter circuits. For example, in an arrangement similar to Fig. 11-11, the control leads from the transfer on (P) may enter the circuit of the counting relays on the last relays, (NF) and (NB), and proceed through break-contacts of a chain of transfers to control the various relays in sequence. Also, in a circuit similar to Fig. 11-12, certain relays may be released after the counting has progressed past them.

Counting Circuits Employing Common Relays. As has been discussed, relay pulse-counting circuits must provide means for detecting
the appearance and termination of each pulse as well as means for recording the count. In the majority of the circuits discussed in preceding paragraphs, each stage of the circuit not only recorded the count but individually detected the presence of its respective pulse. This is true even of the circuit of Fig. 11-14, since relay (P) merely repeats, rather than detects, the incoming pulses. One method of reducing the amount of equipment required is to provide a common means of detecting each pulse which will act in response to every pulse, and an additional means for recording the count as pulsing progresses. From the combinational aspect, most of the former circuits use only a few of the possible combinations and no relay operates more than once. It seems reasonable that some orderly scheme can be developed using fewer relays by permitting several relays to operate a number of times.

A pulse-frequency divider such as that of Fig. 11-4 is a convenient means for detecting the arrival and passage of each pulse of a series. Analyzing the action of this circuit, relay (W) operates at the beginning of the first pulse and releases at the beginning of the second pulse of a

\[
\begin{array}{ccccccc}
\text{p} & 1 & 2 & 3 & 4 & 5 & 6 \\
\text{RELAY (W)} & & & & & & \\
\text{RELAY (Z)} & & & & & & \\
\text{ODD} & & & & & & \\
\text{EVEN} & & & & & & \\
\text{RELAY (1)} & & & & & & \\
\text{RELAY (2)} & & & & & & \\
\text{RELAY (3)} & & & & & & \\
\text{RELAY (4)} & & & & & & \\
\text{RELAY (5)} & & & & & & \\
\text{RELAY (6)} & & & & & & \\
\end{array}
\]

Fig. 11-15 "Odd and Even" Counter with One Relay per Stage
considering any of the pulse-counting circuits of Fig. 11-10 to 11-14, note that two relays act in response to each pulse. In Fig. 11-14, specifically, the (-F) relays operate at the beginning of a pulse [when (P) operates] and the (-B) relays operate at the end of a pulse [when (P) releases]. Thus, if the relay (P) is replaced by relay (W) of the pulse divider, the (-F) relays will count the odd-numbered pulses and the (-B) relays will count the even-numbered pulses. A circuit shown in Fig. 11-15 employs one relay per counted pulse plus two relays for the common pulse-divider circuit. Although Fig. 11-15 shows a shunting
two-relay arrangement, the more rapid three-relay pulse-divider circuits of the type shown in Fig. 11-5 can also be used. Modifications of this circuit, similar to those performed on the basic prime counter of Fig. 11-10, can be carried out as desired.

The "odd and even" counter of Fig. 11-15 can be arranged to recycle if the transfer chains are reversed in direction and if each relay in operating opens the locking path of a preceding relay. In such a circuit, illustrated in Fig. 11-16, relay (1), for example, locks to a break-contact on (3). However, for some applications it might be desirable to lock each relay to the relay immediately following, relay (1) locking to a break-contact on (2) rather than on (3) as in Fig. 11-16. Such operation requires a somewhat different treatment of the locking path from that of Fig. 11-16 to prevent introduction of a hazard.

![Diagram of OPERATING LEADS](image)

As an illustration of this hazard, relay (2) of Fig. 11-16 operates through a make-contact on (1); and if (2) in operating is to release (1), relay (2) must lock itself before opening the locking circuit of (1) so that a buzzing condition will not occur. To effect this, a preliminary make-contact might be used as a locking contact on each relay. However, a simpler expedient is shown in Fig. 11-17 in which the locking ground is obtained through a chain of break-contacts on continuity-transfers. Each continuity-transfer locks its own winding to the chain before breaking the path to the previous relay. With the exception of the locking paths, the completed circuit is similar to that of Fig. 11-16. Note that in Fig. 11-16, a number of variations may be made in the wiring of the transfer chains composing the operating networks of the relays. For example, wiring X may replace the connection from the make-contact on relay (6) to the break-contact of relay (2) in the "odd" transfer network [thus eliminating the transfer-contact of relay (6)], and wiring Y may be used in the "even" chain.

The principles of the prime counter of Fig. 11-12 may be applied to develop still another type of common-relay counting circuit. In the former circuit, the operation of each prime relay, following the end of a pulse, releases the preceding pair. After the Nth or last pulse, the (N) and (N') relays are the only ones operated, and the action of the
(N') relay completely disconnects the pulse lead from the circuit. The Nth stage can now be connected back to the first stage in a manner similar to that in which the first stage is connected to the second. This permits the (N + 1)th pulse to actuate the first stage a second time, and the re-operation of the (1') relay can release the Nth stage. Thus, the modified circuit recycles repeatedly on continuous chains of pulses, progressing once over the N counters for each set of N pulses.

![Diagram of modified circuit](image-url)

Fig. 11-18 Modification of Prime Counter Circuit Using Common Prime Relays
Note that the prime relays in this circuit serve to steer the pulses to the proper nonprime relays and also to hold these relays operated for the required intervals. A variation which is suggested by this mode of operation is that of supplying a small group of prime relays which will control a larger group of count-recording nonprime relays. Analysis indicates that a circuit can be developed with a minimum of three prime relays plus as many count-recording relays as are desired.

Such a circuit is illustrated in Fig. 11-18 in which are provided three common prime relays, designated (P1), (P2), (P3), and an additional nonprime relay for each pulse to be counted. The nonprime relays are in three groups, each group associated with one of the prime relays. Transfer-contacts on the (P-) relays connect, or steer, the pulse lead to the three groups in rotation, and the operation of an individual nonprime relay in a group not only connects itself to its respective prime relay but also prepares an operating path for the succeeding group which is to record the appearance of the next pulse. The operation of the circuit can easily be followed by inspection of the sequence diagram of Fig. 11-18.

The circuits of Figs. 11-15 through 11-18 have a point of similarity in that a group of common relays recycles in each. In all these circuits except Fig. 11-18, two relays (a pulse-divider unit) recycle and a relay per pulse records the count, while in the circuit of Fig. 11-18 the three (P-) relays recycle to control a relay for each pulse. However, in the first scheme the (W) and (Z) relays recycle under their own control, while in the second the recording relays take part in the control of the recycling (P-) relays. Numerous schemes with recycling common relays may be worked out. In such schemes, several relays recycle repeatedly through the same set of combinations, but when all of the relays of the circuit are considered, some new combination must come into existence at each stage in order that new actions can be performed. Particular caution should be exercised where race conditions exist due to several relays acting in close but indeterminate sequence.

Circuits for counting a continuous train of pulses in any system of base notation may be developed along the lines indicated above. Each recycling of a counter represents an added place or position in the system of notation, while the number of stages in the cycle corresponds to the number of units in the corresponding place. Arrangements for recycling a counting scheme usually require careful study.

Partially Recycling Circuits. The recycling circuits that have been discussed may be extended to count continuous pulse trains of any maximum length as determined by the number of circuit elements provided. In many applications, the length of a train of pulses is limited to some
relatively small number; specifically, trains consisting of any number of pulses from one to ten, but no more, must often be counted. Circuits for this purpose are developed in this section. These circuits do not completely recycle but re-use certain relays in order to count to a maximum of ten with a reasonable amount of apparatus. If it is assumed that no more than ten pulses will appear in any given train, the action of the circuit on pulses beyond the tenth is of no consequence. Some external means is normally provided for releasing the circuit after a train has been counted.

If it is required to count a maximum of ten pulses with no count-recording relay recycling more than once, a circuit comprising six counters is possible. The first six pulses can activate these counters in sequence, each counter restoring the previous one to normal as pulsing progresses. After the sixth pulse, the sixth counter will remain operated and the circuit will provide means for holding this counter operated during any pulses beyond the sixth. The seventh pulse reoperates the first counter, the eighth pulse reoperates the second counter and releases the first, and so on, the fourth counter being reoperated by the tenth pulse. The counters operated after each pulse, then, are as shown in Table 11-2, the pulse number being the sum of the designations of the operated counters. Note that the counters are, in effect, operating in a base-6 system.

The circuit arrangement of Fig. 11-12 can easily be adapted to this scheme. Six prime counters must be provided and may be designated numerically from (1) through (6). The last counter, (6), locks to direct ground (that is, circuit off-normal ground) and thus will remain operated after the sixth pulse. The connecting circuit for the pulse input lead may enter the circuit on the armature spring of relay (5') in the manner shown dotted on relay (2') in Fig. 11-12. The transfer on relay (6') is then not required. The circuit is released by removing all holding grounds by external means.

An "odd and even" partially recycling counter, the arrangement of which is similar to that of Fig. 11-16 with the modification of Fig. 11-17 added, is shown in Fig. 11-19. It employs six relays to count ten pulses. The relays of this circuit are designated according to the pulses after which they remain operated. The operating sequence is as shown in Fig. 11-19A. Note that, in order to simplify the operating networks,
relay (5) is arranged to hold during the sixth pulse and release during the seventh pulse when relay (1) re-operates. A make on relay (6) recloses the operating path of relay (1) to the "odd" control lead during the sixth pulse (an even-numbered pulse) so that it may operate during the seventh pulse (an odd-numbered pulse). The transfer on relay (1) in the "even" control lead is necessary to prevent a back-up from the locking ground on the (6) relay. The circuit may be released following any pulse by removal of the holding grounds on relays (5) and (6). An external means may be provided for detecting the end of a pulse train so that the pulsing lead may be steered to another circuit or the relays may be restored to normal to count a subsequent train of pulses.

Counting With Electron Tubes. Several of the basic electron tube circuits discussed in Chapter 10 are applicable to the problem of counting a train of pulses. As an example from that chapter, a cold-cathode two-tube arrangement is illustrated in Fig. 11-20. In this circuit, the first pulse received fires the first tube, T1, and the second pulse fires the second tube, T2. This is accomplished by providing the enabling bias on the starter anode of the tube T2 from a tap on the cathode resistor of tube T1. Thus, tube T2 cannot fire until T1 is ionized. A capacitor across the cathode resistor of T1 prevents the bias on T2 from
increasing, during the application of the first pulse, to such a value that this first pulse can fire tube T2. The interval between pulses must be long enough to allow the bias to increase sufficiently to permit the second pulse to ionize T2. Resistor R is placed in the anode circuit of T1 to limit the conduction current through T1 while the cathode capacitor is uncharged.

Additional tubes can be added to this basic circuit to provide means for counting additional pulses. These tubes may be connected in cascade in the same manner as tube T2 is connected to tube T1, the enabling bias for each tube being provided by the tube immediately preceding it in the chain. In such a circuit, each tube, once ionized, remains ionized until its conducting path is opened.

For practical application, however, it is desirable to limit the circuit current drain as far as possible. Therefore, a more acceptable arrangement would be one in which each tube in firing would extinguish the preceding tube in the counting chain. This may be accomplished by
replacing the current-limiting resistances R, in series with each main anode, with a single resistance R1 in the common anode battery-supply lead, as shown in Fig. 11-21. At the instant a tube fires in this circuit, current through it is limited only by the anode resistance R1, with the result that the potential of all anodes drops, in a particular case, to about +75 volts with respect to ground. Since the cathode of the preceding tube is held momentarily to about +60 volts with respect to ground by its cathode capacitor, it is extinguished. This method of de-ionizing one tube when another fires has been discussed in Chapter 10.

The ST (start) tube in Fig. 11-21 is placed in the counting chain to provide tube T1 with an enabling bias until the arrival of the first pulse. When the anode battery supply is first connected to the circuit in preparation for counting pulses, the starter anode of tube ST is driven positive, due to the presence of the series capacitor C, thus ionizing the tube. The cathode capacitor of ST, in charging, raises the starter anode bias of T1 to a value sufficient to allow the first pulse to fire T1. When T1 fires, tube ST is extinguished by the process described in the preceding paragraph. Since the potential of the starter anode of ST decays to zero volts as capacitor C charges, tube ST cannot again be fired until the anode battery voltage is removed and then reconnected to allow a later chain of pulses to be counted.

The values of the cathode capacitors are critical, as is evident from the discussion above, and depend upon the durations of the pulses and the intervals between pulses. Then, too, as in all such electron tube circuits, the pulse amplitudes must not be great enough to ionize a tube unless its enabling bias is also present. Because of the variations in operating voltages for similar tubes and in the values of other circuit elements and supply voltages, the selection of bias potentials to insure reliable operation may be difficult. That is, not only must the enabling

![Fig. 11-22 Section of Counting Chain Showing Varistor Bias Control](image-url)
bias not reach a magnitude sufficient to fire a tube with no pulse present, but there should be no possibility of firing a tube in the absence of the bias. A modification to reduce the chance of false operation involves the addition of varistor or diode rectifiers inserted in the starter anode circuit of each tube, as shown in Fig. 11-22. The varistor elements prevent the enabling bias from rising appreciably above the +48 volt bias supply, measured from starter anode to ground. Thus there is small possibility of the enabling bias falsely firing a tube.

11.6 SEQUENCE CIRCUITS

A sequence circuit may be defined as a circuit which enables a series of events in a definite sequence. An example is the establishment of connecting paths between circuits in a fixed order as illustrated in Fig. 11-23. In this figure, a connecting path or a group of paths is cut through (A), from a common circuit to several individual circuits; (B), from several individual circuits to a common circuit; or (C), from several individual circuits to several other individual circuits. Sequence circuits are also referred to as walking, progress, or steering circuits.

Although the definition indicates several possible applications for the sequence circuit, the circuit arrangements themselves are fundamentally the same for all applications. That is, the primary function of the sequence circuit is to set up certain conditions in sequence. If the condition is a connection, as shown in Fig. 11-23, the connecting function itself may be performed by a separate circuit under control of the sequence circuit, or by a contact network constructed on the relays of the sequence circuit itself, depending upon the requirements to be fulfilled. In this section, only the sequence function is discussed without regard to the problems involved in the use of the circuit.

The operation of a sequence circuit can be represented by the diagram of Fig. 11-24. The intervals A1, A2, A3, and so on, represent the periods during which paths 1, 2, 3, and so forth respectively, must be closed. The intervals B1, B2, B3, and so on, are open periods which separate the A- intervals. The nature of this diagram indicates that the sequence function can be accomplished by a form of counting circuit in which either the A- or B- intervals are detected and counted, and a unique output condition is established for each interval detected. Here, in effect, the counting consists of designation rather than enumeration, the designations indicating a defined order.

The control pulses which actuate a sequence circuit correspond to those counted by a counting circuit. In general, these are supplied over a single lead, although there are cases in which each external circuit to be connected provides its own control lead.
As an illustration of the manner in which a counting circuit may be adapted to perform the sequence function, consider the block diagram of Fig. 11-23A. For this particular example the following requirements are to be fulfilled:

1. The circuit must close disjunctive paths between the common circuit and individual circuits 1, 2, 3, and so on, in sequence, upon the receipt of control impulses.
2. The period of closure to each of the individual circuits is represented by the intervals A-, in Fig. 11-24.
3. The control pulses are repetitive and occur during the A- intervals of Fig. 11-24.

---

**Fig. 11-23 Arrangements Involving Sequence Circuits**
Although several of the counting circuits discussed earlier in this chapter are applicable in this example, the circuit of Fig. 11-10 is frequently used for requirements similar to those above. This arrangement, as employed here, consists of one two-relay prime counter per individual circuit, the prime relays operating immediately following control pulses and the nonprime relays operating during control pulses. If the path from the common circuit to each individual circuit is to be closed during the respective A- interval and may also be closed during the preceding open interval, a logical contact network is a transfer chain on the prime relays with outputs from the break-contacts, as shown in Fig. 11-25 (Network A). This has the advantage that the path to any individual circuit is prepared in advance of the interval in which it is needed. If circuit requirements permit, however, the common-to-individual circuit network could be placed on the nonprime relays, as shown for Network B in Fig. 11-25. In this case, closure of each succeeding path is delayed beyond the start of the corresponding A- interval by the operating time of the nonprime relay. After the final connecting circuit is served, it is necessary for the common circuit to open the path holding the counters operated, restoring the circuit to normal. This particular sequence circuit can be extended indefinitely by adding additional prime counters. By proper choice of control conditions, it can be made to fit many of the situations requiring sequential operation. Other forms of prime counter circuits can also be adapted to fulfill particular sequence requirements.

A desirable objective in circuits to perform the sequence function is a circuit which requires but one relay per stage. This has been approached in the counting circuits discussed earlier in this chapter, but always at the additional expense of a group of common relays. A true relay-per-stage circuit can be designed, but is not often used for counting since it requires a heavy spring load on each relay for control and, to be completely reliable, may require two input pulsing leads.

The requirements for the circuit will be the same as those established for the previous sequence circuit with the additional provision that it be able to recycle, or act like a ring circuit. The attempt will be made to drive the circuit with a single pulse lead. A suitable operating plan to meet these requirements is shown on Fig. 11-26. Analysis of this plan indicates that the conditions for operating a typical relay are
that the pulse be present, the preceding relay be operated and the second preceding relay be released. The relay must hold to off-normal ground during the interval following the end of the pulse and to the pulse ground during the next succeeding interval. A circuit designed to meet these conditions is shown on Fig. 11-27.

The operating ground from the (CON) relay is carried to the winding of each relay through the break contact of a continuity-transfer in order to prevent a back-up of locking ground onto the lower branch of the pulse lead. The latter is used to hold the preceding relay during the pulse. An alternate method of preventing this back-up is a secondary

![Diagram](image-url)

**Fig. 11-25** Sequence Circuit Based on Prime Counters
winding per relay for holding. If a pair of input leads, \( p_1 \) and \( p_2 \), are practicable, neither of these methods is necessary.

The locking path for each relay is taken through a continuity-transfer on the succeeding relay in order to prevent momentary opening of the locking path when shifting from off-normal ground to pulse ground at the time this next relay operates. However, use of this continuity-transfer introduces an operating hazard in that the release of a relay momentarily connects ground back on the pulse lead during the bunching time of the continuity. If there is unfavorable contact stagger, this ground pulse is applied to the winding of the next relay and, unless minimum operating time definitely exceeds maximum bunching time, may
operate the relay falsely. Again, this hazard can be avoided by the use of two input leads.

Consideration of the operating plan indicates that some method of pre-operating the first relay is necessary for the circuit to function. This can be provided by a series path to ground through break contacts on all relays except the first, as shown by the top row of contacts on the circuit of Fig. 11-27.

If the recycling requirement is not specified, the circuit can be modified by substituting transfer chains for the separate makes and breaks shown for the operating paths in the circuit and for the output paths.

The design of sequence circuits to fulfill other sets of requirements may be carried out in a manner similar to the examples given above. Usually it is found that one of the conventional counting circuits may be used with little or no modification as a basis on which the required circuit arrangement may be constructed.

**PROBLEMS FOR CHAPTER 11**

11-1 A six-pulse relay counting circuit is to be constructed, using pulse-frequency dividers of the type shown in Fig. 11-4 as a basis for the design. The counter is to recycle following the termination of the sixth pulse. Relays to control the pulse dividers may be used where necessary. Assume a slow pulse rate. (This can be done with six relays including two pulse dividers.)

11-2 Design a three-pulse relay counter, employing stages of "prime-pairs". The circuit is to recycle after the third pulse.

11-3 Design a counting circuit to count a maximum of five pulses using the minimum numbers of relays. The circuit should not recycle after the fifth pulse. (Four relays)

11-4 A fast-acting counting circuit to count a maximum of six pulses is to be designed. The circuit arrangement is to include five relays but no resistors, since "shunt-down" operation would make the circuit undesirably slow. The circuit should lock upon the final pulse and furnish an output indication until released.

11-5 (a) On a main street in a certain city there is a series of five traffic lights. These five lights are to be controlled by a relay circuit in such a manner that the green and red lights regulating the main street traffic are lighted for one and a half minutes and one minute, respectively. In order to insure accurately-timed intervals, a source of ground pulses of one second duration with a rate of two pulses per minute is made available.

The five lights are to be controlled progressively: that is, the lights do not all change at once; the light at the second corner should change 30 seconds after the light at the first, and so on.

Design the required relay circuit. (This can be done with either five or ten relays, depending upon the type of sequence circuit used.)
(b) A sixth traffic light is added to the arrangement of part (a) at the corner following the fifth light. This new light is normally off. When a non-locking control key is operated, this sixth light should function in a manner similar to the other lights, following in sequence 30 seconds after the fifth light. However, the light should not begin to function until the beginning of its "green-light" interval.

A second operation of the key should discontinue operation of the sixth traffic light.
Add this feature to the circuit of part (a). (This can be done with four additional relays.)

11-6 An interrupter make-contact operates continuously in a cycle of one second closed, one second open. Design a circuit to deliver two complete one-second ground pulses after momentary operation of a single make-contact start-key. The output pulses should be the first two full pulses after the momentary start-key closure. Following the two output pulses, the circuit should automatically restore to normal. The start-key will not be reoperated during the acting time of the circuit. (This can be done with four relays.)

11-7 If a coin is tossed a number of times, it may be expected that the number of "heads" will be approximately equal to the number of "tails". A relay circuit is to be designed to aid in the study of this phenomenon.

To control this circuit, two non-locking keys (H) and (T) are provided, each with a single make-contact having one spring grounded. After each toss of the coin, one of these keys is operated: key (H) if the coin falls "heads", and key (T) if the coin falls "tails". If, after any toss, the total number of "heads" obtained since the beginning of the test run exceeds the number of "tails" by three, a lamp designated (3H) is to light. Similarly, if at any time the total number of "tails" exceeds the total number of "heads" by three a lamp designated (3T) is to light. The total number of tosses is not of interest to the circuit.

After either the (3H) or (3T) lamp lights, a new test run must begin. The circuit is cleared, and the lighted lamp extinguished, by operation of a reset key. If, however, the operator fails to clear the circuit and, instead, operates key (H) or key (T) after either lamp (3H) or (3T) is lighted, an alarm lamp (ALM) is lighted and remains lighted until the reset key is operated. (This can be done with nine relays.)

SPECIAL PROBLEM

This problem is useful and instructive as a combined circuit design and laboratory construction exercise. Although the circuit is not specifically of the counting type, the principles covered in this chapter are useful in its solution.

The problem is to design and build the circuit portion of an electric combination door or safe lock. The complete lock consists of a relay circuit, a set of control keys, and two lamps, one representing an electromagnetic door latch and the other, an alarm bell. The key and lamp arrangement to be used is shown on the figure below. The circuit should employ no more than five single-winding general-purpose relays.

The "lock lamp" (L) and the "alarm lamp" (ALM) should light under the following conditions:
Lock lamp (L):

When control keys (A), (B), (C), (D) are depressed and released in the sequence D-B-C-B-A. (The assigned sequence may be varied, as A-B-C-B-A, B-D-B-A-C, C-A-D-A-D, or any other sequence of five.)

Alarm lamp (ALM):

1. If an incorrect sequence of control keys is pressed.
2. If two or more keys are depressed simultaneously.
3. If the same key is depressed twice consecutively, unless required by the specified sequence.

When either lamp (L) or lamp (ALM) has lighted, no further operation of keys (A), (B), (C), or (D) should affect the output conditions. Operation of a release key (RL) at anytime in the cycle should restore the circuit completely to normal. Key closures of duration less than the acting time of a relay need not be considered as input signals.
Chapter 12

CODES AND TRANSLATING CIRCUITS

The input signals to large switching systems often represent either data which are to be processed by the system, or instructions which specify the procedures to be followed. These signals should be thought of as "information" rather than as remote-control means for setting switching devices. The information is recorded in the system by operating relays or other switching devices in combinations, and is passed from point to point by combinations of signals on groups of interconnecting leads. A set of combinations of elements in which each combination represents an item of information is a "code". Depending upon circumstances, one set of combinations may be more appropriate than another to represent particular information, and thus several codes for the same information may exist. A circuit arrangement which converts from one code to another is a "translating" circuit. This chapter will discuss various methods of representing information in the form of a code, and will also discuss the design of translating circuits.

Numbers are used as a universal "language" for recording and transmitting information in switching systems. They are used in three ways: (1), to represent quantity; (2), to specify order; (3), as arbitrary symbols or designations. Numbers representing quantities are used extensively in problems solved by digital computers. There are also many instances where switching systems count objects or events and retain a record of the quantity. An example of numbers used to specify order is where a group of leads or switch terminals are numbered in the order of their physical location. The number of a particular lead or terminal then defines its location with respect to the other leads of the group. Numbers may be used as convenient arbitrary symbols to represent instructions given to a switching machine, or to identify entities which have no numerical characteristics of quantity or order. For example, telephone subscribers are assigned numbers. In some systems the numbers correspond to the location of this subscriber line on the terminals of selecting switches, and the number consequently contains intrinsic information which describes the location of the line. In other systems, however, there is no systematic correspondence between subscriber number and line location. The number is purely an arbitrary symbol identifying a particular subscriber. These telephone systems are provided with electrical "cross-reference" circuits in which are
recorded the line locations of all subscribers. This information for a particular line is made available when that line number is received.

Where numbers are used as an orderly set of designations, natural groupings of the designated items often correspond to a system of enumeration. Take for example a group of ten switches, designated 0 to 9, where each switch has ten terminals which in turn are designated 0 to 9. An individual terminal can be identified by a two-digit decimal number, where the first digit gives the number of a switch and the second digit gives the terminal on this switch.

If there were ten groups of switches each containing ten switches, the system could be extended to three digits where the additional digit in the "thousands" place is the group designation. In practical examples, the number of terminals on a switch and the method of grouping often does not conform to a decimal system of designations. The system of enumeration is adapted to the size and grouping of the switches and may be not only non-decimal, but also mixed (that is, the "base" for each digit position is not the same).

An example is a case where 10,000 terminals on the banks of twenty 500-point switches must be identified individually. The twenty switches are divided into five groups, each group containing 2,000 terminals and consisting of four switches, each switch containing 500 terminals. The switches are constructed so that the 500 terminals of each switch are divided into five groups of 100 terminals. These in turn are divided into ten subgroups of ten terminals each. A particular terminal is identified by specifying successively smaller groups and subgroups in which it is located. The first and largest group is one of the five groups of four switches each; the second is the particular one of these four switches. Having specified a particular switch containing 500 terminals, the particular terminal can be identified as being within one of the five groups of 100 terminals on this switch. Within this group, it is located in one of ten subgroups and is a particular one of the ten terminals. Thus the terminals may be identified by five-digit numbers, where the bases of the digits are 5, 4, 5, 10, and 10.

A non-decimal numbering system of special importance in the switching art is the binary or base-2 system. As discussed in the previous chapter, in this system each "place" has but two values, 0 or 1. The importance of the system is that it may be represented directly by the two-valued elements used in switching. Thus, for example, a relay may be assigned to represent each place in binary notation and the relay may be operated or not, depending on whether the "value" of this place for the particular number is 1 or 0.
12.1 HOW INFORMATION IS REPRESENTED

The digit in each "place" of a multidigit number may be represented in switching circuits by providing apparatus which is capable of assuming a number of distinctive "positions" or "conditions". A single decimal digit may be represented, for example, by a ten-point switch which is set on the point corresponding to the digit value. When relays are used to represent multivalued digits, several relays are provided for each digit place and these relays stand operated or released in specified combinations to represent the digit values. For example, four relays may operate in sixteen different combinations (including all released), and ten of these combinations may be chosen to represent the digits 0 to 9. Such a set of combinations which have defined meanings is a "code".

The "elements" of a code are those items which are available for choice in forming the combinations composing a code. In the above example, where four relays are operated in combinations to represent the decimal digits, the code is a four-element code, each relay corresponding to an element. Most of the discussion in this chapter is concerned with two-valued elements where one of the two "values" is specified as an active condition. Some of the two-valued elements used in switching for representing codes are shown in Table 12-1.

<table>
<thead>
<tr>
<th>Code Element</th>
<th>Active Condition</th>
<th>Inactive Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relay</td>
<td>operated</td>
<td>released</td>
</tr>
<tr>
<td>Circuit input or output lead</td>
<td>grounded</td>
<td>open</td>
</tr>
<tr>
<td>Circuit input or output lead</td>
<td>battery on</td>
<td>open</td>
</tr>
<tr>
<td>Circuit input or output lead</td>
<td>lead</td>
<td></td>
</tr>
<tr>
<td>Gas tube</td>
<td>fired</td>
<td>extinguished</td>
</tr>
<tr>
<td>Position on paper</td>
<td>hole punched</td>
<td>no hole</td>
</tr>
</tbody>
</table>

Table 12-1 Typical Code Elements

In addition to numerical information, codes are also used to represent other forms of symbolism. For example, a five-element code (maximum 32 combinations) is used to represent the letters of the alphabet in teletype and other applications. Also, in a switching circuit where a variety of input control signal combinations each cause a different circuit response, a tabulation of the input signal combinations with the corresponding circuit actions is the equivalent of a code, and certain parts of the circuit design may be similar to that of translating circuits. Since codes representing the integers 0-9 occur most frequently in switching systems, the discussion in this chapter will be confined to these codes.
12.2 SIMPLE COMBINATIONAL CODES

Any set of ten different combinations of a number of elements may be arbitrarily assigned as a code to represent the integers 0 to 9. The combinations forming a code sometimes arise naturally due to particular circuit actions. For example, in the single decade counter of Fig. 11-19, the digits 1, 2, ... 9, 0 are represented by the operation of either one or two of the six relays in the circuit. The combinations form a code for representing the ten digits as shown in Table 12-2.

At least four code elements are necessary to represent the ten integers 0 to 9. Four two-valued elements will give a total of sixteen combinations, and any ten of these may be used to form the code; but it is desirable to assign the combinations to represent the integers in a systematic manner. Most of the standard codes in switching systems are "additive", that is, the elements of the code are given numerical designations and the combinations are chosen so that the sum of the designations of the activated elements (for example, operated relays) gives the value of the decimal digit represented by that combination. This makes the code combinations easy to remember, a characteristic of considerable practical importance. Four-element codes of this type are shown in Table 12-3, in which the designations of the code elements are given at the top of each code column and the elements activated for each digit value are tabulated in the column.

When applying these codes in practical problems, special consideration must often be given to the combination to be used for zero. In many cases it is satisfactory to let the "blank" condition of all relays released represent zero. However, the zero condition is then the same as the normal condition when the circuit is idle. When it is desired to let zero be represented by an active combination, the additive combination for ten may be used or one of the unused combinations may be assigned arbitrarily (e.g., 1-4 in the 1-2-4-5 code). The choice of a combination to represent zero often depends on whether zero occurs as the first or the last of the integers 0 to 9. For example, if digits are represented by sequences of pulses, the digit "0" will be represented by ten pulses. If it is then to be converted to a code it probably should be represented by an active combination.

Each of the four-element codes has characteristics which makes it more suitable for some applications than others. The 1-2-4-8 code

<table>
<thead>
<tr>
<th>Digit</th>
<th>Designation of Relays Operated</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1)</td>
</tr>
<tr>
<td>2</td>
<td>(2)</td>
</tr>
<tr>
<td>3</td>
<td>(3)</td>
</tr>
<tr>
<td>4</td>
<td>(4)</td>
</tr>
<tr>
<td>5</td>
<td>(5)</td>
</tr>
<tr>
<td>6</td>
<td>(5), (6)</td>
</tr>
<tr>
<td>7</td>
<td>(1), (6)</td>
</tr>
<tr>
<td>8</td>
<td>(2), (6)</td>
</tr>
<tr>
<td>9</td>
<td>(3), (6)</td>
</tr>
<tr>
<td>0</td>
<td>(4), (6)</td>
</tr>
</tbody>
</table>

Table 12-2 Code for the Single-Decade Counting Circuit of Chapter 11
corresponds to the binary system of notation and is often called the "binary" code. The values of the elements are powers of 2, that is, $2^0$, $2^1$, $2^2$, and $2^3$; and these values can be added together in only one way to give a particular digit. Thus there is no ambiguity in determining additive combinations. Although only ten combinations are used, the code retains its additive nature for all of the sixteen possible combinations.* It can easily indicate the odd or even character of a registered number since the element "1" appears only in odd digits.

When "0" is considered as the first one of the ten integers, the 1-2-4-5 code has the characteristic that the digits below five (0, 1, 2, 3, and 4) do not contain code element "5" in their combinations, while all of the remaining digits (5, 6, 7, 8, and 9) contain element "5". Note that the non-additive combination of 1 and 4 for zero (instead of $1 + 4 + 5 = 10$) conforms with this use. When "0" is considered as the last of the integers, the 1-2-4-6 code has similar characteristics. The element "6" can be used to distinguish between the first five integers (1, 2, 3, 4, and 5) and the last five (6, 7, 8, 9, and 0).

The four-element codes of Table 12-3 have the common characteristic that a single "error" may convert one valid combination of the

Table 12-3 Four-Element Additive Codes

<table>
<thead>
<tr>
<th>Digit</th>
<th>Code Elements 1-2-4-8</th>
<th>Code Elements 1-2-4-5</th>
<th>Code Elements 1-2-4-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1-2</td>
<td>1-2</td>
<td>1-2</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>1-4</td>
<td>5</td>
<td>1-4</td>
</tr>
<tr>
<td>6</td>
<td>2-4</td>
<td>1-5</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>1-2-4</td>
<td>2-5</td>
<td>1-6</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>1-2-5</td>
<td>2-6</td>
</tr>
<tr>
<td>9</td>
<td>1-8</td>
<td>4-5</td>
<td>1-2-6</td>
</tr>
<tr>
<td>0*</td>
<td>2-8</td>
<td>1-4</td>
<td>4-6</td>
</tr>
</tbody>
</table>

*Digit 0 may be represented by a blank combination in any of these codes.

* This supposes that the blank condition is used for zero although, as indicated in the table, 2-8 is often used.
code to another valid combination in the code. In the case of relays, an "error" consists of either the failure of a relay to operate when it was intended to operate, or the false operation of a relay when it was intended to remain unoperated. For example, in the 1-2-4-5 code, if the intention is to indicate digit 3 by operating relays designated (1) and (2), an open lead which causes relay (1) to fail will falsely indicate digit 2. Again, if digit 3 is intended and a trouble ground causes relay (5) to operate as well as relays (1) and (2), then digit 8 is falsely indicated.

12.3 SELF-CHECKING CODES

By choosing the combinations of a code so that a single error produces a combination which is not valid in the code, the occurrence of an error can be detected. Codes of this nature are called "self-checking" codes. At least five elements are necessary to provide ten self-checking combinations for the decimal digits. A five-element self-checking code is shown in Table 12-4. This uses the ten combinations of five elements taken two at a time, and is known as a two-out-of-five code. The elements are designated 0, 1, 2, 4, and 7, and the combinations are additive except for zero which is 4 and 7. With this code, any single error resulting in the false operation or failure of a relay to operate will leave either one or three relays operated and can be easily detected.

A second self-checking code shown in Table 12-4 uses five relays designated 0 to 4, and two additional relays designated 00 and 5, which operate respectively when the digit is below six or above five. This corresponds to the biquinary system of notation and is useful in the design of circuits to perform arithmetic calculations. This code is, therefore, often used in computers.

The two codes of Table 12-4 are sometimes abbreviated by omitting the "zero" elements. This, of course, destroys the self-checking nature of these codes but reduces the number of code elements required. This simplified representation of the biquinary code is widely

<table>
<thead>
<tr>
<th>Digit</th>
<th>Two-out-of-Five 0-1-2-4-7</th>
<th>Biquinary 00-5-0-1-2-3-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-1</td>
<td>00—1</td>
</tr>
<tr>
<td>2</td>
<td>0-2</td>
<td>00—2</td>
</tr>
<tr>
<td>3</td>
<td>1-2</td>
<td>00—3</td>
</tr>
<tr>
<td>4</td>
<td>0-4</td>
<td>00—4</td>
</tr>
<tr>
<td>5</td>
<td>1-4</td>
<td>5—0</td>
</tr>
<tr>
<td>6</td>
<td>2-4</td>
<td>5—1</td>
</tr>
<tr>
<td>7</td>
<td>0-7</td>
<td>5—2</td>
</tr>
<tr>
<td>8</td>
<td>1-7</td>
<td>5—3</td>
</tr>
<tr>
<td>9</td>
<td>2-7</td>
<td>5—4</td>
</tr>
<tr>
<td>0</td>
<td>4-7</td>
<td>00—0</td>
</tr>
</tbody>
</table>

Table 12-4 Self-Checking Codes
used in computers since it produces quite simple circuits for performing addition. For this reason it is preferred over either the four-element codes which require one less code element, or the two-out-of-five code which is self-checking but not suited to calculating circuits. The full seven-element biquinary code is both self-checking and convenient for use in calculating circuits.

12.4 THE THEORY OF ERROR-DETECTING AND ERROR-CORRECTING CODES

The self-checking quality of the two-out-of-five code depends on the relations between the various combinations chosen to represent the items of the code. The combinations are so related that at least two errors (false operation or non-operation of a relay) are necessary to convert one valid combination to another valid one in the code. The principle can be generalized and extended to detect not only the occurrence of single errors but also, by increasing the number of elements (relays or leads), to detect multiple errors and to indicate which elements have failed when an error occurs. That is, a code can be made not only "self-checking" but also "self-correcting," and the error-detecting and error-correcting ability can be extended to any number of simultaneous errors. This, of course, is accomplished at the expense of increasing the number of elements in the code.

In order to explain the general principle, let the "distance" between two combinations be defined as the number of changes of relay positions (operated and released) necessary to convert one combination to another. For discussion purposes, combinations may be represented as in switching algebra, where 0 and 1 represent respectively the operated and released positions of a relay, and the symbols are written in a prescribed order for a given set of relays. For example, the particular combination of three relays, A, B, and C, in which A is operated, B released, and C released may be written 011. To illustrate the meaning of "distance" the combinations 111, 001, and 010 are all at a distance of one "change" from the combination 011. The distance between combinations 011 and 101 is two changes, and so on.

A single error in a given combination will produce a combination at a distance of one from the original. Two simultaneous errors will make this distance two; three errors will produce a distance of three; and so forth. In order to produce a set of combinations which is self-checking for single errors, it is necessary only to select a set of valid combinations in which the distance between combinations is at least two. An error then produces a combination which can be recognized as not being one of those used in the code. This is illustrated for three relays in Fig. 12-1 which represents the eight possible combinations in
their proper relation by the corners of a cube.* Distances between combinations are measured along the edges of the cube. The four combinations, 000, 011, 101, and 111, indicated by heavy dots in the figure, are all at a distance of two from each other.

To produce a "single-error-correcting" code, the combinations are chosen so the minimum distance between combinations is three. A single error then produces a combination at a distance of one from the intended combination but at a distance of two or more from any other combination of the code. Thus single errors can be corrected by changing to the nearest code combination. This is illustrated in the cube of Fig. 12-1 by the pair of combinations 000 and 111, which are separated by a distance of three. A single error in 111 results in one of the combinations 110, 101, or 011, each of which is closer to 111 than to 000. Only a single pair of self-correcting code combinations is possible with three elements, but these may be any of the pairs at opposite ends of a diagonal of the cube. To represent a larger number of items by a self-correcting code, the number of elements must be increased.

A set of single-error-correcting combinations fails in the presence of two errors. The occurrence of two errors produces a combination which is nearer to some combination other than the correct one, and thus an attempt to correct in this case results in the combination being "corrected" improperly. If the minimum distance between code combinations is increased from three to four, a double error will bring the combination to a point halfway between two (or more) of the code combinations and thus the double error can be recognized. Codes having a minimum distance of four are able to correct single errors and to detect the presence of double errors. By increasing the distance between combinations, codes can be developed to correct or detect the presence of any number of errors.

The "distance" concept is helpful in understanding the theory of error-detecting and error-correcting codes. However, in practice the

* Similar figures for a larger number of elements correspond to cubes of higher dimensions. The four-dimensional "hypercube," for example, has sixteen corners corresponding to the combinations of four elements. These geometrical representations were discussed in Section 8.7, the appendix to Chapter 8.
distance requirements are arrived at indirectly, and it is difficult to see intuitively that the various combinations have the desired distance relations. Mathematical proofs are not given here but actual trial of a few examples will show that the codes do permit errors to be detected and corrected, and therefore the distance requirements are met.

In developing a code, a set of combinations having the required characteristics can be chosen so that a system of "even-or-odd" checks of the elements indicates the presence of errors. Certain elements of the code may carry the "information" while other elements provide the checks and are chosen according to the combination of the information elements. This is illustrated by Table 12-5, which shows a check element X and four information elements A, B, C, and D. All of the sixteen possible combinations of A, B, C, and D appear in the table. For each combination, X is chosen to be 0 or 1 to make the total number of 0's an even number. A single error in any element will result in an odd number of 0's (either 1, 3, or 5) and thus can be easily detected. Ten of the combinations have exactly two 0's, and these are used in practice to represent the decimal digits in the two-out-of-five self-checking code which has been discussed. This permits a somewhat better check than a straight odd-even check since two errors must be of an opposite nature to escape detection. That is, one relay must falsely release and another falsely operate. In the additive 0-1-2-3-7 code in Table 12-4, the zero element is purely a check element since its inclusion in an additive combination does not affect the total.

To indicate the location of an error so that a coding system is self-correcting, additional odd-even checks are necessary. One method is to check a group of coded digits by providing a supplementary
"check digit" whose combination of 0's and 1's is determined from the
group of digits which it checks. The method is explained by referring to
Table 12-6, which shows the development of a check digit for the six-
digit number 825423. The six digits are represented in the 0-1-2-4-7
code as shown in the table. The 0's and 1's of the check digit are chosen
for these particular six digits so that there is an even number of 0's in
each of the five columns of the table. Errors are located by a combina-
tion of "row" and "column" checks. Each digit corresponds to a row in
the table, and a failure of the "even" (or two-out-of-five) check of zeros
in this row indicates an error in one of the elements of this row. Fail-
ure of the "even" check of the seven elements of a column indicates an
error in that column. Thus the element at the intersection of the row
and column is in error and if 0, should be changed to 1, or if 1, should
be changed to 0.

This illustrates the basic principle of self-correction. Each ele-
ment is checked by several checks of the odd-even type in such a way
that no two elements are checked by the same combination of checks.
In the above example, each row and column of the table has its own odd-
even type check, and the intersection of a row and column indicates a
particular element.

In the correcting scheme illustrated by Table 12-6, a single check
digit can serve for a series of digits of any length. The possibility of a
double error, however, increases as the number of digits increases.
The scheme recognizes the presence of a double error but cannot cor-
rect or in some cases even determine which digits are in error. Two
errors in general cause two row checks and two column checks to fail
and cannot be corrected with certainty. If two errors occur in the same
column, two row checks and no column checks will fail. Thus there is
no indication of which column is in error. Similar effects occur for
double errors in the same row.

A coding method, developed by R. W. Hamming, will now be de-
scribed which permits individual code combinations to be corrected for
single errors. By this method, seven code elements can be arranged in
sixteen self-correcting combinations. Four of the seven elements are
information-carrying elements and may be used in any of their sixteen
possible combinations. The remaining elements are check elements and
their value (0 or 1) is determined according to a system of odd-even
checks.

The system is explained by Table 12-7, where the seven positions
(columns) correspond to the elements of the code. Positions 1, 2, and 4
are occupied by check elements, and the remaining positions, 3, 5, 6,
and 7, contain elements designated A, B, C, and D which carry the in-
formation. Three odd-even checks are applied to subgroups of the seven
elements. As indicated by asterisks in the table, the first check, which
will be referred to as the X1 check, is applied to the set of four elements located in positions 1, 3, 5, and 7. For a particular code combination of the A, B, D elements, the value of the check element in position 1 is chosen to be 0 or 1 so that the total number of 0's in the four positions is even. The second check, X2, applies to positions 2, 3, 6, and 7. Element 2 is made 0 or 1 to satisfy an even check of 0's in these positions. In a similar manner the third check, X3, is made on positions 4, 5, 6 and 7.

Before explaining the significance of this subdivision of elements into check groups, the application of the above process will be illustrated using the combination 0101 in the information positions. The values of the elements in the three check positions are determined as follows:

| Position: | 1 2 3 4 5 6 7 |
| A B C D |
| - - 0 - 1 0 1 |

To have an even number of 0's:

- Position 1 must be 0
- Position 2 must be 1
- Position 4 must be 0

Since A is 0, and B and D are 1's, the first position must be filled with an 0 in order to make an even number (two) of 0's in the four positions 1, 3, 5, and 7. In a similar manner positions 2 and 4 are found to be 1 and 0 respectively. The complete seven-element combination, when the three checks are included, is 0100101.

The significance of this subgrouping of elements into check groups is that each of the seven elements is in a different combination of check groups. For example, element 1 is in check group X1 only, element 2 in X2 only, element 3 in X1 and X2 but not in X4, and so on, element 7 being the only element in all three check groups.

To determine whether an error has been made in a particular seven-element code combination, the three odd-even checks X1, X2, and

<table>
<thead>
<tr>
<th>Check</th>
<th>Code Positions Checked</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>*</td>
</tr>
<tr>
<td>X2</td>
<td>*</td>
</tr>
<tr>
<td>X3</td>
<td>*</td>
</tr>
</tbody>
</table>

Table 12-7 Check Arrangement for Self-Correcting Code
X3 are made. If all checks are even, then no error has been made. If one or more checks indicate odd, then the error was made in the position checked by this combination of checks. The checks which fail indicate in an additive code the position of the element which is in error. For example, a failure of X1 and X2 indicates that the position 1 + 2 = 3 is in error; failure of X2 and X4 indicates position 6; all three, position 7; and so on. To show the correction procedure, assume that the combination 0100101 which was developed above is received with an error which results in the combination 0100001. The checks are applied as follows:

<table>
<thead>
<tr>
<th>Position</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Number of 0's in check:

- X1 = Odd
- X2 = Even
- X4 = Odd

Checks X1 and X4 show an odd number of 0's and therefore the element in the fifth place is in error. Since this is 0, it should be changed to 1 to give the correct combination. Note that this procedure will correct errors in the check elements in positions 1, 2, and 4 as well as the information elements. If only one of the checks gives an odd count the error is in the corresponding check position. A table of the entire sixteen self-correcting combinations based on the above procedure is given in Table 12-8.

Ten of the sixteen combinations in Table 12-8 may be arbitrarily chosen to represent the integers 0 to 9; if desired, the information elements A, B, C, and D may be assigned values according to one of the four element additive codes. The code may be varied by substituting odd checks for even checks in any one or more of the three checks.

If more than sixteen self-correcting combinations are required, the number of information elements of the code must be increased and this in turn necessitates an increase in the number of check elements. The addition of a fourth check symbol in position 8, following position 7, will
permit the code to be extended to elements in a total of fifteen positions, eleven of which are information positions whose elements may assume any desired combination. This fourth check, X8, will be associated with positions 8 through 15 as a group, and the previous checks, X1, X2, and X4, will be extended through positions 9 to 15, repeating the same pattern as in positions 1 to 7. In this manner the code may be extended to as many positions as desired, n-check elements being able to indicate single errors in \((2^n - 1)\) positions.

The correcting feature of this code fails when two errors occur. Applying the check procedures will result in one or more odd checks, but applying the indicated correction will result in a combination which is not the one originally intended, although it is one of the established combinations of the code. This can be verified by trial, using any of the combinations in Table 12-8. One way to indicate the occurrence of two errors is to add an eighth element to the seven-element code. This "master check" element, M, may be placed in position "zero" ahead of position 1. It is made 0 or 1 in a particular combination so that the total number of 0's in the eight positions is even. Failure of this check then indicates the occurrence of a single error. The correcting procedure may be deduced as follows:

<table>
<thead>
<tr>
<th>If Master Check of All Elements is:</th>
<th>And If Group Checks X1, X2, X4 are:</th>
<th>Then the Indicated Condition is:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Even</td>
<td>All Even</td>
<td>No Errors</td>
</tr>
<tr>
<td>Odd</td>
<td>Some Odd</td>
<td>Single Error: Correctable</td>
</tr>
<tr>
<td>Even</td>
<td>Some Odd</td>
<td>Double Error: Not Correctable</td>
</tr>
<tr>
<td>Odd</td>
<td>All Even</td>
<td>Error in Master Check Position: Correctable</td>
</tr>
</tbody>
</table>

This eight-element code gives erroneous indications only when three or more errors occur. Further reliability, if demanded, would require the use of additional check elements.

Error-correcting codes of the above type permit the correct information to be reconstructed on a digit-by-digit basis when errors occur. For example, when information is recorded as punched holes in a paper tape, errors due to mutilation or flaws in the paper could be corrected at the time the information is read. Various advantages may be illustrated by a circuit arrangement indicated in Fig. 12-2. The input consists of eight leads which include three check leads, four information leads, and a master check lead for double-error detection.
These leads are normally grounded in the combinations of a code which corrects single errors and detects double errors. When no errors occur, the input combinations are repeated on the output leads. In case a single error occurs in a combination presented to the input leads, the correct combination will be produced on the eight output leads and, in addition, some combination of the X1, X2, and X4 leads is grounded to indicate which input lead is in trouble. Thus the apparatus on the output could continue to function properly and, if desired, the trouble information could be recorded and the trouble cleared at some later time. In case of two errors in the input combination, the double-error lead (DE) is grounded and no other output leads grounded. This indicates an inoperative condition which cannot be automatically corrected.

The output leads, instead of merely repeating the input code, could present the information in any other code arrangement desired. That is, the circuit may translate as well as correct. Theoretically, this circuit requires only eight relays, one on each input lead, and may be designed by ordinary combinational methods. However, it is evident that the contact networks will be rather elaborate, and auxiliary relays are necessary if relays of practical size are to be used.

12.5 TRANSLATING CIRCUITS

A translating circuit is one which converts or translates information received in one code to the same information expressed in some other code. As indicated in the block diagram of Fig. 12-3, the circuits are basically combinational. Input conditions operate relays according to one code, while contacts on these relays ground (or otherwise activate) output leads according to the combinations of a second code. A given input combination always produces the same output combination.
regardless of the sequence in which input combinations occur. Translating circuits may work between any two codes, the circuit usually being a "one-way" device. That is, a circuit which translates from a code K1 to a second code K2 will not also perform the inverse translation of K2 to K1.

In some translating circuits the input leads directly control relays whose only purpose is to perform a translating function by means of a network of contacts. In other cases the relays carrying the translating network may be intimately involved in some other function, and render it difficult to isolate actual input leads to the translating circuit. The input conditions then are considered to be the operation of the relays rather than conditions appearing on specific input leads. An example might be a counting circuit, with the sequence shown in Table 12-2, arranged to ground one of ten output leads according to the number of pulses counted. The counting function is the means for causing the relays to establish successive combinations for counting the input pulses, while the translating circuit network causes these combinations to ground one of ten output leads.

12.6 DESIGN TECHNIQUES FOR TRANSLATING NETWORKS

Although the design of a translating network involves only the usual principles of contact network design, certain characteristics of the code combinations and the practical requirements which must be met by the network indicate lines along which circuits may be developed. If the input code combinations include all of the possible combinations in which the relays may operate, then the action of the circuit for all conditions is completely specified and the design of a network is entirely routine. However, when some of the possible combinations are not used, as is more often the case, decisions must be made as to whether these invalid combinations may represent closed paths in the contact network. By taking advantage of the invalid combinations it is often possible to reduce the required number of contacts in the network. Consider as an example any one of the four-element codes of Table 12-3 where "0" is represented by operated relays. Of the sixteen different combinations in which four relays may operate, one is the "normal" or "all relays released" condition, ten combinations represent the decimal
digits in a four-element code, and five combinations remain unused. In most circuit applications, these five invalid combinations never occur in normal circuit operation, so that allowing the network to close one or more of its output leads for these combinations is theoretically satisfactory and may permit a simpler network to be devised.

An invalid combination will occur only because of a trouble condition such as a defective relay or contact, or trouble crosses, grounds, or opens in interconnecting leads. If the nature of a circuit and its surroundings is such that false combinations are extremely unlikely, or if the occurrence of a false combination causes only local reactions, the best procedure is to design the simplest possible network which satisfies the code conditions and to permit output leads to be open or closed as convenient for the invalid combinations. Where false output conditions will cause objectionable reactions in associated circuit units, or will cause appreciable portions of a system to be made inoperative for a period of time, then it is advisable to impose the strict requirement that output leads be grounded only when the input combinations are those specified in the code. Although this may result in more complex contact networks than when the less strict requirements are imposed, the design work is often easier since the requirements are completely stated and it is not necessary to try various arrangements with invalid combinations to determine which produces the simplest network. The extension of the requirements to include a signal on an alarm output lead when a false combination occurs is obvious. The self-checking codes, of course, make these trouble protection measures more effective.

Another requirement which may be necessary and which influences the choice of contact network arrangements is that ungrounded output leads may not be crossed during normal circuit actions. This condition is imposed when crosses or "back-ups" between ungrounded
leads may cause false actions in circuits connected to the outputs. An example is indicated in Fig. 12-4 where a particular input condition to the translating circuit grounds output lead X as shown and also connects leads Y and Z together. If relay (Y), for some reason, has been previously operated and locked, as indicated, the locking ground will "back-up" through lead Y to lead Z and falsely operate relay (Z). This trouble can be corrected by redesigning the translating network to eliminate crossing of leads. An alternative solution which does not involve the translating circuit is to remove the grounding condition in the external circuit. In the example shown, this can be done by using a separate locking winding on the relay, or by operating and locking the relay through a make-before-break transfer.

If it is necessary that output leads should not be crossed, care must be exercised in combining the networks of individual output leads. Satisfactory circuits are determined by inspection and analysis, assuming that all possible situations occur, and checking for back-up paths.

One method of preventing the appearance of grounds on output leads in case of a false input combination is to provide a "checking" network as indicated in Fig. 12-5. The checking network is closed for all valid input combinations of the code, but is open for all other combinations. The translating network can then be any network which closes the output leads according to the required code. Sometimes the form of the checking network is obvious, as in the case of the two-out-of-five code in Table 12-4 where a simple symmetric network may be used. The function of the checking network is to close the circuit path only when the combination in which the relays stand is one of those specified in the code. Whether this is, in reality, a "check" of the validity of the information depends on the nature of the code. To illustrate: when a "self-checking" code is used, a single error produces a combination
which is not in the code and consequently a checking network will detect the error by opening the circuit path. However, for other codes, where certain errors convert one combination to another in the code, an error is detected only when the resulting combination is one of those not used in the code. A checking network of this type can only determine that the registered combination is one of those specified in the code and not that it is the one originally intended.

\[ \text{Fig. 12-6 Translating Circuit for Pulse Counter Relays} \]

### 12.7 EXAMPLES ILLUSTRATING DESIGN TECHNIQUES

Several of the forms a translating circuit may take will be illustrated and compared by developing circuit arrangements which perform the same translation. The codes are based on the operating combinations of the counting circuit given in Table 12-2. The translating circuit consists of a network of contacts on the six relays which grounds one of ten output leads according to the digit registered on the relays.

A simple circuit can be easily developed from an analysis of the conditions shown in the table. The output lead for digit 1 is grounded when relay (1) is operated, provided also that relay (6) is released. The lead for digit 7 is grounded when (1) and (6) are both operated. Therefore, a path through a make on (1) and a break on (6) is closed for digit 1, while a make on (1) in series with a make on (6) is closed for digit 7. Similar reasoning gives paths for the other eight digits, each path containing a make or a break on (6). The ten paths are combined to use a single transfer on (6) in the contact network shown in Fig. 12-6.

If a trouble condition causes the relays to operate improperly, the contact network of Fig. 12-6 does not protect against false output signals. For example if relays (1) and (2) are operated at the same time, output leads 1 and 2 are both grounded. Whether this circuit is satisfactory will depend on the likelihood of false action of the relays and the consequences of false signals given to the connecting circuits.
Although this is not a self-checking code, some degree of checking is possible. One procedure is to construct for each output lead a path which passes through a series chain of six contacts, one on each relay—a make or break depending on whether the relay is operated or released. Each path is closed for only the correct one of the 64 possible combinations in which the six relays might operate. Typical paths are shown in Fig. 12-7 for digits 1, 3, and 7. A complete circuit for all digits is shown in Fig. 12-8 where individual paths have been combined. This circuit opens all output leads in the event a trouble condition causes the relays to operate in some combination not in the code.

A third type of network for this example may be developed according to the plan of Fig. 12-5 where the checking network closes only for valid combinations of the code. Analysis of the code in Table 12-2 shows that the first five relays are operated one at a time. An indication that exactly one of these relays is operated is sufficient to show that the relays are operated in one of the correct combinations of the code. The one-out-of-five symmetric circuit shown in Fig. 12-9 will give this check and may be placed in the ground lead of the translator, Fig. 12-6. The resulting network grounds an output lead only if the six relays are
The Design of Switching Circuits

Fig. 12-9 A Check Circuit for Use with Fig. 12-6

operated in one of the ten correct combinations of the code. Note, however, that this circuit checks only single errors in the first five relays and does not insure that the particular grounded output lead is actually the one intended. For example, if digit 7 is intended and relay (6) fails to operate, the combination for digit 1 is established and lead 1 is grounded.

The circuit of the combined Figs. 12-6 and 12-9 does not effect a great saving in contact springs over Fig. 12-8, but distributes the spring load more uniformly over the relays. A possible advantage of Fig. 12-8 is that crosses between output leads cannot occur since all paths are disjunctive. The other arrangement, however, will cross two or more output leads under certain trouble conditions. For example, if relays (1) and (2) are erroneously operated at the same time, no output leads are grounded since the symmetric circuit, Fig. 12-9, is open; but two pairs of output leads are crossed, lead 1 to lead 2 and lead 7 to lead 8 (see Fig. 12-6). Choice between the various circuit arrangements for this translating problem will depend on the particular conditions under which it is used.

The design of translating circuits can be further illustrated by circuits which perform the inverse translation to that above. These

Fig. 12-10 Three Arrangements for Translating Digits 1 and 7
circuits have ten input leads which are grounded one at a time to cause six output leads to be grounded one or two at a time. Referring to Table 12-2, the "Digit" column will now correspond to inputs, and the "Relays Operated" column to outputs. If ten relays are used, one connected to each input lead and designated accordingly, the relays (6), (7), (8), (9), and (0) must each ground lead 6 and one of the leads 1, 2, 3, 4, and 5. Relays (1) to (5) require one make-contact each, while relays (6) to (0) each require two. These are connected as shown in Fig. 12-10A for the (1) and (7) relays. The fact that relay (1) and the similar relays (2) to (5) each contain only a single make suggests that these relays may be omitted and the corresponding input leads connected directly through to the outputs. This scheme, shown in Fig. 12-10B, has the possible disadvantage that operating relay (7) places ground back on input lead 1. This can be eliminated by a break-contact on the relay as in Fig. 12-10C. A translating circuit using this scheme requires five relays.

A circuit arranged according to Fig. 12-10A can be made to discriminate against false input combinations by providing a one-out-of-ten symmetric circuit on the ten relays and supplying all grounds through this network, as in Fig. 12-5. A similar check cannot be made with Figs. 12-10B and 12-10C since relays do not appear on all inputs.

12.8 FURTHER EXAMPLES BASED ON STANDARD CODES

The general procedure for the design of a translating contact network for combination codes will be illustrated by a circuit which translates from the 1-2-4-5 four-element code to the 0-1-2-4-7 self-checking code. Four relays designated 1, 2, 4, and 5 operate according to the

<table>
<thead>
<tr>
<th>Digit</th>
<th>Input Relays (1) (2) (4) (5)</th>
<th>Output Leads Grounded L0 L1 L2 L4 L7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 1 1 1</td>
<td>X X</td>
</tr>
<tr>
<td>2</td>
<td>1 0 1 1</td>
<td>X X</td>
</tr>
<tr>
<td>3</td>
<td>0 0 1 1</td>
<td>X X</td>
</tr>
<tr>
<td>4</td>
<td>1 1 0 1</td>
<td>X X</td>
</tr>
<tr>
<td>5</td>
<td>1 1 1 0</td>
<td>X X</td>
</tr>
<tr>
<td>6</td>
<td>0 1 1 0</td>
<td>X X</td>
</tr>
<tr>
<td>7</td>
<td>1 0 1 0</td>
<td>X X</td>
</tr>
<tr>
<td>8</td>
<td>0 0 1 0</td>
<td>X X</td>
</tr>
<tr>
<td>9</td>
<td>1 1 0 0</td>
<td>X X</td>
</tr>
<tr>
<td>0</td>
<td>0 1 0 1</td>
<td>X X</td>
</tr>
</tbody>
</table>

Table 12-9 Combinations for Translating Circuit
four-element code. Contacts on these relays are then required to ground five output leads (L0, L1, L2, L4, and L7), two at a time in the proper code combinations. Since a large portion of the sixteen possible input combinations are used, the methods of switching algebra may be employed to advantage in developing the network. (Eleven combinations are used, ten for the digits 0 to 9 and an eleventh for the normal condition of all relays released.) In order to introduce a partial safeguard against output errors, the use of invalid combinations will not be allowed.

The code combinations are given in Table 12-9, where the symbols 0 and 1 represent operated and released relays respectively. The two output leads grounded for each digit are indicated by X’s in the table. The first step in the design is to develop a network for each output lead individually by considering the conditions which must ground this lead. For example, output lead L0 is grounded for digits 1, 2, 4, and 7. The corresponding relay combinations are tabulated and the circuit for this lead developed by switching algebra as follows:

\[
f(L0) = \begin{pmatrix}
0 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 \\
1 & 1 & 0 & 1 \\
1 & 0 & 1 & 0
\end{pmatrix} = [2' + (1 + 4')(1' + 4) + 5'](1' + 2 + 4').
\]

To illustrate further: lead L4 is grounded for digits 4, 5, 6, and 0; and the circuit is developed as follows:

\[
f(L4) = \begin{pmatrix}
1 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 \\
0 & 1 & 1 & 0 \\
0 & 1 & 0 & 1
\end{pmatrix} = 2' + (4 + 5')(4' + 5).
\]

The circuits for the remaining leads, L1, L2, and L7, are developed in a similar manner, and the five networks are then examined for possible simplifications. By properly arranging the order of the elements in the individual networks, and by making use of the disjunctive nature of transfer contacts, the five networks may be combined as in Fig. 12-11. In this figure the paths of \( f(L0) \) and \( f(L4) \) developed above are shown in heavy lines for easy reference.

It should be noted that translation from a simple combination code to a self-checking code, as in this example, does not permit checking to a greater extent than is possible in the original simple code. An error in the four-element input code which converts a code combination to another valid combination will simply produce a two-out-of-five output combination indicating the wrong digit. Errors which cause more or
less than two output leads to be grounded correspond to invalid input combinations which can be detected without performing a translation. The purpose of translating to a checking code would be to detect troubles during later circuit actions.

In some translations a characteristic of the code combinations may indicate a procedure for designing the network. For example, in translating from the two-out-of-five code to a code consisting of ground on one of ten output leads, the translating network may consist entirely of make-contacts, since only two of the five input relays are operated at any given time. A symmetric check network may be used ahead of the translating network to prevent false output grounds if a trouble condition causes more than two relays to operate. A circuit of this kind is shown in Fig. 12-12. The translating network may be arranged in a

![Diagram](image-url)
The number of ways which result in different distributions of spring loads on the five relays. The one shown in the figure distributes the overall load of the check and translating networks by favoring in the translating network those relays which have heavy loads in the symmetric check network.

The check does not guard against crosses on ungrounded output leads. For example, if digit 1 is registered by operating relays (0) and (1), output lead L1 is grounded as required; and leads L4 and L5, though not grounded, are connected together. In many applications this is of no consequence but, if necessary, these crosses can be eliminated at the expense of more contacts. One possible solution is to provide an independent path consisting of two make-contacts in series for each output lead of the translating network. In Fig. 12-12 this would require the addition of two makes on the (1), (2), and (4) relays, making a total of twenty contacts, or four per relay, in the translating network.

12.9 SYMMETRIC CHECKING NETWORKS

The checking network for a self-checking code is usually some form of symmetric network. The use of a two-out-of-five symmetric network for checking the action of relays operating in a two-out-of-five code is illustrated in Fig. 12-12. This network is closed only for the ten combinations representing the digits. In the biquinary self-checking code a one-out-of-two circuit on the (00) and (5) relays placed in series with a one-out-of-five circuit on relays (0), (1), (2), (3), and (4) may be used. When some of the possible combinations which close a symmetric circuit are not used in a code, a symmetric circuit can often be modified to reject these combinations.
As an example, certain applications may require that information in addition to the ten numerical digits be represented by two-at-a-time code combinations. A sixth element, designated "10", can be used with the usual 0, 1, 2, 4, and 7 elements to form five additional combinations. If some of the fifteen combinations are not used, the checking network may consist of a two-out-of-six symmetric circuit which is modified by omitting contacts so that the path is closed for only the desired combinations. Modified symmetric circuits which are closed for twelve and thirteen particular ones of the fifteen two-at-a-time combinations are shown in Fig. 12-13A and 12-13B. It should be recognized that the unmodified two-out-of-six symmetric circuit will also give a single error check in these examples, as will a circuit which indicates only that an even number of relays is operated. However, a circuit which is closed for only those combinations that are used gives a check against certain multiple-error conditions.

12.10 INTERMEDIATE TRANSLATION

When there is a limited number of translating contacts on the relays or switch elements, it sometimes is not possible to translate directly to the desired output code. In these cases, auxiliary apparatus may be employed to perform an intermediate translation. An example of this is where digits are registered on the verticals of a crossbar switch having only two available make contacts at each crosspoint. The particular crosspoint (0 to 9) which is closed represents the value of the digit registered, and the two contacts of this crosspoint must ground the desired output leads. When the output code is one of the four-element codes of Table 12-3, there is occasion for three output leads to be grounded. This can be done by means of an auxiliary relay as shown in Fig. 12-14. In effect, the digits are translated by the interconnections of the crossbar switch contacts into an intermediate five-lead code in which no more than two leads are grounded simultaneously. The auxiliary relay converts this to the desired four-lead code.

![Fig. 12-13 Modified Symmetric Check Circuits](image-url)
The method may be extended by adding a second relay as shown in dotted lines in the figure. The four output leads can then be grounded in any of the fifteen possible combinations by grounding either one or two of the resulting six input leads of the auxiliary translator.

12.11 TRANSLATION BETWEEN NUMBERING SYSTEMS

In some translation problems a large number of input combinations must be translated to a corresponding number of output combinations with a systematic correspondence between the input and output combinations. An example is where a multidigit number in decimal notation is translated to an output code corresponding to some other numbering system or orderly method of designation. The procedure in designing these circuits is to determine the way in which the two systems correspond and use this as a clue in constructing the translating network. If it is often possible to determine a systematic method of expressing output combinations in terms of input combinations. Equivalent contact networks are then easily developed. Suitable choice of the codes in which the original information is expressed will often lead to quite simple translating circuits.

The principle can be illustrated by a circuit which translates from a two-digit decimal number to a three-digit mixed-base number where...
Table 12-10 Correspondence Between the Decimal Numbering System and a Mixed-Base Numbering System

Fig. 12-15 Translation from Decimal to a Mixed Base System
the bases are 5, 4, and 5.* Table 12-10 shows the correspondence between these two numbering systems. The first and third digits of the mixed-base number can be determined from the first and second digits respectively of the decimal number. The second digit of the mixed-base number is determined from both digits of the decimal number. As shown in the table, this digit depends on whether the first decimal digit is even or odd and on whether the second decimal digit is or is not below five. If a choice of codes can be made, the 1-2-4-8 code is an obvious choice for the first digit and the 1-2-4-5 code for the second digit. The second (base-4) digit of the mixed-base number can then be determined from the "1" element of the 1-2-4-8 code and the "5" element of the 1-2-4-5 code. The circuit can be developed by conventional methods, and it is shown in Fig. 12-15 where the first digit relays are designated (A-) and the second digit relays (B-).

Two factors contribute to the simplicity of this circuit. One factor is that the codes chosen for the first and second decimal digits fit into the mixed-base system. If some other codes are used, such as two-out-of-five, for example, the indication of odd-even and below-five is more complex. The other factor is that there is a simple systematic correspondence between the two systems of enumeration. For example, a translation of a multidigit decimal number to the base-2 system, or vice versa, is complicated because simple relations do not exist.

12.12 LARGE TRANSLATORS

A major translating problem occurs in some switching applications when a large quantity of information must be translated to output information which has no systematic correspondence to the input information. An example occurs in large digital computers where frequently used mathematical formulae are assigned identifying index numbers. When one of these numbers is presented to the input of the translator the necessary information to permit the computer to solve this formula is delivered at the translator output. Another example is in the common-control type of telephone switching system where the first three digits dialed by the subscriber, which indicate the office being called, are translated into instructions which permit the machinery to locate and establish a call over one of the trunks to this office.

These translators act as "files" of information where the input signals, in effect, are arbitrary "index" numbers which identify various complex sets of information and permit any given set to be produced in response to its identifying index number. In general, the nature and

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* This is recognized as part of the system of designations of 10,000 terminals on the terminals of 500-point switches, which was mentioned in the introductory paragraphs of this chapter.
form of the input information bears no relation to the particular output information. To permit flexibility in their use, these translators are often arranged so that the output information corresponding to an input index number or "code" can be changed easily.

The general principle of a large translator is to provide translating devices which can indicate to output circuits the complete output information corresponding to an input index code. The translator must then contain a means for selecting and activating a particular translating device when its assigned code is presented to the input. Large translators may employ relays, varistors, gas tubes, or special electromechanical devices. A typical arrangement of a relay-type translator is shown in Fig. 12-16. This consists of a selecting circuit which places ground on a code point corresponding to the input code. A code point is provided for every possible input code. The code points are cross-connected to translating relays which, in turn, have their contacts cross-connected to output terminals according to the desired output information. The output information may be in numerical form using one of the combination codes, or may merely be in the form of control signals which cause associated apparatus to perform specific actions.

The cross-connections make this system of translation very flexible. Each translating relay can indicate any set of output information, and the code-point cross-connections permit this set of information to be associated with any one or more input codes. Physically the cross-connections are made between banks of wiring terminals and may be easily changed.

The selecting circuit may take various forms, several of which are discussed in the following chapter.
PROBLEMS FOR CHAPTER 12

12-1 Design a simple translating circuit which will translate from the 1-2-4-8 code (0 = 2-8) to the 1-2-4-5 code (0 = 1-4). The input and output elements consist of leads on which the active condition is indicated by ground. (This can be done with 41 contact springs.)

12-2 Design a simple translating circuit which will translate from the 1-2-4-8 code (0 = 2-8) to the 0-1-2-4-7 code. The input and output elements consist of leads on which the active condition is indicated by ground. (This can be done with 53 contact springs.)

12-3 Design a simple translating circuit for the relays of the decade counting circuit of Fig. 11-19 (Chapter 11) which will deliver output indications by grounding five leads in the 0-1-2-4-7 code. (This can be done with 32 contact springs.)

12-4 Design a translating circuit which will translate from the 0-1-2-4-7 code to the 1-2-4-8 code. Provide a means for grounding an alarm lead (AL) if an error occurs in the input combination. (This can be done with 46 contact springs.)

12-5 Two base-ten digits are registered on ten relays using the additive two-out-of-five code. Design a translating circuit with two groups of output leads, the first group containing twenty leads and the second group containing five leads, such that one and only one lead is grounded in each group for each combination of the two input digits. This provides a mixed-base output, with the first digit base-20, and the second, base-5. Keep the number of relays employed in the circuit to a minimum, and do not use more than 24 contact springs on any relay. (This can be done with 103 contact springs.)

12-6 The minutes of the hour from 00 to 59 are indicated in a self-checking code by a group of relays which operate continuously. The units digit (0 to 9) is represented by five relays, U0, U1, U2, U4, and U7 in the conventional two-out-of-five code. The tens digit (0 to 5) is represented by four relays, T0, T1, T2, and T4 in two-at-a-time additive combinations where zero is represented by T2 and T4 operated. Design a circuit which will light a red lamp (R) on the hour, the half-hour, and the quarter-hours, (i.e. when the numbers indicated are 00, 15, 30, and 45), and will light a green lamp (G) at five-minute intervals within each quarter-hour (i.e. when the numbers indicated are 5, 10, 20, 25, 35, 40, 50, and 55). In addition to the above, the circuit must light an alarm lamp (AL) at any time that an error occurs in the operation of the indicating relays. (This can be done with 62 contact springs.)

12-7 What is the minimum number of code elements required to provide fifty code combinations in which the presence of single errors can be detected?

Assuming that each code element is represented by a relay, design a check circuit on the contacts of these relays which will light an alarm lamp when an error occurs.

12-8 The operation of certain teletype apparatus requires code combinations representing the twenty-six letters of the alphabet and additional combinations to control the operation of the printing typewriter as follows:

1. Shift to Figures
2. Shift to Letters
3. Space
4. Carriage Return
5. Line Feed
Outline the method of constructing a code representing this information which is capable of correcting single errors. Cover the following items in the outline:

1. Total number of code elements
2. Number of information elements
3. Number of check elements
4. Method of evaluating check elements for a given combination of information elements when constructing the code. (Give example)
5. Method of checking and correcting errors. (Give example)
6. Effect of double errors. (Give example)

How can the code be extended to detect double errors?
A "selecting" circuit is specifically defined as a switching circuit arrangement which chooses from a number of similar items a particular one identified by predetermined information. There is no freedom of choice in selecting; the specified item must be picked, and no other can be used.

In switching systems the many types of items from which selections are made include particular paths or terminals of an interconnection network, specific circuit units necessary to exert or accept control action, one of several information sources such as register circuits, and one of many code points in a large translating circuit. In practical selecting circuits, the selectable items are most often represented by single terminal points or single leads, although in some cases each individual item may comprise several terminals. The output of a selecting circuit is most often a control lead or leads by means of which the associated selectable circuit unit may be seized, controlled, or made busy. The act of "picking" in a selecting circuit usually involves marking the specified point or lead by applying to it ground or battery or other identifying circuit condition.

The discussion in this chapter will be confined to selecting arrangements which make use of relays or similar two-valued devices. However, selecting can, of course, also be accomplished by other means such as a multipoint switch which is directed by appropriate control to connect to a specified one of its output terminals.

The chief characteristics of a selecting circuit are indicated in the block diagram of Fig. 13-1. This shows a signal (for example, ground) which is applied to a particular one of the selectable outputs according to the existing conditions on the control leads. For each output there exists a particular combination of control conditions, and a given output is selected by setting up the corresponding control combination. In effect, then, there must exist some "code" which associates control combinations with particular outputs. Thus, in its broadest aspect, a selecting circuit is a translating circuit which selects one of n outputs by translating from a combinational control code to a one-out-of-n output code. However, because of the magnitude of the usual
selecting problem, it is not practical to handle it by the techniques developed in the preceding chapter.

13.1 ELEMENTARY RELAY SELECTING CIRCUITS

The simple multiple of relay contacts shown in Fig. 13-2 is a very elementary selecting circuit. Grounding a particular control lead causes a connection to be established to the corresponding output. The restriction that only one control lead be grounded at any given time is implied. This is a trivial example of selecting, since the original control is on a one-out-of-n basis. However, in later sections of this chapter, this arrangement will be used as a unit in assembling more elaborate selecting circuits.

A more important example of a simple selecting arrangement is the transfer tree arrangement of relay contacts. An example is shown
in Fig. 13-3 which uses three relays to select one of eight outputs according to the combination in which the relays are operated. A characteristic of transfer trees which is typical of more elaborate selecting arrangements described later is that the selection is accomplished in "stages." As can be seen from Fig. 13-3, each stage of a transfer tree consists of the contacts of a single relay, and the operation or release of this relay subdivides the possible choice of outputs until the final stage connects to the particular one.

![Diagram of transfer tree](image)

Fig. 13-3 Selecting by Means of a Transfer Tree

A property of transfer trees which is a result of the exclusive use of transfer spring arrangements is that all paths are disjunctive and the input (ground) can be connected to only one output terminal at any given time. Although it is implicit in selecting circuits that one and only one output be activated at any given time, it is not always a characteristic of selecting circuits that all paths are disjunctive. For example, see Fig. 13-2. Since every input combination selects an output in a fully developed transfer tree, any sequence of action of the control relays when selecting a particular output may cause the selection to pass over several of the outputs momentarily. If this introduces objectionable consequences, it is necessary to hold open the output path until the desired combination of control relays has been established. Since the n relays of a transfer tree can make a selection among $2^n$ outputs, it is the most efficient selecting arrangement possible in terms of numbers of relays required. However, due to the impracticability of constructing relays with large numbers of transfers, this type of circuit is not used in large selecting circuits.
13-2 MULTIBRANCH TREES: GENERAL

The transfer tree provides two branches toward the output at each junction point, or fork, of the circuit paths. The number of branches can be increased by using a multiple arrangement of the type shown in Fig. 13-2. With this arrangement, the contacts of all relays are make-contacts; and if each fork contains n branches, there must be n relays at each stage. As an example, a two-branch tree of three stages using only make-contacts is shown in Fig. 13-4. Each stage contains two relays, and one and only one relay in each stage is operated to make a selection. Fig. 13-4 is the make-contact equivalent of the three-stage transfer tree of Fig. 13-3. A two-stage three-branch tree would permit selecting one of nine outputs. It is not necessary that the number of relays (or branches) in each stage of a multibranch tree be the same. Fig. 13-5 shows a two-stage tree with a two-branch first stage and a four-branch second stage. A one-out-of-eight selection is made by operating one of the two first-stage relays, \( A_0 \) and \( A_1 \), and one of the four second-stage relays, \( B_0 \), \( B_1 \), \( B_2 \), and \( B_3 \).

When the number of outputs is large, a selecting tree may be arranged in many ways. Optimum conditions in a particular case will depend both on theoretical considerations and on practical aspects.
having to do with available apparatus and control requirements. These several aspects of the selecting problem will be discussed separately.

13.3 THEORETICAL CONSIDERATIONS OF MULTIBRANCH TREES

A generalized diagram of a multistage multibranch selecting tree is shown in Fig. 13-6. From a consideration of this and the examples of Figs. 13-4 and 13-5, a number of statements relating to the size of these arrangements can be made.

1. If the number of stages is \( n \), and the number of branches at each stage is respectively \( R_1, R_2, \ldots R_n \), then the maximum number of outputs, \( M \), is: \( R_1 \times R_2 \times \ldots \times R_n \).

2. The number of relays in each stage is equal to the number of branches of that stage. Hence, the total number of relays in \( n \) stages is: \( R_1 + R_2 + \ldots + R_n \).

3. The number of paths into any stage is equal to the outputs of the preceding stage. Therefore the number of inputs to the \( k \)th stage is: \( R_1 \times R_2 \times \ldots \times R_{k-1} \).
4. The number of contacts on one relay of a given stage is equal to the number of paths into that stage.

5. Since each path into a stage connects to one contact on each relay of that stage, the total number of contacts in a given stage is the product of the paths into that stage by the number of relays of that stage. This also is the number of outputs of that stage.

6. From statements 3, 4, and 5 it can be deduced that the total number of contacts of an n-stage tree is: \((R_1 + R_2 + \cdots + R_n)\).

A selecting tree to reach a given number of outputs may consist of a few many-branched stages or a larger number of stages with less branches at each stage. The optimum circuit, neglecting practical considerations for the moment, can be arrived at by investigating several arrangements, comparing for each of the arrangements the total number of relays and the total number of contacts. The general procedure is to separate a given number of selectable outputs into a set of factors. Each factor represents a stage of the selecting tree, and its numerical value corresponds to the number of relays in its respective stage.

<table>
<thead>
<tr>
<th>No. of Stages</th>
<th>Relays in Each Stage</th>
<th>Total Relays</th>
<th>Contacts per Relay in Stage</th>
<th>Total Contacts</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3—12</td>
<td>15</td>
<td>1 3</td>
<td>39</td>
</tr>
<tr>
<td>2</td>
<td>6—6</td>
<td>12</td>
<td>1 6</td>
<td>42</td>
</tr>
<tr>
<td>3</td>
<td>2—3—6</td>
<td>11</td>
<td>1 2 6</td>
<td>44</td>
</tr>
<tr>
<td>3</td>
<td>3—3—4</td>
<td>10</td>
<td>1 3 9</td>
<td>48</td>
</tr>
<tr>
<td>4</td>
<td>2—2—3—3</td>
<td>10</td>
<td>1 2 4 12</td>
<td>54</td>
</tr>
</tbody>
</table>

Table 13-1  Arrangement of Selecting Stages for Selecting of Thirty-Six Outputs

Table 13-1 shows several arrangements for thirty-six outputs. The number, thirty-six, was chosen for an example since it may be factored in several ways. It is an obviously necessary condition that all factors be integers, and for this reason a tree having a capacity greater than the required output is often used with some possible outputs omitted. The table illustrates several conditions which are generally true for arrangements having the same number of possible outputs. As the number of stages is increased, the trend is for the total number of relays to decrease and the total number of contacts to increase. However, a single four-branch stage is better than two two-branch stages since the number of relays is the same in each case, but the single stage requires fewer contacts. Actually, for a given number of
relays, the greatest number of outputs can be obtained when each stage contains three relays or as close to three as possible. For example, twelve relays may be arranged in three stages of four relays each to give a selection of \(4 \times 4 \times 4 = 64\), or in six stages of two relays each, to give \(2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64\). If arranged in four stages of three relays each, the number of outputs in \(3 \times 3 \times 3 \times 3 = 81\).

When all stages do not have the same number of relays, those with the larger numbers of relays should be placed in the last stages in order to keep the total contacts at a minimum. This is evident from the mathematical expression given in statement 6 above. In an example from Table 13-1, if the three-stage selector, \(3 \times 3 \times 4 = 36\), which uses 48 contacts, is changed to place the four-relay stage first, that is, \(4 \times 3 \times 3 = 36\), the total number of contacts is \(4 + (4 \times 3) + (4 \times 3 \times 3) = 52\), an increase of four contacts.

A conclusion which can be drawn from the table and which is generally true is that for a given number of stages and outputs the most efficient arrangement, from the standpoint of numbers of relays, is when the number of relays in each stage is the same or as nearly as possible the same. This is shown by both the two-stage and three-stage examples of the table.

13.4 PRACTICAL CONSIDERATIONS: SIZE OF RELAYS

The theoretical considerations discussed above should be used as a guide in developing selecting schemes, but certain practical aspects often make it impossible to approach, even approximately, conditions determined by pure theory. One important restriction in the application of the theory is the number of make-contacts provided on practical types of relays. General-purpose and multicontact types of relays are used for selecting. The general-purpose relays may carry as many as twelve, and in some cases a few more, make-contacts. Existing types of multicontact relays carry as many as sixty makes, often arranged in two sets of thirty contacts. A separate operating magnet is provided for each set of thirty contacts so that a sixty-contact relay is, in effect, two thirty-contact relays.

As indicated by the preceding section, the number of relays in each stage, including the last, should be low. However, if a large number of outputs must be handled, this means that each final-stage equivalent relay must be made up of several individual relays with windings multiplied together. For example, 1000 outputs could be accommodated by five 200-contact equivalent relays, each made up of four 50-contact relays with windings multiplied together. The preceding selecting stages must provide 200 inputs to this final stage. Now, if the same final-stage relays are treated as a group of twenty individual 50-contact relays, the preceding stages need only supply fifty inputs to the final stage.
This is clearly a more economical arrangement and indicates that multiplying of several relays to provide one big relay is undesirable.

Subject to restrictions imposed by control codes, a good approach to the design of a practical selecting circuit therefore is to start with the final stage. Since each final output lead connects to a contact on a relay of the last stage, the number of relays of a given size necessary for the last stage can be found by dividing the number of final outputs by the number of contacts on one relay. When the number of outputs is not exactly divisible, the next higher number of relays is required. Depending upon circumstances, it may be desirable either to place the same number of outputs on each relay, thus leaving some contacts of each relay idle, or to fill all relays except one. The number of contacts on the final-stage relays should be as great as possible, consistent with equal division into total number of outputs and the ability to control the resulting final stage. The lower stages collectively must select a particular contact of an operated final-stage relay so that the number of outputs of the next-to-last stage is equal to the number of active contacts on one final-stage relay. The number of relays of the next-to-last stage can be determined in a manner similar to that described for the final stage, and the processes repeated for lower stages. In this way both the number of stages and the size of each stage is determined. Choosing different sizes of relays at succeeding stages will produce a different over-all arrangement, and it is usually desirable to investigate several arrangements before choosing a particular one.

As an example of the above process, consider a case where one of three hundred outputs is to be selected. If thirty-contact relays are used in the final stage, the total number of relays required will be 300 ÷ 30 = 10. There are thirty paths into this stage, and this number of outputs must be provided by the next-to-last stage. By the methods of the preceding section, it is determined that thirty outputs can be selected in three stages: 2 x 3 x 5 = 30; and the required spring loads per relay are within the capability of general-purpose relays. Thus the selecting tree is in four stages: 2 x 3 x 5 x 10 = 300. The numbers of contacts on the individual relays of each stage are, respectively, 1, 2, 3, 6, and 30. The number of relays is 2 + 3 + 5 + 10 = 20, and the total number of contacts is 338.

An alternative arrangement using general-purpose relays in the final stage, each with ten contacts, requires thirty such relays in this stage. The resulting selector is in three stages, 2 x 5 x 30 = 300. It requires thirty-seven relays and a total of 312 contacts. Although this is twenty-six contacts less than the four-stage arrangement, it requires seventeen more relays. All other factors being equal, the four-stage arrangement is preferred since the twenty-six additional contacts are less expensive than seventeen relay structures.
13.5 PRACTICAL CONSIDERATIONS: CONTROL CODES

Since the function of a selecting circuit is to pick a particular output according to the control information, consideration must be given to the form or code in which this information occurs. In some cases, the code can be arranged to conform with a particular type of selecting arrangement, while in other cases there is little freedom of choice in the form in which identifying information is represented, and the selecting circuit must be adapted accordingly. The over-all problem of choosing a selecting circuit for a particular application is often a compromise influenced by the control information code and the size of practical relays as well as by theoretical considerations.

The control information for a selecting tree is similar to a multidigit number where each digit corresponds to a stage of the selection. For example, consider a case where one of a thousand outputs is to be selected. This may be done in three ten-branch stages. Each stage corresponds to a digit of a three-digit base-10 number. It contains ten relays and is controlled by ten leads. The thousand outputs can be designated from 000 to 999, and connected so that grounding one control lead in each stage selects the individual output having the corresponding designation.

Numbering systems (see Chapters 11 and 12) other than base-10 may be used in selecting circuits. The three-stage selector of Fig. 13-4 is a base-2 system. The subscripts in the output lead designations

![Diagram](image-url)
correspond to the digits 0 to 7 in binary notation. Mixed-base systems are also used. In Fig. 13-5 the bases of two and four are used in selecting one of eight outputs. In the output designation subscripts, the A digit has either of two values, 0 and 1, while the B digit ranges through the four values, 0, 1, 2, and 3. Other examples corresponding to mixed bases are given in Table 13-1.

It is not necessary that the order of the stages of a selecting tree be the same as the order of the digits of the numbering systems used in the control. Rearrangement of the order of selecting stages has the effect of rearranging the distribution of the output leads on the final stage relays. If the control leads for two similar sized stages of a selecting tree are interchanged, the same tree configuration can be used and it is necessary only to redesignate the output leads.

When selecting according to a numbering system, it is usually desirable, for practical reasons, to have the outputs appear on the final-stage relays in consecutive numerical order. In this case the selecting stages are in reverse order to that of the numerical digits, the final stage corresponding to the first or highest ranking digit. This is indicated in Fig. 13-7 for a two-stage selection of one out of one hundred. Each second-stage relay connects to ten outputs having the same tens digit, while the first stage selects one of ten paths according to the units digit.

Much of the information used in switching systems is in the form of decimal numbers. However, a selecting circuit based entirely on decimal stages is usually not the most economical. For this reason selecting arrangements controlled by decimal digits through simple translating circuits are of interest. The biquinary code is frequently used. For example, when selecting one out of one hundred, each of the two decimal digits may be factored, 2 x 5, to make a selection in four stages, 2 x 5 x 2 x 5 = 100. Changing the order of the stages of a selector does not affect its ability to make a selection, so that this selector may be arranged in several ways each of which results in a different grouping of leads on the output stage. Since fewer contacts are required if the larger selecting stages are placed toward the output, the arrangement: 2 x 2 x 5 x 5 = 100 may be used. This is shown in Fig. 13-8 where the first and third stages are controlled by the units digit, and the second and fourth by the tens digit. As indicated in the figure, this requires fourteen relays and a total of 126 contacts. This is a satisfactory selecting arrangement from a theoretical standpoint, but the grouping of the leads on the final-stage relays and in the interconnections between stages is not as orderly as might be desired.

Other codes may be used to make the grouping of leads more orderly. The biquinary code used in Fig. 13-8 is an "add 5" code since
a "1" in the binary stage indicates that adding 5 to the quinary value gives the decimal value. Another way of breaking down the decimal values is to divide the units into five groups of two, 0-1, 2-3, 4-5, 6-7, and 8-9, and specify either the odd or even one of each pair. This, in effect, is a "quibinary" code, and is frequently used in selecting circuits. If the tens digit in Fig. 13-8 is changed to "quibinary" or "odd-even" notation, with no changes made in the configuration of the tree, twenty consecutively numbered leads will appear on each final-stage relay. For example, the first (0) relay of the last stage connects to the twenty leads 00 to 19. This can be verified in Fig. 13-8 by redesignating the control leads for the tens digit, odd-even for the two-branched stage and 0-1, 2-3, 4-5, 6-7, and 8-9 for the five-branched stage. Paths may then be traced through the selecting network, and the leads redesignated accordingly; for example, "tens below five" becomes "odd tens," and so on. This code is also used in the following example.

When a selection is to be controlled by information in the form of decimal digits, it is sometimes advantageous to arrange certain stages of the selector to be controlled jointly by two or more control digits. This is illustrated by Fig. 13-9, which shows a three-stage selector, $5 \times 4 \times 5 = 100$. The second stage is controlled by the add-5 element of the units digit and the odd-even elements of the tens digit, through an auxiliary translator relay. Note that this arrangement allows all leads to be grouped in numerical sequence.
13.6 MODIFICATION OF SELECTING TREES

Multibranched selecting trees may often be modified to reduce the amount of apparatus required or to adapt them to particular applications. These modifications are possible when the signal condition (for example, ground) applied to the selected output by the selecting network is the same as the control condition applied to the control leads to operate the relays of the selecting stages.

The simplest modification is the omission of the relays in the first stage. Since each first-stage relay contains a single make-contact, the control leads may be substituted directly for the selecting output leads of this stage. For example, in Fig. 13-9, the five leads 0 to 4 from the units digit translator may be connected directly as inputs to the second stage in place of the five selecting leads coming from the contacts of the first-stage relays. Thus the first-stage relays are not required.

![Diagram of Three-Stage Selector for One Hundred Outputs](image)

Fig. 13-9 Three-Stage Selector for One Hundred Outputs
In general, the selecting leads into a stage and the control leads of that stage may be interchanged when the selecting signal and the control signal are the same, for example, ground. This is indicated in Fig. 13-10. Fig. 13-10A shows the normal arrangement where m selecting input leads from the preceding stage are multiplied over the contacts of n relays operated by the control leads of that stage. This arrangement has m·n outputs and contains n m-contact relays. Since the stages place ground on only one of the selecting input leads, these may be used to operate the relays, and the control leads for this stage may then be multiplied across the contacts of these relays as shown in Fig. 13-10B. This arrangement makes the same selection as Fig. 13-10A. It has corresponding output leads, m·n in number, with a different grouping, and it requires m n-contact relays. A selecting circuit may have the selecting and control leads of every stage transposed as indicated in Fig. 13-11. The output leads of each stage operate the relays of the following stage, and there is no tree of selecting paths extending through the stages. In a multistage selector a variety of arrangements can be obtained by transposing the selecting and control leads of some stages and retaining the normal tree interconnection in other stages.

13.7 USE OF SELECTING CIRCUITS IN LARGE TRANSLATORS

It was stated in Section 12.12, Large Translators (Chapter 12), that large translators often utilize a selecting circuit as an integral
part of the translating means. This is particularly true when several terminals are required for each translator output, and where the relationship between the multidigit control code and the output information must be capable of rapid change.

There are two general solutions to this problem which are based on the use of selecting trees. In one, the selecting circuit establishes a path to an individual code point per control code. A relay is associated with each code point, and contacts on the relay are cross-connected to give the desired information. A simplified sketch of this arrangement was shown as Fig. 12-16 in Chapter 12.

A second method, practical when relatively few independent terminals per translator output are required, is to carry all output paths through the selecting circuit itself. In general, the switching of output paths should be confined to the final stages of the selecting circuit.

An example of a selecting circuit used for code-point selection is shown on Fig. 13-12. This circuit is used in a common-control telephone switching system to translate the three-digit office code dialed by a telephone subscriber into the information required to set up the connection to the correct central office. Since in this system no office code has an initial 0 or 1 (except the single digit "0" operator's code), the circuit permits a selection of any one out of 800 outputs. The output or code points, in turn, are wired to the translating relays which furnish the requisite information. The "ZO" output corresponds to a single "0" received by the circuit.

Each digit of the office code is initially stored on a two-out-of-five basis. Individual translator relays per digit convert to the decimal
code suitable for selecting circuit control. The selecting circuit itself employs the principle of transposed stages shown in Fig. 13-11. As indicated in Fig. 13-12, there are eight relays, AT2 to AT9, operated by the eight leads from the A-digit translator, each relay having ten working contacts. There are eighty BT relays controlled by the B digit through the contacts of the AT relays. The C digit selects one out of ten contacts on the BT relays. A total of 88 relays and 880 contacts is used.

A second example of a selecting circuit used for large-scale translation is shown on Fig. 13-13. In this circuit a selection of one-out-of-1000, is provided, with three cross-connectable translation terminals per output. The circuit offers the additional feature that independent information potentials instead of a common ground are applied to the selected output. Selection again is on a decimal basis, and the design is straightforward. Note that the units (U-) stage of relays is necessary only to separate the three information leads.
Fig. 13-13 Connecting Three Leads Through a Selecting Circuit to One-out-of-1000 Points
13.8 SELECTION BY SEQUENTIAL CONTROL

When a selecting circuit is required to operate a selectable relay rather than simply ground an output terminal, techniques somewhat different from the selecting-tree network can be employed. A minimum of one relay (or similar device) per output is required, and the relay assists in its own control.

The theory of the method is best illustrated by example. On Fig. 13-14 is shown a circuit for selecting one relay out of a hundred. The relays are divided into ten groups of ten relays each. Each group of relays is numbered in accordance with one of the tens digits, and each relay within a group is assigned an individual units designation. The operating paths of all relays in each group are multiplied together and connected to the corresponding tens control lead as shown on the figure. In similar fashion, locking contacts on all relays with the same units designation are multiplied together and connected to the corresponding units control lead.

In operation, the control lead corresponding to the tens digit of the desired selection is first grounded. The group of relays in the appropriate tens group all operate. Then the control lead corresponding to the units digit of the desired selection is grounded, and the ground is removed from the tens lead. Nine relays release, leaving operated only the relay represented by the grounded tens and units leads.

This method can be extended to as many groups, subgroups, and relays as required. However, for each additional selection, an additional
winding and locking contact must be incorporated in each relay to prevent back-ups.

The chief disadvantages of this selecting method are the large number of relays required and the relatively slow speed of operation. However, the method is easily applicable to inexpensive high-speed electronic devices which have independent operating and holding mechanisms. For example, a circuit might employ multi-anode cold-cathode gas tubes with an appreciable margin between breakdown and sustaining potentials and with suitable anode-to-anode transfer characteristics. By multiplying anodes in accordance with the units, tens, hundreds, etc. designations (comparable to the multiplying in Fig. 13-14), and applying successively to the control leads a breakdown potential and transfer-sustaining potentials, any one tube may be selected. Additional anodes may then furnish output information.

13.9 SELECTION WITH NON-SYMMETRICAL VARISTORS

The function of selecting can be performed by many techniques aside from the relay circuits described in this chapter. Many of these techniques are adapted to specific applications and do not have the flexible and general utility offered by relay circuits. However, a selecting circuit employing rectifying varistors, useful where high speed is required, is worthy of discussion.

Basically, the relay-selecting tree is an "and" or coincidence circuit. A particular selection is made by a series contact network controlled by relays which are operated or released in accordance with code signals corresponding to the desired selection. As pointed out in Chapter 10, rectifying varistors are well adapted to use in "and" circuits, and therefore are inherently capable of performing the same type of selecting function.

![Varistor "And" Circuit](image-url)
The basic varistor "and" circuit is shown on Fig. 13-15. The output lead is held close to ground potential by the voltage drop across high-resistance R as current flows to ground through any one of the varistors. A positive output signal appears only when a positive input signal is applied simultaneously to all input terminals, or the ground connection is removed simultaneously from all input terminals. In a varistor selecting circuit, each output path is similar to that of Fig. 13-15, with the varistors connecting to the several groups of selection control leads. The varistor connections are arranged so that a particular input combination will permit the potential of only one corresponding output terminal to rise to signal value. In general, as with relays, the control groupings and varistor connections can be arranged in a variety of ways, some more economical in use of varistors than others.

A typical circuit is shown on Fig. 13-16. This is a circuit closely similar to the relay transfer tree of Fig. 13-3, with the exception that unselected outputs are grounded and the one selected output has a positive potential. The circuit is arranged so that inputs A and B can select a pair of outputs (0-1, 2-3, etc.), and input C can select the odd or even one of the pair. This is illustrated in Table 13-2 where it is assumed that the ('') mark indicates the normal condition of an input control contact.

![Diagram of selecting with non-symmetrical varistors](image-url)
It can be seen that inputs A and B in combination place ground on any three of the four pairs of outputs. For example, A'B grounds outputs (4-5), (6-7), (0-1), (4-5) and leaves ungrounded outputs (2-3). Input C then grounds set (1-3-5-7) or (0-2-4-6) depending upon its condition. If inputs A'BC are activated, output lead 3 is the only one that rises to a positive potential.

An equivalent circuit which requires the same number of varistors can be obtained by connecting each of the six input control leads to a different set of four outputs through individual varistors (the same numerical assignments can be employed as in Table 13-2). However, in larger selecting circuits, arrangements similar to that of Fig. 13-16 are usually more economical.

Although key or relay contacts are shown as control inputs on Fig. 13-16, any device of satisfactory impedance which can shift potential between two distinct levels is suitable for control purposes. Control can also be on a pulse basis.

### PROBLEMS FOR CHAPTER 13

#### 13-1 Design a selecting circuit to apply a tone signal to any one of a thousand terminals numbered from 000 to 999.

The output terminals must be in numerical order on the final-stage relays, and control is restricted to a decimal, biquinary, or quinary basis or any combination thereof. Label control leads plainly.

Relays can be: General-Purpose (1 to 12 makes); Multicontact (1 set of 30 contacts per magnet, 1 set of 25 contacts per magnet, or 1 set of 20 contacts per magnet).

All magnet and coil structures cost the same.

A make contact costs the same on any relay.

One magnet and coil equals twenty make-contacts in cost.

Design the most economical circuit subject to the above restrictions.

#### 13-2 A selecting circuit must apply ground to any one of five hundred terminals when control leads are actuated in the appropriate manner. Control information is available from the contacts of twenty-five relays which record any number from 0 to 499 in decimal code. Design an economical selecting circuit, using relays as listed in problem 13-1. Design also a suitable translating circuit for the twenty-five recording relays in order to control the selecting circuit. The translating and selecting circuits must both be taken into account in order to produce the most satisfactory over-all circuit.
13-3 A selecting circuit of the multibranch tree type, using only make contacts, is to be designed to connect a single input lead to one of 294 outputs.

(a) Make up a table similar to Table 13-1 of the text showing:
   1. Number of successive selecting stages
   2. Number of relays required in each stage
   3. Total number of relays (assuming relays that can carry any desired number of contacts)
   4. Total minimum number of contacts required in each complete selecting circuit to give only the 294 outputs

(b) Choose the circuit arrangement which requires the theoretical minimum number of relays. If two or more arrangements offer the same minimum number, choose between them on the basis of minimum contacts.

(c) Assuming the use of only general-purpose relays, which accommodate up to twelve make contacts each, choose a selecting circuit which requires the minimum number of relays to connect one input with any one of 294 outputs. Indicate the number of relays and contacts required.

13-4 Design a varistor selecting network to select any one of thirty-two outputs. Economy in the use of varistors should be kept in mind.
A switching system often consists of a number of functional units each of which performs certain of its actions in an independent manner but must from time to time interchange control signals or information with other functional units of the system. The particular units associated for this interchange will depend upon the conditions existing in the system at a given time. Permanent interconnecting paths between the various units do not provide the necessary flexibility in associating units, and therefore the usual procedure is to provide the interconnecting paths between the units only during the time required. The functional units which establish these interconnecting paths for information and control signaling purposes between otherwise independent system units are called "connecting" circuits.

The circuit performing the connecting function is often a physically distinct unit having its own relays or equivalent apparatus whose sole function is to close and open interconnecting paths. In other instances a true connecting function is performed by contacts on relays which also perform some other basic function such as counting, selecting, or sequence control. For example, the relays of a sequence circuit, in addition to the contacts provided to control the sequence action, may carry a network of contacts which connect an input circuit path to one of several output circuits according to the stage of the sequence at a particular time.

Connecting circuits are of greatest importance in large switching systems which contain many semi-independent units. When a large amount of information must be transmitted rapidly between units by combinatorial signals, the connecting circuit must carry many leads, perhaps a hundred or more. On the other hand, when the amount of information is small or the information can be transmitted sequentially, very few leads are necessary. In general, the type of connecting circuit depends to a great extent upon the number of leads. The configuration of a connecting circuit is also affected by the number of units that must be interconnected. One extreme is illustrated by a circuit which interconnects a few major control units over a large number of leads; the opposite extreme by a telephone office switching network which interconnects thousands of subscribers with three leads per connection.
A connecting circuit consists only of a means for establishing interconnecting paths and does not include within itself the means for operating the relays or switches whose contacts provide the paths. It is basically a passive network and must be directed from some outside source to close the proper paths at the proper time. The control may be derived from one or both of the circuits involved in a connection, or may come from another functional circuit such as the selecting circuits described in the preceding chapter or one of the locating circuits discussed in a later chapter. In the general discussions which follow, the circuit units served by a connecting circuit will be referred to as "terminal circuits."

![Fig. 14-1 One-to-One Connecting Circuit](image)

The three basic situations in which connecting circuits are used are indicated in Figs. 14-1, 14-2, and 14-3. The first of these, Fig. 14-1, illustrates a one-to-one connection where the leads between two terminal circuits are closed or opened by means of a connecting circuit. This very simple condition where the same two functional circuits are always connected together requires little discussion. The connecting circuit may consist of a single relay (or several operating in parallel to provide sufficient contacts) which closes the interconnecting leads. The many-to-one connection of Fig. 14-2 finds more important applications in switching systems. In this arrangement any one of several terminal circuits on the left may be connected through to the single circuit on the right. A final and more complex arrangement is that of Fig. 14-3, where any one of the terminal circuits on the left may connect to any one of the terminal circuits on the right.

![Fig. 14-2 Many-to-One Connecting Circuit](image)
Fig. 14-3 Many-to-Many Connecting Circuit

Fig. 14-3 applies to two conditions. The first is where only one connection between a circuit on the left and one of the circuits on the right is established at any given time. The second case, which occurs more often in practical systems, permits several simultaneous but independent connections to be established. That is, after one connection has been established, any of the remaining circuits on the left may be connected to any of those remaining on the right, and so on. As will be shown later, the complexity of the circuit increases as the capacity for simultaneous connections increases. For economic reasons, the size of connecting circuits is often limited so that after some maximum number of connections has been made, no further connecting paths are available, although further connections between terminal circuits may occasionally be requested. This condition where connections cannot be made because of an "all paths busy" condition of the connecting circuit is known as "blocking".

From a connection standpoint a connecting circuit has no "direction." For example, in Fig. 14-1 there is no significant difference whether the connection is thought of as progressing from left to right or from right to left. The act accomplished by the connecting circuit is that of connecting the terminal circuits together. Many-to-one connections as in Fig. 14-2 or their reverse, one-to-many, are also essentially nondirectional. There are, however, two factors which seem to give direction to a connection although these are primarily properties of the connected circuits rather than the interconnecting paths themselves. One of these is the control of the connection and the point from which the initial request or need for a connection originates. Usually one terminal circuit in the course of its actions reaches a state where it is necessary to connect to some other circuit. It then initiates a request for a connection which results in operating the connector relays directly or through some intermediate control circuit. The other directional
aspect of a connection is the direction in which information or signals are passed through the connecting paths. This is obviously determined by conditions in external circuits rather than in the connecting circuits.

14.1 THE MULTIPLE

A fundamental arrangement used in connecting circuits is the "multiple" shown in Fig. 14-4. Any of the individual functional circuits T₁, T₂, ..., Tₙ may be connected to the common circuit Tₑ by operating the corresponding connector relay (C-). The diagram shows the connection of only two conductors, but this obviously may be extended by increasing the number of contacts on each relay. This arrangement is called a multiple because all the (C-) relays have corresponding contact terminals on one side multiplied together to a set of common leads to the common terminal circuit. A multiple is usually represented on schematic drawings by showing only a single relay as in Fig. 14-5, indicating the multiple connection by means of a symbol, also shown in Fig. 14-4, and an explanatory note. When using multiples, some feature in the associated control circuits must insure that only one connector relay operates at any time in order to prevent double connections of two or more individual circuits to one common circuit.
14.2 MANY-TO-MANY CONNECTIONS BY MULTIPLES

The principle of the multiple can be extended to many-to-many situations by providing additional multiples. This is illustrated in Fig. 14-6 which shows a second multiple arrangement to provide connections to a second common circuit, $C_m$. For simplicity, the figure shows only one contact on each relay. The contact terminals on both sides of the
relays are multiplied to different sets of relays. Fig. 14-6 may be considered in one direction as two multiples of three relays each, and in the other direction as three multiples of two relays each. The circuit is represented more clearly by redrawing Fig. 14-6 to place the relays in rows and columns as shown in Fig. 14-7. This will be called a "crosspoint array". In a crosspoint array each terminal circuit on one side is associated with a row of connecting relays, while each terminal circuit on the other side is associated with a column of connecting relays. A connection from a circuit on a particular row to a circuit on a column is made by operating the connector relay at the intersection of this row and column. The similarity to a crossbar-type switch is obvious. To prevent double connections, the control circuits must insure that only one relay in any row or any column is ever operated. A crosspoint array is often indicated by showing only a single relay and indicating the multiples as in Fig. 14-8.

A crosspoint array to connect n circuits on one side to m circuits on the other must contain a total of n-m relays. The total possible number of simultaneous and independent connections is equal to either n or m, whichever is smaller. This arrangement is widely used for connecting where it is important that the associated terminal circuits be used to their full capacity. There is no possibility of blocking due to an "all paths busy" condition of the connecting circuit. Each circuit on one side has an individual means of reaching every circuit on the other side, and thus the only reason that a particular connection cannot be made is that all the terminal circuits on one side are busy.

Fig. 14-7 Rearrangement of Fig. 14-6 to Show the Cross-Point Characteristic
14.3 LINKS

An arrangement which provides a means for making one connection for many-to-many conditions is shown in Fig. 14-9 and is known as a "link". This consists of two multiples connected back-to-back so that any T circuit may reach the common link by means of a multiple on one set of relays, and may then reach any one of the C circuits by a similar multiple on a second set of relays. If there are n terminal circuits in one group and m circuits in another, an interconnecting link requires a total of n + m relays. Two relays, one in each multiple, must be operated to establish a connection, and a single link provides a means for making only one connection between the two groups of terminal circuits at any given time.

In order to accommodate several simultaneous connections, additional links may be provided as shown in Fig. 14-10. The simplified form of this arrangement is shown in Fig. 14-11. The multiples at each end of the links here become crosspoint arrays, and the total number of relays is equal to L(n + m), where L is the number of links. This interconnecting scheme provides a means of adjusting the amount of apparatus according to the demand for simultaneous connections between two groups of terminal circuits. It serves a purpose similar to that of the full n-m crosspoint array of Fig. 14-7. Choice between

Fig. 14-11 Crosspoint Arrays at Each End of Links
these two many-to-many connecting arrangements is usually influenced by economic considerations and the expected maximum requirements for simultaneous connections. Each of the two arrangements, Figs. 14-7 and 14-10, provides full access since any circuit in one group may connect to any circuit in the other group. In providing the means for full access, the crosspoint array of Fig. 14-7 also provides a means for handling the maximum possible amount of traffic or number of simultaneous connections. A connection is always possible until one set of terminal circuits is completely busy, and for this reason the crosspoint connecting arrangement is used where the traffic load is heavy.

The link arrangement of Fig. 14-10 is usually more economical where only a small number of simultaneous connections is required but complete access is necessary. This is often the case when the holding time of a connection is short, and terminal circuits which request a connection during busy intervals can wait until a link becomes idle. The link scheme uses apparatus inefficiently when the required number of simultaneous connections becomes large. Suppose there are \( n \) terminal circuits having access to \( m \) terminal circuits through a connector, \( m \) being greater than \( n \). The maximum possible demand on the connector is \( n \) connections, since this number will make the small group of circuits all busy. If these connections are made by means of links, there will be the same number of links, \( n \), as there are circuits in the small

Fig. 14-11 Simplified Representation of Fig. 14-10
group; and these circuits could be wired permanently to the respective links. Thus the connecting relays in the crosspoint array at one end of the links can be omitted, and the connector becomes a simple m x n crosspoint array. The omitted relays, which formed an n x n crosspoint array, served no useful purpose except to permit every link to have access to each of the n circuits. This is usually unnecessary but occasionally is done so that a trouble condition on one connector multiple will not completely block access to one of the n circuits.

It is evident that as the number of links in Fig. 14-10 is increased, a point is reached where the total number of relays required is sufficient to construct a single crosspoint array, of the type represented by Fig. 14-7, which will serve the two sets of circuits more efficiently. This point may be determined as follows:

Relays in crosspoint array = m*n
Relays in link circuits = L(m + n)

Making these equal:

m·n = L(m + n)  
Or:  
L = \frac{m·n}{m + n}

When the number of simultaneous connections required is greater than the value of L in the above expression, a crosspoint array can be constructed with fewer relays. The relative value of L can be better understood by considering the case where m and n are equal. Then:

L = \frac{n^2}{2n} = \frac{n}{2}

This shows that when two equal groups of circuits are connected together, as in Fig. 14-10, by a number of links which is equal to half the number of circuits in one group, then the same number of connector relays could be rewired per Fig. 14-7 to permit all of the circuits of one group to be simultaneously connected to the circuits of the other group.

14.4 MULTISTAGE CONNECTING

A connecting circuit which closes paths at two or more points to establish a connection is called a "multistage connector". The links described in the last section are examples of two-stage connecting arrangements. Connectors may be extended to more stages as illustrated in Fig. 14-12. Each set of relays providing a means for closing the paths at one point is known as a "stage". A "link" as previously defined is a path between stages having a means for multiple access at both ends. A "channel" is a particular path through a multistage connecting circuit consisting of links connected end-to-end by operating a relay (or other connecting device) in each stage.
Perhaps the most outstanding application of multistage connections is in automatic telephone systems of the common-control type, where a multistage interconnecting network is used to connect subscriber lines to trunks. This requires many connections with only a few leads (three or four) involved in any one connection. In these systems, the selecting actions necessary in locating the lines and trunks which are to be connected are performed by the common-control circuits. The interconnecting network serves merely as a means for establishing connecting paths.

The general principle is to arrange a group of switching devices in a "switch block" which provides a means for connecting a number of inputs individually to any one of a number of outputs. These switch blocks may consist of groups of relays arranged in a crosspointing array, or they may consist of a single crossbar type switch. The number of input and output circuits is usually of the order of ten or twenty. To accommodate large numbers of inputs and outputs it is necessary to provide several stages of switch blocks connected together in an arrangement called a "grid network" or simply "grid".

The general two-stage grid using switch aggregates of the crossbar type is shown on Fig. 14-13. The input switches of the grid are usually designated primary, and the output switches secondary. The basic interconnection requirement is that each primary switch group have access via at least one link to each secondary switch group. The link spread between switch groups is almost invariably laid out in a uniform fashion for ease of control. For example, in Fig. 14-13, note that the 0 outputs* of all primary switch groups connect to the 0 secondary switch group, the 1 outputs connect to the 1 secondary switch group, and so forth. Thus, in allocating secondary terminations of links, the output terminal number on the primary switch designates the secondary switch number. Furthermore, it is customary for the primary switch number to designate the secondary switch terminal.

The grid provides a means for any input to reach any output. However, since normally only one link is provided between an input switch group and an output switch group, only one connection between

* In grid-type networks it is customary to start numerical designations with 0 instead of starting with 1.
the inputs and outputs of these particular switch groups can be established at any one time. If the outputs consist of several groups and it is satisfactory to use any circuit within the group when a connection to the group is required, the outputs can be grouped as shown in Fig. 14-13. The input groups on a switch block then can reach a circuit in a particular output group through any one of its available links. This network, when there are ten switch blocks in each stage and each switch block consists of a ten-by-ten crossbar switch (or its equivalent in relays), provides a satisfactory method of connecting a hundred inputs to ten groups of outputs, each output group containing ten circuits.

To extend the grid to provide adequate connecting paths to a particular output without excessive probability of blocking, it is only necessary to add a third stage, the links to which will duplicate the link spread between the first two stages. This is shown symbolically in Fig. 14-14 where each stage is assumed to be ten switch blocks high.

![Diagram of Basic Primary-Secondary Grid](image-url)
In this network any single network input can be connected to any single network output over ten channels consisting of pairs of links in the "A" and "B" link groups. The links forming channels between primary switch group 0 and tertiary switch group 0 are indicated by heavy lines in Fig. 14-14. Note that, between particular input and output switch groups, the available "A" links match in a one-to-one manner with the available "B" links. Each "A" link must be used with a particular "B" link. The group of "A" links originating at a particular primary switch group can be matched as a whole with the group of "B" links terminating on any one of the tertiary switch groups.

A connecting network with increased input and output capacity can be constructed by employing a number of two-stage grids which are interconnected in the same general pattern as the links within a grid. This produces the four-stage network shown in Fig. 14-15. When built full size with ten-by-ten switch blocks, this network will provide connections between a thousand inputs and a thousand outputs with ten channels available for any given connection. The network consists of ten input grids and ten output grids. Each input grid has a group of ten links called "junctors", to each of the ten output grids. Thus a maximum of ten connections between particular input and output grids can exist at the same time. A channel between an input and an output consists of an "A" link, a "B" link, and a "C" link. These links are in groups of ten which match so that on a given connection a particular "A" link must be used with a particular "B" link which in turn must be used with a particular "C" link.

14.5 CONTROL OF CONNECTING CIRCUITS

A connecting circuit establishes paths under external control. The control is often provided by associated functional circuits such as the
selecting circuits which have been described, or the lockout, hunting, or finding circuits discussed in other chapters. However, a general consideration of connecting circuits and the conditions in which they are used indicates a number of basic control requirements which must be met.

Usually the "request" for a connection originates in one or the other of the two circuits which are to be connected together. At times, both circuits take part in the control, either directly or through some intermediate functional circuit.

Fig. 14-15 A Network of Grids
One of the first requirements in the control of a connecting circuit is that double connections must be prevented, that is, only one originating and one terminating circuit must be associated in any single connection. In most of the many-to-one and many-to-many arrangements which have been discussed, the false operation of one or more connecting relays will result in a double connection. For example, in Fig. 14-4, if a desired connection is made between circuits $T_1$ and $T_c$ by operating relay ($C_1$), a false operation of relay ($C_2$) would cause circuit $T_2$ also to be connected to $T_c$. The simple control arrangement indicated in Fig. 14-16, where the initial requests originate in the individual terminal circuits and directly operate the connecting relays, will result in double connections if the individual circuits are allowed to act independently. An intermediate control function is usually provided as shown in Fig. 14-17. The individual circuits present their requests for a connection to the control circuit which co-ordinates the requests and permits only one connection at a time to be made. In this example, when one connection has been established, other individual circuits requesting connections must wait until the original connection is released. The control circuit then acts to establish a connection for one of the waiting circuits, the particular one being determined by some plan incorporated in the detailed design of the control circuit. One of the most severe situations for this type of control circuit is where simultaneous requests for connections are made by two or more individual circuits. The control must quickly and reliably decide upon one of the requesting circuits, establish a connection for this circuit, and cause the others to wait.

The connector control circuit is more complex and often becomes a multifunction circuit when the connector has multiple access on both sides as indicated in Fig. 14-18. Here the $T$ circuits on one side originate requests for the $C$ circuits on the other side. These requests may embrace one of two conditions: (1), the individual $T$ circuit may request a connection to a particular one of the $C$ circuits; or (2), the $T$ circuit may request a connection to any available one of the $C$ circuits, the
particular one being of no consequence. In the first case the T circuits must transmit information to the control circuit which identifies the particular C circuit desired, and the control action must include a selecting function in making the proper connection. In the second case the control circuit must contain within itself a means for deciding upon one of the C circuits and making the proper connection. In either of these cases the control circuit must have a means for determining the busy or idle condition of the C circuits in order to avoid establishing double connections to these circuits. In some arrangements the C circuits indicate their busy-idle condition directly to the control circuit for its guidance, as indicated in Fig. 14-18. In other cases the control circuit may determine the busy-idle status of a C circuit by examining the connector relays serving this circuit. For example, in Fig. 14-7, if any one of the \((C_i)\) relays serving circuit \(C_1\) is operated, this circuit is busy on a previously established connection.

When a multistage connector such as Fig. 14-14 or Fig. 14-15 is to be controlled, the control circuit must contain a means for selecting those groups of links which are candidates for use on a given connection between an input and an output. The control circuit must then make a busy test of these links and select a set of matching idle links which can be connected together to form a channel.

Connectors of the type indicated in Fig. 14-18 usually provide paths which permit connections between more than one pair of circuits to exist at the same time. However, the control circuit often acts on a one-at-a-time basis in setting up connections. In the case of several simultaneous requests, the control circuit will serve one request and cause the others to wait until the circuit paths for this connection have been closed through. The control circuit will then serve another waiting request. The circuits associated on a connection must provide means for holding the connection once it has been set up by the control circuits. This corresponds to "supervision" in a telephone connection.
14.6 COMBINED SELECTING AND CONNECTING

The selecting and connecting functions are often closely associated in switching systems. Many of the selecting networks (for example, the various types of trees), perform their selecting function by establishing a single connecting path from an input point to one of a number of output points. In order to provide several paths, it is only necessary to duplicate the selecting network for each path by providing additional contacts on the selecting relays. This is often done in cases where the number of paths required is small, and although it generally requires more contacts than an equivalent arrangement using a separate set of connecting relays, it is sometimes more economical in practical cases.

Other than this rather obvious combination of selecting and connecting, certain arrangements combine selecting and connecting principles so closely that the two functions cannot be readily isolated. A good example is where a number of common circuits select and connect to a number of individual circuits, and it is desired that several common
circuits be able within limits to select and connect simultaneously in the group of individual circuits. This example is shown in Fig. 14-19. The figure shows two originating circuits, \( T_0 \) and \( T_1 \), having access to four terminating circuits, \( A_0 B_0 \), \( A_0 B_1 \), \( A_1 B_0 \), and \( A_1 B_1 \). The access is through two stages of selecting and connecting relays and can obviously be extended to serve a larger number of circuits. The selecting action of this arrangement is directed by the originating circuits. Each of these has a set of four selecting control leads, \( A_0 \), \( A_1 \), \( B_0 \), and \( B_1 \), and will ground one each of the \( A \) and \( B \) leads according to the designation of the terminating circuit to which a connection is desired. The \( A \) leads, through co-ordinating control circuits which prevent double connections, operate the first-stage connecting relays. The operation of one of these relays extends the \( B \) control leads as well as the intercircuit communication leads, and gives the corresponding \( T \) circuit exclusive use of a part of the second stage and the corresponding group of terminating circuits. The originating \( T \) circuit operates one of the second-stage relays over the \( B \) leads, and connection is thus established to the specified terminating circuit. When one originating circuit has obtained a connection in the terminating group corresponding to the \( A_0 \) lead, the second may obtain a connection in the \( A_1 \) group without interference.

**PROBLEMS FOR CHAPTER 14**

14-1 A group of one hundred \( T \) circuits must have full access to a common group of five \( C \) circuits. The \( T \) circuits can be divided into subgroups of ten, and the holding time per connection is so short that a single connection from a subgroup of ten \( T \) circuits to a \( C \) circuit at a time is satisfactory. Fifty leads per connection are required. Design the connecting circuit.

14-2 (a) A primary-secondary grid arrangement is to be designed using four-by-four switches of the crossbar type. The design should allow for connecting sixteen inputs to sixteen outputs, using a consistent numbering arrangement in associating the switch elements with the links.

(b) Show an arrangement using two input grids and two output grids of the above type for connecting thirty-two inputs to thirty-two outputs with a possibility of thirty-two simultaneous connections. Each input shall have access to every output.

(c) What is the minimum number of through connections in each of the above two arrangements, parts (a) and (b), which can block a particular idle input from reaching a particular idle output?

14-3 Using ten-by-ten switches of the crossbar type, devise an arrangement for connecting one hundred input terminals to fifty output terminals by means of a primary-secondary grid. Provide fifty links and arrange these so that the access from the primary switches to the secondary switches is equalized.
Chapter 15

CIRCUITS FOR LOCKOUT

A major consideration in the design of means for establishing connections between units of a switching system is that of controlling the connecting circuits which close through the desired paths. As discussed in the preceding chapter, certain basic requirements are generally imposed upon the control of each connecting circuit. For example, where any one of a group of individual terminal circuits is to be given access to one common circuit through a connecting arrangement, provision must be made in the control of the connector to prevent more than one connection to the common circuit at any time. If the terminal circuits are to function independently of one another, the design of a control circuit must be such that, even in the presence of simultaneous requests for connection on the part of two or more of the terminal circuits, no possibility of double connections exists. The action of the control circuit, then, must be to allow one individual terminal circuit to gain connection to the common circuit, locking all others out until this connection is dropped. In this way, the terminal circuits requesting connection to the common circuit can be served one-by-one.

Circuits for obtaining this lockout function are considered in the present chapter. Defined broadly, a "lockout" circuit is one which, under the control of a number of external circuits, provides an output indication corresponding to one and only one of these circuits any time. The nature of the control maintained by the external circuits and of the output indications furnished by a lockout circuit depend upon the particular application involved. However, since there are certain commonly-encountered situations where lockout action is necessary, it is profitable to examine these situations to determine the general requirements which a lockout circuit must satisfy.

Three switching arrangements involving lockout circuits are shown in block form in Fig. 15-1. In each of these arrangements, any of a number of individual terminal circuits, acting independently, may request service by placing ground on a control lead to a lockout circuit; the lockout circuit insure that the requests are served one at a time.*

* A rotary switch symbol is shown in each lockout block to indicate this characteristic, although it should not be inferred that a switch is necessarily used in the circuit configuration.
Fig. 15-1 Arrangements Involving Lockout Circuits
Fig. 15-2 Evolution of the Double-Transfer Lockout Circuit
The particular service requested depends upon the operation of the over-all switching system. In Fig. 15-1A, the terminal circuits are in effect requesting permission to carry out some circuit action which may be performed by only one circuit at a time. The lockout circuit, upon receipt of one or more requests, grants one request by placing a ground indication on the output lead to the favored terminal circuit. When this terminal circuit has completed its action, it removes its control ground to the lockout circuit, and the lockout is free to return a ground indication to some other requesting circuit.

In Figs. 15-1B and 15-1C, the individual terminal circuits request connection to a common circuit, the situation discussed in the introductory paragraph. The lockout circuit itself may perform the connecting function as in Fig. 15-1B if the number of leads involved is small; or instead, the circuit may control a separate connecting circuit, as in Fig. 15-1C. In the former case, the output indication established by the lockout is the closing of a connection between the common circuit and a requesting circuit, the connection being maintained until the requesting circuit involved removes its request ground. In the arrangement of Fig. 15-1C, the lockout circuit serves requests by cutting through grounds to operate the connector relays in the connecting circuit.

Inspection of the three block diagrams of Fig. 15-1 indicates that the lockout circuits perform identical functions in all three cases. Each lockout is controlled by ground signals originating in the individual terminal circuits. Under the control of these signals, the lockout circuit establishes a unique path to one of the output leads corresponding to a requesting terminal circuit. This path is maintained as long as the corresponding request signal is present.

15.1 THE DOUBLE-TRANSFER LOCKOUT CIRCUIT

The design of a circuit which would serve requests in the order in which they originate appears, ideally, to be the most satisfactory approach to the lockout problem. However, lockout circuits which assign preference dependent upon time of request are generally uneconomical in the number of relays or other switching elements required. Moreover, it is unnecessary in many cases to establish a definite order of service, especially where the number of concurrent requests is relatively small.

If it can be assumed that, for a particular application, the order of serving concurrent requests is not of importance, then the basic requirement is that the lockout circuit close a path to the output lead associated with an input request lead when that request lead is grounded, with the restriction that one and only one of the input request leads that may be concurrently grounded can be served at a particular time. The
connection is to be maintained until the ground is removed from the request lead that was served. Thereafter, a second output path can be closed if an input request is still waiting.

As a first step in the design of an appropriate circuit, each control lead is connected to the winding of a corresponding relay. The connecting paths to the output leads can be closed through a contact network placed on these relays. A suitable configuration for this network, however, is not immediately apparent from an inspection of the lockout requirements; the requirements must be restated in terms of the conditions of the controlling relays.

One such restatement which apparently satisfies all the original lockout requirements is that, with the relays arranged in a row, a relay in operating shall close through a path to its corresponding output lead if all preceding relays are released. This suggests a simple transfer chain as shown in Fig. 15-2A. For any combination of requests over the control leads, a corresponding combination of relays is operated, but only the output lead corresponding to the operated relay farthest to the left in the figure is connected to ground.

As a result of this characteristic, there is established in the circuit an inherent order of preference. That is, in the case of simultaneous requests, the output lead which is grounded corresponds to the requesting terminal circuit which holds a position to the left of all other requesting circuits. Thus, the terminal circuit farthest to the left in Fig. 15-2A has the highest or first preference, and the circuit farthest to the right has the lowest or last preference. Accordingly, the controlling relays are sometimes called preference relays. In the figure, the preference relays (P), the control leads a, and the output leads b, are designated to indicate the order of preference; the subscripts "H" and "L" denote highest and lowest preference, respectively.

As specified by the original lockout requirements, the circuit of Fig. 15-2A gives service to a terminal circuit by connecting the corresponding output lead to ground as long as that circuit requests service, with one important qualification; i.e., that no terminal circuits holding higher preference initiate requests during this time. Requests from lower-preference circuits have no effect, but a request from a higher-preference circuit will steal the connection, transferring the ground signal from the output lead originally grounded to the output lead corresponding to the new request. It is necessary to prevent this transference by some modification of the circuit since, as stated in the lockout requirements, the output path corresponding to a requesting circuit must be maintained until the control lead from that circuit is ungrounded.

Since the undesired transference can only occur from an output lead of lower preference to one of higher preference, a logical modi-
lication of the circuit of Fig. 15-2A is that of causing the operation of any preference relay (P−) to prevent the subsequent operation of any higher-preference relay. This might be achieved by causing each relay, in operating, to open the input control leads to all higher-preference relays. Such a scheme, however, involves large contact loads on the lower-preference relays, particularly if the total number of preference relays in the lockout circuit is great.

A more practical alternative, shown in Fig. 15-2B, is that of supplying battery to the relay windings through a chain of break contacts in such a manner that the operation of any relay removes battery potential from the windings of all higher-preference relays. However, means must be added to this arrangement to supply battery potential to a relay, once operated, independent of the possible subsequent operation of lower-preference relays. This may be done, as in Fig. 15-2C, by allowing each relay, in operating, to connect its winding directly to battery.

Although the arrangement of Fig. 15-2C completely satisfies the requirements for the lockout function, it permits an inadmissible circuit action to arise under certain conditions. As an illustration of this, assume that preference relays (PH) and (PL) are operated. Then, if the control lead of one of the intermediate (P−) relays is grounded, this relay will start to operate through its own break-contact, since battery is supplied from the make-contact on relay (PH). A "buzzing" situation is thus established until relay (PH) is released. This circuit action, however, may be eliminated by replacing the break-make transfer contacts in the battery chain with continuity transfers, as in Fig. 15-2D.

This final arrangement, the so-called double-transfer lockout circuit, as illustrated in Fig. 15-2D, is widely used in applications where economy of relays is important and where the inherent pattern of preference is acceptable.

In general, it is undesirable to place direct battery on the contact springs of relays since these springs may be inadvertently shorted to ground during maintenance activities. Where the double-transfer lockout circuit is to be employed in applications where the failure of a fuse may cause large portions of a switching system to become inoperative, the basic arrangement of Fig. 15-2D may be modified to remove battery from the preference relay springs. An obvious modification is that of replacing the battery connections to the preference relays by ground connections, and employing battery (through a protective resistance) as the control condition passed from each requesting terminal circuit.

Where separate connector relays are required to handle a group of output paths, the connector relays can be integrated with the lockout control. One such arrangement, shown in Fig. 15-3, is a modification of Fig. 15-2B. Here, the operation of a preference relay (P−) operates
The Design of Switching Circuits

OUTPUT LEADS TO INDIVIDUAL TERMINAL CIRCUITS

TO COMMON CIRCUIT

THROUGH OTHER RELAYS

CONTROL LEADS

Fig. 15-3 Lockout Circuit Employing Two Relays per Input

its associated connector relay (CON_) through a transfer chain. The connector relay, in operating, opens the battery path to all higher-preference (P_) relays. Thus, only one of the connector (CON_) relays may be in an operated state at any time; and, as a result, the output paths may be established through single make-contacts rather than through chains of transfer-contacts. The preference characteristics of this dual-purpose lockout circuit are similar to those of the double-transfer circuit.

The preference pattern of the double-transfer lockout may be of considerable consequence. In cases where the frequency of requests is low and lockout-enforced delays in obtaining service occur only occasionally, the preference pattern set by the lockout has small importance. However, where the frequency of requests is high, and where it is likely at any time that several requests will be waiting for service, the preference pattern has a significant effect on the over-all operation of the system. Evidently, the highest preference request which successfully operates the corresponding preference relay is a controlling factor. All subsequent requests of lower preference also operate their preference relays, and requests of lower preference will therefore be
served before any subsequent requests from higher-preference terminal circuits. The preference relays associated with these higher-preference requests cannot operate until all lower-preference relays have released. When the last of the lower-preference relays has released at the end of a lockout round, all waiting requests start to operate their respective preference relays simultaneously. Since each relay acts to remove the battery path to all preference relays higher than itself, high-preference relays which are slow in operating may not succeed in closing their direct locking paths to battery, and will, therefore, not remain operated. It is possible that several such slow relays may be left unoperated by a rapidly-acting low-preference relay. Since it is not practical, in most cases, to insure that the preference relays will be sufficiently identical in speed to avoid the hazard just described, the double-transfer lockout circuit may occasionally allow certain terminal circuits to monopolize the service, while other terminal circuits with slow preference relays are denied service for relatively long periods.

15.2 END-RELAY LOCKOUT CIRCUITS

The monopolization occurring as a result of the race at the end of a lockout round can be eliminated by providing additional relays which, at this critical point, allow all preference relays to remain under the control of their input leads until every preference relay corresponding to a waiting request has had time to operate. The lockout action is then permitted to continue in its usual fashion until all relays are again released. The added relays preventing the race may be called "end" relays, since they act at the conclusion of each round of service.

The circuit of Fig. 15-3 is well-adapted to this modification. Inspection of the figure indicates that the end relays should function to delay the operation of any (CON_) relay after the all-preference-relays-released condition occurs, since it is the operation of a (CON_) relay which prevents higher-preference (P_) relays from operating. The delay should be sufficient to allow the slowest of the (P_) relays to operate.

A suitable end-relay arrangement is shown in Fig. 15-4. Starting with this circuit normal, the operation of any (P_) relay operates its associated (CON_) relay and, also, the end relay (A) which is in series with the transfer chain ground supply. Relay (A) locks itself to ground and operates relay (B). Although operation of (B) opens the initial ground supply, the locking ground for (A) maintains the operating path for the (CON_) relay. Lower preference (P_) relays are permitted to operate if their control leads are grounded, and the associated (CON_) relays operate in preference order as each preceding (P_) relay is released.
Relay (A) remains operated until the release of the last-operated (P__) relay. When (A) releases, the ground available for the operation of the (CON__) relays is removed from the transfer chain on the (P__) relays. Waiting requests can cause the operation of their corresponding (P__) relays, following the release of the last-operated (P__) relay, but no (CON__) relay can operate until the slow-release relay (B) has provided operating ground through its break-contact. When (B) has released, this ground is cut through, and the (CON__) relays can thereafter operate in preference order. Continuity-transfers are employed in the operating path of relay (A) to hold it operated during the release of any (P__) relay preceding the last one of the cycle.

This scheme provides full prevention of monopolization and is subject only to a relatively minor hazard. Outputs to two terminal circuits may be established simultaneously for short intervals due to the overlap time between the operation of one (CON__) relay and the release of another. This may be avoided where necessary by selecting relays with suitable acting-time characteristics, or by using a break-connected preference chain for the output network.

OUTPUT LEADS TO INDIVIDUAL TERMINAL CIRCUITS

OUTPUT LEADS

CONTROL LEADS

Fig. 15-4  Lockout Circuit Using End Relays
Another method of eliminating the undesirable effects of monopolization is that of controlling the access of request signals to the lockout circuit, permitting the request signals to enter the lockout simultaneously and then preventing additional requests from affecting the lockout until all requests in the original group have been served. When the last request in the lockout circuit has been served, the control paths from the individual terminal circuits to the lockout are closed to allow all waiting terminal circuits to operate their corresponding preference relays. The access paths are then reopened until the end of the round, when all preference relays have been released. In addition to preventing monopolization, this type of circuit provides a closer approximation to service in order of request than the circuits previously described.

A lockout circuit operating in this manner is shown in Fig. 15-5. In this arrangement, a slow-operating relay \( G \) functions as a gate to control the operating paths for the preference \( P_\) relays. This gate relay operates when one or more \( P_\) relays operate, opening all direct control leads to the preference relays. The preference relays which are in an operated state when \( G \) operates are held under the control of their corresponding terminal circuits. Therefore, these terminal circuits are served according to the wired preference of the \( P_\) relays, and all new request signals are forced to remain outside the gate. Upon the release of the last-operated preference relay, the gate relay \( G \) is released, allowing all waiting request signals to operate their preference relays before relay \( G \) again opens the direct control paths. The
output ground source of the lockout circuit is passed through a make-
contact on the gate relay to prevent false outputs during the simulta-
neous operation of the preference relays in the gate-released interval.

In the arrangement of Fig. 15-5, the gate relay (G) must be made
sufficiently slow in operating to insure that the slowest preference relay
will have sufficient time to close its locking contact before the gate
opens the direct operating path. This requires only that the operating
time of the gate relay be slower than the difference between the operat-
ing times of the slowest and fastest preference relays. This follows
since the fastest preference relay will energize the gate relay; the gate
relay, in turn, must not operate until the slowest preference relay has
been given adequate time to operate. Otherwise, the slower preference
relays could not lock operated when in competition with the faster re-
lays. Where, in certain applications, this situation is tolerable and a
fast gate relay might therefore be selected to speed over-all circuit
operation there is a possibility that a "pumping" condition may arise as
a result of contact stagger time. This condition may occur when the
locking contact on a preference relay does not close until after the con-
tact operating the gate relay has closed. The gate relay may then oper-
ate quickly enough to open the direct operate path of that preference
relay before the locking contact has been closed. To avoid this hazard,
a preliminary make-contact is used to lock the preference relay before
the fast-operating gate relay is energized. Such an arrangement insures
that at least one preference relay will be locked operated whenever the
gate relay opens all control leads.

Fig. 15-6 Gate Lockout Circuit with Check on Preference Relays
Although the gate lockout circuit has the advantage of serving its associated terminal circuits in a relatively close approximation to the order of requests, there is a somewhat greater possibility of failure in this type of circuit. The request lead from each terminal circuit passes through a break-contact on the gate relay, and, if one such contact should become permanently open as a result of dirt or some other trouble condition, the corresponding terminal circuit will be denied service until the trouble is remedied.

The basic gate lockout circuit of Fig. 15-5 may be modified to facilitate checking that all terminal circuits have access through the gate relay contacts. In this modified circuit, shown in Fig. 15-6, paths are set up at the beginning of each round to operate all preference relays through the respective break-contacts on the gate relay, after which those relays not associated with requesting terminal circuits are released. A so-called splitting relay serves to connect all input control leads together, so that when any control lead is grounded by its terminal circuit, all preference relays (P_) operate. When all (P_) relays have operated, the actuating ground for the (P_) relays is passed through a chain of make-contacts to a "prime pair,"* consisting of relays (T) and (T'). Relay (T) operates and, in turn, operates the splitting relay (SP) which acts to release those preference relays not associated with grounded control leads. The operation of (SP) removes the energizing ground from the prime pair so that relay (T') operates. Relay (T') connects ground from paralleled make-contacts of the operated (P_) relays to the gate relay (G). Relay (G) then operates and ground is cut through to an output lead as determined by the highest-preference (P_) relay operated. The gate relay is held operated as long as any (P_) relay remains operated. Release of the last-operated (P_) relay restores the circuit to normal, releasing the gate relay (G), the prime-pair relays (T) and (T'), and the splitting relay (SP). Circuit means may be added to provide an alarm in case of trouble preventing the operation of a preference relay at the beginning of the lockout sequence.

15.4 VARYING THE ORDER OF PREFERENCE IN LOCKOUT CIRCUITS

Although, in most practical cases, preferential order is not a characteristic which is required in the functions to be performed by a lockout circuit, preference in serving terminal circuits is evidently an inherent quality of relay lockouts. In all lockout circuits discussed so far, the assignment of preference has remained fixed, determined only by the relative positions of the preference relays. Where monopolizing is prevented, the order of preference has little effect upon the average

* Chapter 11, Circuits for Counting
Fig. 15-7 Lockout Circuit with Preference Reversing

Fig. 15-8 Lockout Circuit with Changing Preference
waiting time of any individual request, so that fixed preference is generally satisfactory. However, lockout applications occasionally arise where it is desirable to change the order of preference periodically, either to minimize adverse effects of a trouble ground on a control lead or to equalize usage of circuits assigned by a lockout circuit.*

The simplest method of changing lockout preference is to reverse the order of preference on alternate uses of the circuit. This can be accomplished by a pulse-frequency divider which shifts the output paths between two transfer chains facing in opposite directions. This is shown in Fig. 15-7. The pulse-divider in this arrangement is actuated by the gate relay each time the circuit operates and returns to normal. In another, more elaborate method, the relative order of preference is maintained but the assignment of highest preference is changed on each return of the lockout circuit to normal. A lockout circuit operating in this manner is shown in Fig. 15-8. Here the preference chain is closed in a ring, and the point of entry is established by a sequence circuit. This sequence circuit is stepped by the lockout circuit (in particular, by the gate relay operation) and thus shifts the position of highest preference. Where it is not economical to provide a sequence circuit large enough to shift the starting point of the preference, in turn, to each preference relay, a sequence circuit may be used containing fewer stages controlling fewer points of entry.

15.5 MULTISTAGE LOCKOUT CIRCUITS

In those cases where a large number of individual circuits must compete for service through a lockout, provision of one preference relay per terminal circuit may prove to be uneconomical. Instead, the lockout action can be carried out in stages, so that groups of circuits first compete with each other in a group lockout stage, and then the individuals of the successful group compete in a second lockout. In such a two-stage lockout, there must be two separate indications from each terminal circuit: one associated with the group containing that circuit, and the other associated with the individual circuit itself within the requesting group. These indications may be provided by two separate leads per terminal circuit, as shown in Fig. 15-9A. One set of leads is connected in common to form a single group-identifying lead, and the others, not multiplied, identify the individual circuit making the request. If, for some reason, only one lead per individual terminal circuit is available, these leads can be multiplied together in groups through contacts of a splitting relay (SP), as shown in Fig. 15-9B. When a group is served through the group lockout, this splitting relay is operated to separate the leads for use in controlling the individual lockout stage.

* Examples of this latter application are given in Chapter 16, Circuits for Finding and Hunting.
In a two-stage lockout arrangement, it is usually necessary to include a set of connecting relays to carry to the second lockout stage the individual request leads of the group being served. Fig. 15-10 shows a two-stage lockout controlled by sixteen individual circuits, employing four first-stage double-transfer lockout relays \((AP_0)\) to \((AP_3)\), four connector relays \((C_0)\) to \((C_3)\), and four second-stage double-transfer lockout relays \((BP_0)\) to \((BP_3)\), or a total of twelve relays. This number compares favorably with the sixteen relays which would be necessary in a corresponding single-stage lockout. Some of the relays, however, are heavily loaded; and, because of the sequential action of the two stages, the over-all circuit operation is somewhat slower than that of a single-stage lockout. Any type of lockout circuit may, of course, be substituted for the double-transfer circuits illustrated in the figure, and the number of stages may be increased to suit the particular application.

15.6 ELECTRONIC LOCKOUT CIRCUITS

Certain electronic devices are inherently adapted to use in lockout circuits. It will be recalled that if a number of cold-cathode gas tubes are provided with a common inductive impedance, any attempt to fire the tubes simultaneously or sequentially will result, with small
probability of failure, in the ionization of only one.* Once a tube has been fired, it will remain ionized until its conduction current is reduced below the sustaining value. Thus, an electronic lockout circuit can be constructed of gas tubes with a common impedance, one tube for each individual terminal circuit, as illustrated in Fig. 15-11. Requests for service consist of positive voltage conditions placed on the control leads by the terminal circuits. Indication that a lockout tube (L-) has been ionized is derived from a load resistance $R_L$ in the conducting path of the tube; the indication, then, is a change in the potential of the output lead corresponding to the chosen terminal circuit.

The physical theory accounting for the behavior of parallel gas tubes is beyond the scope of the present text. However, as discussed in

* Chapter 10, Elementary Electronic Switching Circuits
Chapter 10, the lockout phenomenon is, principally, a result of the dynamic negative resistance region traversed by a tube in passing from the non-ionized state to the stable ionized state. In this region, a differential increase in current through the tube is accompanied by a corresponding decrease in potential across the tube. Thus, if two lockout tubes \( (L_\text{a} \text{ and } L_\text{b}) \) attempt to pass through the negative resistance region together, a momentary unbalance between the two tubes will be immediately increased, and only one tube will reach stable ionization. Subsequent requests for service cannot result in ionizing their corresponding tubes, since the common cathode potential is held sufficiently positive by the conduction through the tube originally ionized to prevent the ionization of other tubes.

For this type of circuit to operate with minimum possibility of failure, all tubes attempting to reach stable ionization must be held in the negative resistance region until all but one tube have been extinguished. The time required for this depends upon such factors as the de-ionization time of the tubes, the distributed capacitance in the circuit, and the magnitude of the negative resistance component of each parallel element. The common inductance \( L_C \) is provided to increase the length of time for which the tubes are maintained in the negative resistance region. However, since a high value of inductance decreases the operating speed of the lockout circuit, a compromise must often be made between speed and circuit reliability.
The effective negative resistance exhibited by each parallel leg of the lockout is, in actuality, the negative-resistance characteristic of a tube less the resistance of the series load resistor $R_L$. Since the reliability of the circuit is a function of this effective negative resistance, the load resistances must be as low as possible. One method of decreasing the effect of the load resistors on the over-all negative resistance of each parallel leg is to place a capacitor in parallel with each load resistor. These capacitors serve to lower the positive resistive component (during the transient lockout action) in series with the negative resistance component of the activated tubes.

Reliability of circuit action may also be increased by employing multielement tubes which ionize in successive stages. Corresponding stages can be multiplied to common impedances so that lockout action occurs in traversing the negative resistance region of each stage. If the probability of two tubes passing through one such region is small, the probability of covering all such regions together is infinitesimal.

Since it is the dynamic negative-resistance characteristics of gas tubes which permit their use as lockout elements, it seems evident that any device with similar characteristics could be employed in a lockout arrangement. The effectiveness of the device will, of course, depend upon such factors as the magnitude of the negative resistance, the time necessary for the device to pass from a normal state to an activated state, etc. In general, however, any electronic lockout operating on the basis of negative resistance may be represented as in the block diagram of Fig. 15-12. Each block in the diagram is an element with two stable states, one state considered as the normal or rest condition, and the other state considered as an active condition providing a corresponding output indication. When actuated by a signal on its associated control

![Fig. 15-12 Block Diagram of Electronic Lockout Circuit](image-url)
lead, the element attempts to pass from its normal condition to its active condition through a negative resistance region in its characteristics. As already discussed, the negative-resistance characteristics in conjunction with the common inductive impedance must be such that only a single element may reach the active condition.

This general solution to the lockout requirements may be used in the derivation of numerous electronic circuit arrangements for various specific applications. For example, in a particular instance it might be necessary that a gas-tube lockout circuit give positive-potential signals on the output leads as active indications, and zero potential as the normal indication. These requirements suggest that the output leads be connected to the main cathodes of the gas-tube lockout elements. However, to prevent momentary false indications on the output leads, the main cathodes should not be in the paths through which the tubes are fired. For this reason, tubes with separate control anode-to-cathode gaps might be employed, the ionization of a control gap transferring to the main anode-to-cathode gap. The negative resistance exhibited during this transfer could be utilized to obtain the lockout action.

A lockout circuit operating in this manner is shown in Fig. 15-13. With all tubes extinguished, a positive potential is placed on one or more control leads by their respective terminal circuits. This potential is sufficient to break down the control gaps of the corresponding tubes.
Lockout may or may not occur at this stage of operation, depending upon the characteristics of the tubes and the magnitudes of the circuit components. With the control gap ionized, the discharge transfers to the main-anode-to-starter-cathode gap. Lockout action is obtained during this transfer, since the main anodes and starter cathodes, respectively, are multiplied together and the latter are connected to a common inductive impedance. Only one tube, then, can effect this transfer; and, as the transfer occurs, the multiplied starter cathodes rise in potential sufficiently to extinguish all starter gaps. Finally, the discharge in the one tube remaining ionized is transferred to the main-anode-to-main-cathode gap, actuating the output lead connected to the main cathode of the tube. The potential on the output lead is maintained, and the tube remains ionized until the common battery supply lead is opened, restoring the circuit to normal.

It is necessary in this circuit to maintain the main-anode-to-starter-cathode discharge, after transfer to the main cathode is completed. Otherwise, waiting terminal circuits could again fire their respective starter gaps when the starter cathodes returned to ground potential. Under these conditions, another tube could then be fired since there is no lockout action between main gaps. The resistance ($R_C$) in the starter cathode circuit must be chosen, therefore, to allow sufficient current to enable transfer to the main gap and still maintain the starter-cathode-to-main-anode discharge. Individual resistors in the input control leads are utilized to limit the current on starter-gap breakdown to a low value.

**PROBLEMS FOR CHAPTER 15**

15-1 (a) The control leads of a ten-relay double-transfer lockout circuit (Fig. 15-2D) are designated $a_0$ through $a_9$. Lead $a_0$ controls the relay having first preference in the output transfer chain. Give the sequence in which the output leads are grounded for the following sequence of grounding and opening of the control leads (overscore denotes opening). Some of the requests will be abandoned without obtaining service. Assume sufficient time between changes of control conditions for relays to act, and assume also that all the preference relays are identical in characteristics.

Control sequence: $a_2\overline{a}_3-a_0-a_7-a_5-a_7-a_8$

$\overline{a}_1-a_5-a_2\overline{a}_9-a_0-a_6-a_1-a_7-a_2-a_4-a_6-a_9$

(b) Repeat part (a) for a gate-type lockout circuit (Fig. 15-5).

15-2 Four individual terminal circuits compete in a lockout circuit to establish a connection to a common circuit. It is desired for a particular application that the terminal circuits be served in a sequence closer to that in which they originate requests than can be afforded by either the gate or the double-transfer lockouts described in the text. A plan is adopted, therefore, in which the first request to appear is to be served immediately, and the second request is to establish a preference
condition such that it will be served next, no matter what other requests may appear before the first requesting circuit has released its connection. Requests that are originated while a connection is maintained and after the preference for the circuit to be served next has been established, are not necessarily served in the sequence in which they are originated.

A request for service from a terminal circuit consists of a ground condition on its corresponding control lead. The connection to be established from terminal circuit to common circuit consists of a single lead.

Design a relay lockout circuit to fulfill these requirements. (This can be done with nine relays.)

15-3 Ten primary terminal circuits, designated A₀ to A₉, are associated with ten corresponding secondary terminal circuits, designated B₀ to B₉, through a lockout circuit. Upon request (a control lead grounded) from a particular primary terminal circuit, a single-lead path is connected through from that primary circuit to its corresponding secondary circuit, and all other primary circuits are prevented from obtaining connection to their corresponding secondary circuits. Only one primary circuit is to be connected to its respective secondary circuit at a time. A connection, once established, is to be maintained until the primary circuit involved has terminated its request condition.

Design the required relay circuit to perform the lockout and connecting functions. (This can be done with ten relays.)

15-4 A gate lockout circuit (Fig. 15-5) is used to control the access from a number of terminal circuits to a common circuit. As a means for measuring the traffic handled by the lockout circuit, it is specified that a ground pulse of approximately forty milliseconds duration be supplied to an electro-mechanical counter over a single lead TM each time a terminal circuit obtains connection through the lockout to the common circuit.

Design a relay arrangement controlled by the relays of the gate lockout circuit which will provide the required pulses. It may be assumed that the minimum duration of each terminal-circuit-to-common-circuit connection is approximately 400 milliseconds. (This can be done with three additional relays.)

15-5 Design a lockout circuit to establish upon request an ungrounded one-wire connection from any one of ten input circuits to a single common circuit, meeting the following requirements:

(a) The input request signal from each input circuit is ground on a single control lead. This ground is maintained until the connection is to be opened.

(b) When the lockout circuit is idle, two or more input requests originating simultaneously (within the acting time of a relay) constitute a condition which must light a common alarm lamp and prevent any connection as long as a double request persists.

(c) Any requests occurring while an established connection is maintained must be locked out and must light a common busy lamp which will stay lighted as long as the subsequent request persists. However, if the first connection drops out while a single second request is maintained, the second request shall establish a connection. If two or more requests are waiting when the first connection drops out, they shall be treated as requests originating simultaneously.

(d) The circuit shall employ no resistors or shunting techniques in its solution.

(This can be done with eleven relays.)
Chapter 16

CIRCUITS FOR FINDING AND HUNTING

It is often necessary in switching systems to choose one item out of a group of similar items on some predetermined basis and to give an indication of the choice. The information derived from the circuit that performs the choice, known as a "locating" circuit, is used to determine which of the several items will be used in a particular situation, and most frequently serves to control a connecting circuit, either directly or indirectly.

The broad class of locating circuits breaks down into three functional subdivisions: selecting circuits, finding circuits, and hunting circuits. From the circuit point of view, the selecting circuit, or circuit which picks a particular predetermined item from a group, forms a distinct subgroup which it is convenient to treat independently. This was done in Chapter 13, Circuits for Selecting. The other two functional subgroups, finding circuits and hunting circuits, are closely related circuitwise and are discussed in this chapter.

A "finding" circuit can be defined as a circuit which locates from among a group of items one particular item which has itself initiated a request to be found. In general, the requests are introduced at random, and the finding circuit must act to give a positive indication of only one of perhaps several competing items. An example of the application of a finding circuit is in telephone systems where, when a subscriber initiates a call, his particular line must be found and identified.

A "hunting" circuit, on the other hand, is a circuit which locates any available one of several identical items when it receives a request signal from an external circuit. Availability is usually based on the busy or idle condition of the individual items, with the choice restricted to idle items. A hunting circuit might well be used in a computer system to locate an idle register unit in a set of registers into which to place a number after an arithmetical operation.

A block diagram indicating the relationship between a finding or hunting circuit and its environment is shown on Fig. 16-1. The blocks labelled G- represent the several items or circuits of the group from which the locating choice is to be made. Each G- block is associated with the locating circuit by means of a T- lead which, in the case of a
The Design of Switching Circuits

finding circuit, provides the individual request signal; and, in the case of a hunting circuit, gives a busy or idle indication. The information specifying the located G-item is provided by a signal on one of the output L-leads, which, as mentioned before, may actuate a connecting circuit to associate the corresponding G-item with a common circuit, or may simply make available the location information for use by some other functional circuit. The principal external difference between finding and hunting circuits lies in the origin of the start signal, carried over lead ST of Fig. 16-1. With finding circuits, the start signal is always provided by one of the G-circuits. Analysis of the situation indicates that the start signal can be applied to a common lead, as shown on the figure. This lead may control a connector to associate the finding circuit with the G-circuits, or it may initiate appropriate action within the finding circuit. If the finding circuit is permanently connected to the G-circuits, a signal on one of the T-leads may be sufficient to start action without the necessity of a separate ST lead.

If the locating circuit is used for hunting, the start signal always originates in some external circuit which desires the locating of one of the G-circuits. If the hunting circuit is a common circuit, the ST lead may control a connector to associate the hunting circuit with the desired group of G-circuits; if the hunting circuit and G-circuits are permanently connected, the ST lead may supply ground to a contact network on the hunting relays in order to give an output indication.

16.1 BASIC REQUIREMENTS OF FINDING AND HUNTING CIRCUITS

From a careful analysis of the circumstances under which finding and hunting circuits must work, certain basic requirements can be deduced. These are in addition to considerations affecting the start control and the method of connecting the locating circuit to the associated
group of circuits. Factors such as these latter tend to vary with every design problem. The basic requirements derive from the following considerations:

**Finding Circuits**

(1) Circuits that may request finding action can, in general, initiate requests independently and at any time.

(2) If a request is entered while a previous request is being served, the second request must not be able to interfere with the established output condition of the finding circuit.

(3) Since requests may, in general, be entered at random, the finding circuit must be able to accommodate simultaneous requests and give an output indication corresponding to only one request at a time.

**Hunting Circuits**

(1) The circuits of the group in which the hunting action is taking place are available (idle) or unavailable (busy) independently of each other. A circuit may change from a busy to an idle state at any time. In the usual situation, the hunting circuit is the only circuit with access to the group, and individual group circuits cannot become busy except under control of the hunting circuit itself. However, in multifunctional circuits, which are not discussed here, it is sometimes necessary to permit two or more hunting circuits to work in a single group simultaneously. Under such circumstances, special circuit means are required to guard against adverse effects from group circuits becoming busy from other points during a hunting action.

(2) If, after the hunting circuit has located a particular idle circuit of a group, another circuit becomes idle, the appearance of the second idle circuit must not interfere with the established output condition of the hunting circuit.

(3) When the hunting circuit starts its hunting action, several circuits in the associated group may be idle concurrently. The hunting circuit must be able to give an output indication corresponding to only one of the available circuits at a time.

The analysis of the above considerations leads rapidly to the conclusion that the basic requirements of hunting and finding circuits are identical with the basic requirements for lockout circuits. Therefore, although the circuit concepts are functionally different, in circuit form hunting-finding circuits are identical or similar to the lockout circuits discussed in Chapter 15. This is clearly indicated in Fig. 16-2, where
the configuration of a double-transfer lockout circuit is used for either finding or hunting in a situation where the finding request signal or the idle hunting indication is an active (grounded) condition on one of the T-leads.

If the circuit of Fig. 16-2 is used for finding, the common ST lead is grounded by any one or a combination of the group of circuits which may request finding action. In addition, any requesting circuit concurrently grounds its T-lead. The ST lead operates a connector relay to associate the locating relays with the T-leads, and one or more of the (L-) relays operates. An output indication corresponding to the operated (L-) relay in highest preference appears on one of the L-leads. If simultaneous requests are present, or if subsequent requests appear, they cannot affect the output until the original winning request is withdrawn.

When the circuit of Fig. 16-2 is used for hunting, the ST lead is grounded by an external circuit which requires the locating of an available circuit in a group of identical circuits. With circuit availability indicated by ground on the associated T-lead, the chosen output will again be supplied by the operated (L1) relay in highest preference without possibility of interference from other circuits in the group.

In lockout circuit applications, it is usually convenient to enter the lockout with an active condition on the service request lead. However, in hunting-finding circuits (particularly hunting circuits), circumstances may dictate the removal of an active condition of the T-lead as
the request signal or the idle indication. This requires a change in the basic circuit to prevent the release rather than the operation of a higher preference locating relay during the hunting or finding process. This is easily accomplished by replacing the lockout transfer chain of Fig. 16-2 (which prevents higher preference relays from operating) with the lockup contacts of Fig. 16-3 (which prevent higher preference relays from releasing). In addition, a timing relay is necessary to hold open the output signal until the locating relays have established their condition.

Certain other basic requirements for hunting and finding circuits develop logically from their functional application. For example, when a circuit is located after entering a finding request, external circuit action is initiated which will eventually result in the withdrawal of the original request signal. In similar fashion, when an idle circuit is located in a hunting action, external circuit action will eventually cause the original circuit to be made busy. In both these cases, the change in condition from request to no-request or from idle to busy of a located circuit must not be allowed to affect adversely the subsequent action of the locating circuit. Most frequently, this requirement is met by coordinating the sequential action of the locating circuit with respect to the other functional circuits comprising the multifunctional unit, rather than by modifying the locating circuit itself. However, this must always be kept in mind as one of the crucial factors in the design and application of finding or hunting circuits.
16.2 FINDING CIRCUITS

By their fundamental nature, finding circuits most closely approach lockout circuits in requirements and function. Unless unusual circumstances intervene, the normal input request signal to a finding circuit is an active condition, and therefore a standard lockout configuration is satisfactory. In general, any of the circuits of Chapter 15, including double-transfer, gate, end-relay, multiple-stage, and electronic lockouts, are suitable for finding. The specific type used will depend upon the requirements of the particular application. The use of a common start lead and connector arrangement may or may not be necessary.

16.3 HUNTING CIRCUITS

Hunting circuits tend to vary more in their subsidiary requirements than finding circuits. In addition, the idle-busy indication is frequently the inverse of the usual lockout input signal so that a less familiar circuit form is required. For both these reasons, several circuits, designed to meet different requirements, will be presented in this section.

The first circuit to be designed must meet the following requirements in addition to the basic hunting requirements.

(1) The circuit must respond to ground as a busy signal, and to no ground as an idle indication.

(2) The location indication must be returned on the same T-lead that is used for the individual busy-idle indication. (It may be assumed that the terminus of this lead makes the corresponding circuit busy in addition to indicating its status.

(3) In addition to supplying the location indication, the hunting circuit must close through a group of several leads corresponding to the located circuit.

(4) A relay (HS) must be operated to indicate that the hunt was successful.

(5) A relay (AB) must be operated to indicate that all circuits in the hunt-group are busy.

Consideration of the first three requirements indicates that the final circuit can most conveniently be designed with two relays per input. For example, requirements (1) and (2) imply that a relay per input must first recognize absence of ground on a lead as an idle indication, and then must be able to apply an output ground back on the same
lead. This is difficult to achieve in a straightforward fashion with a single relay per input without loading up the relays with a large number of control contacts. However, with two relays per input, one set of relays can indicate initially by their operated or released condition the busy or idle state, respectively, of the associated circuits. A transfer chain on these relays can operate on a preferential basis an auxiliary hunting relay corresponding to the first idle circuit. By utilizing a transfer chain on each auxiliary relay to open a common operating path and concurrently close its own locking path, the circuit can protect itself against subsequent changes of condition on the input test leads. The circuit is shown on Fig. 16-4. This configuration permits the operated auxiliary hunting relay, (HA-), to return a ground signal on the associated T- lead without any difficulty and by means of a set of make contacts, to close the paths specified in requirement (3).

Requirement (4), that a separate relay, (HS), must be operated to indicate that the hunt was successful, is easily met by a relay in series with all (HA-) relays. Although operation of (HS) could be accomplished just as well by paralleled make contacts on the (HA-) relays, this method saves contacts.

Requirement (5), for an all-busy relay (AB), is met with equal facility. When all circuits in the hunting group are busy, a path through the (H-) relay transfer chain can operate an (AB) relay, which in turn will lock and open the operating ground supply for the (HA-) relays.
The general type of circuit just described can also be adapted to a hunting circuit where ground on a T-lead is an idle indication. Additional requirements on the circuit might be:

(1) The indication of a located circuit must be given over an L-lead independent of the input T-lead. As soon as it has identified an idle circuit, the hunting circuit must permit an immediate change from idle to busy indication on the T-lead corresponding to the located circuit. This requirement must often be met when the hunting circuit output directly shifts the chosen circuit from idle to a busy condition.

(2) The circuit must permit a systematic rotation of preference. This requirement is often imposed in order to equalize the usage of a group of circuits.

The first change required in the basic circuit of Fig. 16-4 to meet these requirements is the inversion of the (H-) relay transfer chain so that an operated instead of an unoperated (H-) relay operates an auxiliary hunting relay. This eliminates the need for the timing relay of Fig. 16-4, which was necessary to cover the operating time of the (H-) relays. In order to meet requirement (1), an adaptation of the gate principle used in lockout circuits may be utilized. That is, as soon as a

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Fig. 16-5 Hunting Circuit: Ground as Idle Indication
hunting relay has locked in to identify a located circuit, the gate can act to open all test leads to obviate effects from subsequent changes in condition. A circuit with these changes is shown in Fig. 16-5. Note that the (CON) relay serves the double function of connector and gate.

The final requirement, that of rotating preference, can be accomplished by the usual method of connecting the preference transfer chain in a ring and tapping on to it at several points. An external sequence circuit can then advance the preference on any desired basis.

Circumstances sometimes make it desirable to associate a hunting circuit permanently with its hunting group rather than through a connector. Reasons for this may be that the hunting circuit has only one group with which to work, or that maximum speed of operation is required. In this case, the relays of the hunting circuit continuously follow the condition of their associated circuits, and the start signal merely applies ground to an output contact network on the hunting relays. Again it is necessary, during the hunting process, to prevent relays from changing condition.

This latter requirement could most simply be met if the start lead input were able to supply the output location signal and lock up operated relays concurrently. This can be done by utilizing secondary windings on the hunting relays to prevent back-ups, as shown on Fig. 16-6. With this circuit, at the moment the start signal appears, the hunting relays are in some particular configuration corresponding to the state of the circuits of the hunting group. [Note that an operated (H-) relay indicates busy]. The ground from the start lead travels through the transfer chain to provide an output at the first unoperated (H-) relay. At the same time, the start ground holds operated all relays higher in preference than the output relay.
A fundamental hazard in this scheme as described is that, if all hunting relays are operated when the start lead is grounded, the circuit will block. This requires provision of the (AB) relay to open the start lead until at least one (H-) relay is released.

In application of the circuit of Fig. 16-6, care must be taken to co-ordinate properly the closure of the (B) contact in the located circuit and the removal of ground from the start lead. If the (B) contact is closed early, the start ground will be advanced to a second L- output. This sequential co-ordination must be accomplished by means external to the hunting circuit.

When the hunting circuit is permanently associated with the hunt-group, the equal distribution of usage to the latter circuits can be achieved without difficulty. A logical method of doing this is to lock operated each hunting relay when its corresponding circuit is located on a circuit usage until all circuits in the hunt-group have been assigned. Then the common locking path is opened, permitting the hunting relays of any idle circuit to release. This requires a timing circuit to control the locking path to insure that a sufficient interval is provided for all idle relays to release. The circuit, with the locking and timing portions drawn with heavy lines, is shown on Fig. 16-7. This circuit also requires careful co-ordination between removal of the start signal and closure of the (B) contact in a located circuit to insure proper operation.

A requirement may sometimes be imposed that a permanently associated hunting circuit must be able to accept an immediate return of busy over the T- lead from a located circuit, before it is possible to remove ground from the start lead. As mentioned previously, this

*CONTACT (B) MUST NOT CLOSE UNTIL ACTIVATING GROUND HAS BEEN REMOVED FROM ST LEAD

Fig. 16-7 Permanently Associated Hunting Circuit which Insures Equal Service
requirement is not met by the circuit of either Fig. 16-6 nor Fig. 16-7. The problem resolves itself into these two phases:

1. Return of the busy signal on the T- lead of a circuit just located must not advance the start ground to a lower preference output.

2. Return to availability of a higher preference circuit during the hunting process must not shift the output of the hunting circuit.

Statement of the requirements in this way suggests combining a double-transfer lockout circuit with a set of hunting relays as shown on Fig. 16-8. Although the hunting relays can change their condition at any time while the start lead is grounded, the characteristics of the lockout circuit permit only one locating output lead to be activated. The circuit requires a separate locking path for the lockout relays to prevent release when the operating path advances.

16.4 ELECTRONIC HUNTING CIRCUITS

The inherent lockout characteristics of gas tubes make this device eminently satisfactory for performing the hunting function when speed is an important consideration. Any of the gas-tube lockout circuits discussed in Chapter 15 is adaptable to hunting, and a typical circuit is shown in Fig. 16-9. In this circuit, an idle circuit is indicated by high positive voltage on the T- lead. When the connector is closed to associate the hunting circuit with the individual circuits, tubes connected to idle circuits attempt to fire across the control gap. The lockout
action imposed by the common impedance connected to all control cathodes permits only one tube to break down and transfer to the main gap. If the current through the control gap is maintained by delaying the shift of the located circuit test lead from the idle indication to busy, the voltage drop in the common anode-cathode resistors will prevent any other tube from firing until the connector releases and reoperates. In the circuit as shown in Fig. 16-9, the output indication is obtained from the operated (L-) relay in the main gap circuit of the conducting tube. If a potential change is satisfactory as an output, it can be obtained directly from a load resistor in the main gap anode or cathode circuit.

The gas-tube hunting circuit has no fixed preference characteristics as had the corresponding relay circuit. Instead, the preference depends upon the indeterminate ionization and transfer-time characteristics of the individual tubes comprising the hunting circuit. If this is unsatisfactory in a particular application, it is possible to impose an effective preference upon a gas-tube circuit by placing R-C networks with graduated delay in the input paths to the control anodes.

Because of the similarity between the lockout and hunting functions, any electronic devices exhibiting negative-resistance characteristics that are suitable for lockout purposes can be used in hunting circuits. Thus, the principles discussed in Section 15.6 (Chapter 15) under the heading "Electronic Lockout Circuits" are directly applicable to the design of electronic hunting circuits.
PROBLEMS FOR CHAPTER 16

16-1 The figure below indicates a connector and control arrangement which give one hundred T-circuits access to five C-circuits. The T-circuits are divided into subgroups of ten, with each subgroup associated with its own connector and control. Within each connector, a fifty-lead single-link connecting path is satisfactory.

Design the connector and control circuit. Connector requests from the T-circuits occur at random, and the control circuits must permit maximum usage of the connectors and C-circuits without any possibility of double connections. (This is a multifunctional circuit which can be done with thirty-five relays per connector.)

16-2 A group of twenty telephonenumber lines are served by a set of ten central office paths, any one of which may be used to complete a connection. Each of the ten central office paths terminates on a non-homing, back-acting 22-point rotary finding switch, whose bank terminals are wired to the telephone lines.

Associated with each telephone line is a line circuit which contains two relays. When one of these two relays operates, one finder switch locates the line and connects the telephone line through to the central office path. The connection is held until the line relay releases. The other relay may be used for control.

Finder switches are assigned to calling lines by a single hunting switch of the same type as the finder switches. The hunting switch should normally assign an idle finder switch prior to receiving a request, although it should be prevented from hunting when all finders are busy.

Design circuits for the line, finder, and hunting circuits. The switches may contain up to six arcs (of which two are reserved for the talking path) and supplementary relays may be used. (This can be done with one relay per finder switch.)
Chapter 17

CIRCUITS FOR TIMING

There are innumerable situations in switching design that call for circuits or apparatus to produce timed intervals. These situations arise from a variety of causes which in practice are either stated in the requirements of the problem or may easily be recognized by the circuit designer.

Many cases of timing in switching systems occur when it is necessary to generate a specific time interval against which a circuit-acting period may be measured. This may be done for trouble detecting, for example. If the circuit action is not completed within the predetermined interval, some alternative action is taken automatically, or an alarm is given, depending upon the specific circumstances.

Other situations requiring timing are engendered by circuit considerations or the characteristics of apparatus. Race conditions between groups of relays often require specific timed intervals to insure correct circuit operation. The crossbar switch is a two-step device which requires a definite interval between operation of a select magnet and operation of a hold magnet. When a circuit tests for a condition which may or may not be present, it is often necessary to provide a definite minimum time for recognition of the condition. These situations frequently can be handled by providing an appropriate sequence of operation instead of generating specific timed intervals. In these cases, the timing function is embodied in the design but does not appear as such.

Another class of timing makes use of circuits which generate recurrent pulse intervals, usually for signaling purposes. These pulsing circuits are discussed in detail in the next chapter.

Since the requirements for timing cover a wide range, both for over-all time and for accuracy, a variety of circuit and apparatus techniques must be used. In some situations a simple device such as a slow-acting relay is adequate. In others, circuits of considerable complexity are necessary. In each particular design problem requiring timing, the factors of over-all time, accuracy, and available control means must be weighed in choosing the type of timing circuit or device.

In this chapter, several important timing methods in current use are discussed, together with their relative advantages and limitations.
Attention is confined to the generation of timed intervals ranging upward from the millisecond range; circuits for timing extremely short intervals are not included.

17.1 STANDARD SLOW-ACTING RELAYS*

The most frequently used timing device in relay switching circuits is the slow-acting relay. The need for this device is so widespread that slow-acting types with a great variety of winding and spring combinations are provided within the framework of commercial general-purpose relays. The slow action is achieved by means of copper or aluminum sleeves around the relay core, and in some cases by special construction of the magnetic structure of the relay.

The circuit requirements for relays which must provide timed intervals vary greatly. In some cases, where it is necessary to cover the operation of fast-acting relays by a timing relay, the operating time of an ordinary general-purpose relay with a heavy spring load and a high-time-constant winding may be sufficient. In this case the timing relay would not necessarily appear on the circuit drawing with an indication that slow action was involved. However, in the majority of situations requiring relay timing it is necessary to choose a relay definitely designed for slow action.

Relays with a heavy sleeve can be obtained with minimum operating times up to the order of 100 milliseconds. Particular combinations of winding, sleeves, springs, and adjustments will give lower minimum operate times up to this value, so that the designer has considerable leeway in choosing an appropriate relay.

It must be remembered, however, that in choosing a slow-operate relay to cover a certain time interval, the relay which gives a desired minimum time is also subject to a much greater maximum operate time. The spread between maximum and minimum times for relays of a particular type is often of the order of three-to-one, so that to obtain a guaranteed time interval of 60 milliseconds, the designer is usually forced to accept a possible time of 180 milliseconds unless special timing adjustments can be specified. This may or may not be of importance in the operation of a particular circuit, but it must always be kept in mind. In certain cases where over-all circuit holding time is of the utmost importance, this large spread makes it questionable to use ordinary slow relays for timing purposes.

* This section applies only to general-purpose relays including a conducting sleeve or slug on the core to increase acting time. Specially constructed slow-acting relays are not discussed.
A simple illustration of the use of a slow-operate relay for timing is shown in Fig. 17-1A. When relay (A) operates to close output (1), there is a delay equal to the operate time of relay (T) before output (2) is closed.

In practice, the release time of general-purpose type relays can be made considerably longer than the operate time.

Relays designed specifically for slow release carry the possible minimum release time up to the order of 500 milliseconds. Again, there is a comparatively large spread between minimum and maximum times for a particular relay type. If time intervals of this order are required, a slow-release relay can usually be employed. Use of a slow-release relay, of course, implies that the relay is operated well ahead of the time when its release characteristics are utilized in the circuit sequence. This is illustrated in Fig. 17-1B.

![Diagrams of Switching Circuits](image-url)
If a longer time interval is required than is possible with the operate or release time of a single relay, relays in tandem can be employed. This is shown in Fig. 17-1C, which uses three pre-operated slow-release relays. The relays can be both operated and released in tandem, or, if relatively fast pre-operation is desired, the relays can be operated simultaneously and released in tandem. The utility of this principle can be extended by employing other cycles of slow-acting relays.

The acting time of relays can be appreciably affected by external circuit elements. A resistor in series with the winding of a relay may be used to increase the operating time by reducing the margin of circuit current over the just-operate current. A low resistance bridged across the relay winding increases the release time by permitting the magnetic flux in the relay structure to decay slowly after the operate or hold path is opened. The same effect can be achieved by bridging across the winding a varistor poled to oppose the operating current.

Another circuit element useful in delaying relay operation is the thermistor. This device has the property of increasing its conductance tremendously by thermal effect after current is permitted to flow through it during a period of time. A properly chosen thermistor in series with a relay winding will hold the current below the non-operate value for an interval after circuit closure. When the thermistor has heated to the critical value, the current will increase to the point where the relay can operate. Two disadvantages of this technique are that the variations in time are large, and that the cooling period required between successive operations is comparatively long.

Obviously, the use of slow-acting relays can be extended to generate almost any desired time interval. It is usually uneconomical, however, to use more than two or three relays for the sole purpose of timing. The major disadvantage of slow relays is the large spread between maximum and minimum times, although in many applications this may be of little consequence.

17.2 INTERRUPTER TIMING CIRCUITS

An important and much used method for obtaining time intervals, particularly those involving several seconds, is to employ a counting circuit actuated by a mechanically or electrically driven interrupter. The most frequently used spring combinations on the interrupters are simple makes and break-make transfers. The time per interrupter cycle may range from a fraction of a second to hundreds of seconds.
The technique in using the interrupters consists of providing a counting circuit which will count any desired number of cycles. A typical circuit to count two cycles of pulses from an interrupter with a transfer-type spring combination is shown in Fig. 17-2. The counting action begins when lead "TM" is grounded. It can be seen by analysis that with this type of circuit, designed to accept the ground closures for C interrupter cycles, the minimum and maximum elapsed times before the last relay operates are:

Minimum time  = (C-1) \( W + X + Y + Z \) + X
Maximum time  = C \( W + X + Y + Z \) + X

where, W, X, Y, and Z are the time intervals of the individual parts of the interrupter cycle. With this type of circuit each pair of relays does not, strictly speaking, time a complete cycle, but rather responds to the beginning of each ground closure within the cycle. However, if period X (the open period following the first effective ground closure) occupies the greater part of the cycle, the approximate effect is that of timing a complete cycle with each pair of relays.

If a circuit for counting pulses from an interrupter with a single make-contact is employed, the minimum and maximum times involved in counting the ground closures of C cycles are:

Minimum time  = (C-1) \( W + X \) + X
Maximum time  = C \( W + X \) + X

where W and X represent the time duration of the closed and open parts of the cycle, respectively.
With both these circuits, the uncertainty in the length of the timed interval is equal in length to the time of one complete cycle. The per cent variation in total time measured depends upon the number of cycles counted and the manner in which the time is allotted to the intervals within the cycle. For instance, in either type of circuit, if the W period occupies the major part of the time within the cycle, and if the ground closures of two cycles are counted, the total variation is of the order of two to one. If X is much greater than W, the variation for two cycles is three to two. Furthermore, as the number of cycles increases, the variation decreases in terms of the ratio of maximum to minimum time. In absolute terms, of course, the time variation is equal to one cycle.

Both these circuits can be used to measure a single cycle of the interrupter if the X period is made the larger and more important part of the cycle. If X is the smaller part of the cycle, the variation in total measured time for one cycle becomes excessive. In general, the counting circuit used as part of the timing circuit may be any of the types discussed in Chapter 11.

In conclusion, it can be seen that the interrupter timing circuit is well adapted to measuring long time intervals, if variations of the time of one cycle can be tolerated. The absolute variation of time can always be reduced to whatever value is considered necessary by decreasing the cycle length and increasing the count.

17.3 CAPACITOR-TIMED RELAYS

When a circuit situation arises where it is necessary to generate a time delay up to a few tenths of a second within relatively close limits (of the order of ±10 per cent), the capacitor-timed relay circuit may be used. This circuit consists of an electrically biased polar relay whose operation is controlled by the charge or discharge current of a capacitor. By holding the circuit variables to close limits, and choosing the operating point at an appropriate location on the R-C exponential curve, the variation between minimum and maximum operating times can be closely controlled.

The basic theory of this circuit embodies the relations involved in the charge and discharge of a capacitor through a series resistance. Including a relay winding in the circuit introduces inductance which modifies the current-voltage relations, but with suitable polar relays the inductance of the windings is minor compared to the circuit resistance and can generally be neglected. Also, mutual inductance between windings is negligible. Therefore the circuit can be treated simply as resistance and capacity.
In Fig. 17-3 is shown a simple R-C circuit with means for applying either a potential E or a short circuit across R and C. When the switch is operated from position a to position b, the variation of current with time is shown as curve A in the figure, and is expressed mathematically as:

\[ i = \frac{E}{R} e^{-t/RC} \]  \hspace{1cm} (17-1)*

The initial current is solely a function of E and R, and the rate of decay is determined by the product RC.

If, after sufficient time to permit essentially full charge on the capacitor, the switch is thrown from b to a, the current flows in the reverse direction and its decay is again represented by curve A.

The voltage across the resistor during the charging period is also represented by curve A where

\[ e_R = E \cdot e^{-t/RC} \]  \hspace{1cm} (17-2)*

During the discharge period \( e_R \) is reversed in polarity.

Since the voltage across the capacitor is the difference between the applied potential and \( e_R \), it follows curve B during the charging time and is equal to

\[ e_c = E (1 - e^{-t/RC}) \]  \hspace{1cm} (17-3)*

However, during discharge, the capacitor voltage decays from E to zero so that \( e_c \) is represented by curve A and is equal to

\[ e_c = E \cdot e^{-t/RC} \]  \hspace{1cm} (17-4)†

It should be noted that, if the voltage across the circuit is reversed, after charging the capacitor, the effect is that of doubling the voltage, since the circuit voltage and the capacitor voltage add together. This can be utilized to increase the effective voltage, where circuit considerations make this desirable.

* If a residual voltage \( e_x \) exists on the capacitor at time \( t = 0 \) when the switch is operated to position b, the current and voltage relations are modified to the more general form given below. Note that positive \( e_x \) aids battery potential E.

\[ (17-1) \quad i = \frac{E + e_x}{R} e^{-t/RC} \]
\[ (17-2) \quad e_R = (E + e_x) \cdot e^{-t/RC} \]
\[ (17-3) \quad e_c = E - (E + E_x) \cdot e^{-t/RC} \]

† If the discharge cycle (switch b to a) is started before the capacitor has reached full charge, the effective capacitor voltage is \( e_x \) instead of E.

\[ (17-4) \quad e_c = e_x \cdot e^{-t/RC} \]
Since in capacitor-timed relay circuits the capacitor charge or discharge current is used directly, curve A and equation (17-1) will be most useful in the discussion.

The simplest and perhaps most useful circuit for obtaining a delayed operate timing action is shown in Fig. 17-4. With the control relay normal and battery-ground connected, the current flow through the two windings holds the polar relay on its back contact. The primary winding is poled in such a way as to attempt to operate the relay, but the secondary winding, poled oppositely, overcomes the primary by a considerable margin to hold the relay non-operated. The capacitor, C, is completely discharged at this time. The relay constants shown are for a typical sensitive polarized relay.

When the control relay operates, current through the secondary winding immediately starts charging the capacitor. This initial charging
current must be great enough to hold the relay on the back contact. However, as the capacitor charges, the secondary current decreases, following curve A of Fig. 17-3, until the secondary ampere-turn value drops below the opposing primary ampere-turn value by an amount sufficient to cause the relay to operate. This is illustrated in Fig. 17-5, where relay ampere turns are plotted against time. The secondary ampere turns \( N_s i_s \) are initially much higher than the primary ampere turns \( N_p I_p \), as indicated by the point where the solid curve crosses the zero time axis.

When the control relay opens the short circuit across the capacitor, \( N_s i_s \) starts decreasing at an exponential rate determined by \( RC \). The relay will operate when \( N_s i_s \) decreases to the point where it is equal to \( N_p I_p - N_i \), \( N_i \) being the ampere-turn value at which the relay is adjusted to operate. If \( i_t \) is the particular value of \( i_s \) at which the relay operates,

\[
N_s i_t = N_p I_p - N_i
\]  
(17-5)

It can be seen from this figure that for a given relay and circuit voltage, the operating time can be controlled by varying \( I_p \), by changing \( RC \), or by changing the balance between \( R \) and \( C \) while keeping \( RC \) constant.

With all these variables to work with, the principal aim of the design job is to choose the optimum values of the circuit constants to
give the required time with minimum variation of time. To meet this objective, of course, circuit elements with low tolerance limits must be used.

**Principles of Design: Factors Affecting Accuracy.** From equation (17-1) the secondary current \( i_s \) through the relay is:

\[
i_s = \frac{E}{R_s} e^{-t/R_s C}
\]  

(17-6)

Solving this equation for the value of "time" \( T \) at which the relay should operate, the result, expressed in terms of base-10 logarithms, is

\[
T = 2.3 \frac{R_s C \log_{10} \frac{E}{i_t R_s}}{}
\]  

(17-7)

where \( i_t \) is the particular value of \( i_s \) at which the relay operates. This means that \( T \) varies directly with \( R_s, C \), and the log of the ratio of \( E \) to \( i_t R_s \). Except when the operating point is chosen so that \( T = R_s C \), there are two values of \( R_s \) which will give the required time. Of these, only

Fig. 17-5 Curves Illustrating the Operating Point of the Timing Relay
The lower value should be used, since the rate of change of discharge current (and hence the accuracy) is greater for the low \( R_s \).

To obtain best results, the secondary timing winding of the relay should be as effective as possible and should have an exact number of turns of wire. This means crowding as many secondary turns as possible on the relay, consistent with permitting the primary winding to function. The primary winding also should have an exact number of turns.

To reduce the variation in the timing obtained by the relay, the lowest possible operate adjustment and the highest possible non-operate adjustment is desirable. This reduces the variation in \((N_p I_p - Ni)\), the ampere-turn level at which the relay operates. Variation of \(Ni\) (operate adjustment ampere turns), causes corresponding variation in time from the specified design value.

The primary ampere turns, which hold the relay operated after the secondary current has died away, must be several times the operate test ampere turns to provide adequate contact pressure. A reasonable value for \(N_p I_p\) is five or six times the operate adjustment ampere turns. Since the operating point \(N_s i_t\) of the relay is equal to \(N_p I_p - Ni\), \(Ni\) should be as small as possible in order not to raise \(i_t\) too high for flexibility of design.

When the variation in circuit elements and the applied voltage is taken into account, equation (17-7) can be modified to represent minimum time and maximum time:

\[
T_{\text{min}} = 2.3 \frac{R_s}{C_{\text{min}}} \log_{10} \frac{E_{\text{max}}}{i_{t(\text{max})} R_{s(\text{min})}} \quad (17-8)
\]

\[
T_{\text{max}} = 2.3 \frac{R_s}{C_{\text{max}}} \log_{10} \frac{E_{\text{min}}}{i_{t(\text{min})} R_{s(\text{max})}} \quad (17-9)
\]

In equation (17-9):

\[
i_{t(\text{min})} = \frac{N_p I_p_{\text{min}} - Ni_{\text{Opr.Tst}}}{N_s} \quad \text{where: } I_p_{\text{min}} = \frac{E_{\text{min}}}{R_{p(\text{max})}}
\]

\[
i_{t(\text{max})} = \frac{N_p I_p_{\text{max}} - Ni_{\text{N.O.Tst}}}{N_s} \quad \text{where: } I_p_{\text{max}} = \frac{E_{\text{max}}}{R_{p(\text{min})}}
\]

\[
(17-10) \quad (17-11)
\]

* Typical examples of the operate (Opr. Tst) and non-operate (N.O.Tst) ampere-turn adjustment values are shown in the table of Fig. 17-4.
Although the effects of variation in the applied voltage tend to cancel out, an increase in $E$ reduces the net operating time through its modification of the value of $i_t$. It should be noted that the direct effect of $R_s$ upon $T$ is greater than that of its effect through the log ratio, and therefore $R_{s(min)}$ is used throughout the equation for $T_{(min)}$, and similarly $R_{s(max)}$ is used in the equation for $T_{(max)}$.

With this information in mind, there are several factors affecting the design which can be demonstrated mathematically. Without going through the mathematical steps, the most pertinent factors are as follows:

(1) For any particular value $i_t$ of the secondary current at which the relay operates, the minimum capacity that can be used to give a time delay $T$ is equal to

$$C = \frac{T}{R_s} \quad \text{where} \quad R_s = \frac{E}{\varepsilon i_t} \quad \left(\varepsilon = 2.718\right) \quad (17-12)$$

Therefore, the lowest possible capacity for a given time with a given relay can be determined by calculating the minimum allowable $i_t$ from equation (17-10), and from equation (17-12) determining $R_s$ and $C$. However, if minimum time variation is desired, other factors make a different approach desirable.

(2) For minimum variation of time, with a given value of $R_sC$, $R_s$ should be as low as possible and $C$ as high as possible.

(3) If the value of $R_s$ is fixed as a starting point, the greatest accuracy is obtained by setting

$$C = \frac{T}{R_s} \quad \text{(to give the operating point at} \ T = R_sC) \quad (17-13)$$

since the charge curve based on this value of $C$ has a greater slope as it passes through time $T$ than the curve for any other value of $C$.

(4) If the value of $C$ is fixed as a starting point, the greatest accuracy is obtained by setting

$$R_s = \frac{T}{2C} \quad \text{(to give the operating point at} \ T = 2R_sC) \quad (17-14)$$

since the charge curve based on this value of $R$ has a greater slope as it passes through time $T$ than the curve for any other value of $R$.

* The derivation of the relationships in items (2), (3), (4), and (5) is based on finding the curve with maximum slope through the operating point. Variations in circuit constants when the slope is maximum will result in least variation in time. These results can be checked roughly by sketching exponential curves for several values of the variable in question on a co-ordinate graph whose abscissa is time instead of $t/R_sC$. 

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(5)* The greater the circuit voltage, the greater is the accuracy that can be attained. However, good accuracy is obtained by using a battery supply of 48 volts.

Items (3) and (4) may appear at first glance inconsistent in that for the two conditions of $R_s$ and $C$ fixed, they indicate two different points on the exponential curve as optimum for minimum variation of time. However, this apparent inconsistency merely means that if a value of $R_s$ is chosen, and then the best value of $C$ is calculated for that $R_s$, from the calculated value of $C$ a new $R_s$ can be determined which will give more accurate results. Successive applications of the rules of items (3) and (4), starting with an arbitrary value of $R_s$ and the desired time $T$, will result in successively lower values of $R_s$ and higher values of $C$, to agree with item (2).

In designing a circuit to meet certain time limits, there are different methods of approach. Since there are several elements that can be varied, something (either $R_s$ or $C$) must be arbitrarily fixed in order to start the design. Economic considerations may determine the maximum capacity that can be used, in which case $C$ is fixed and $R_s$ is calculated to give an operating point such that $T = 2R_sC$. If the closest possible time tolerance regardless of amount of capacity is desired, the smallest $R_s$ (relay winding plus small contact protecting resistance) is taken as fixed and the capacity should be calculated to give the operating point at $T = R_sC$.

Since the latter procedure usually results in a high value of capacity, and does not give an appreciable improvement in accuracy over $T = 2R_sC$, the customary procedure is to set $R_s$ at its minimum value and choose $C$ to put the operating point in the vicinity of $T = 2R_sC$. This is the method that will be used in the example of designing a circuit, given later in this chapter.

It should be noted that if the design is started with a fixed value of $C$, the resultant $R_s$ must fall within certain limits to be practicable. That is, $R_s$ cannot be less than the secondary winding resistance of the relay, and cannot be so great that the level of $I_t$ requires primary amperes turns less than five or six times the operate test value. In either of these eventualities, it is necessary to change the assumed initial circuit considerations.

The considerations thus far have been concerned only with the theoretical aspects of the problem and have assumed zero inductance, zero armature travel time, and no contact chatter. Inductance and travel time generally have negligible effect when the delay time is high, say

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* See the footnote on page 389.
above 0.1 second. Below this level, however, these factors may increase the calculated time by a considerable percentage and may have to be taken into account. There is no hard and fast rule for calculating the magnitude of the extra time incurred by these factors, so that if it is desired to keep the minimum delay time below a fixed level, a laboratory test of the theoretical design is in order.

Of even greater importance is an apparent change that occurs in the operating current value of the relay. The adjustment procedure provides for a sudden application of current. In a working circuit, however, the current change in the relay is gradual as the operating level is approached. The chief result of this is that the relay does not operate instantaneously as the current passes through the theoretical operating point. In effect, the operating adjustment value of the relay is increased. In practice, this effect may be compensated for by adjusting the design value of primary resistance to give the true time as measured in the laboratory.

Still another factor which has to be considered in practice is the change in value of circuit constants with temperature. For instance, if the timing circuit is to be used at frequent intervals, or stands with battery and ground connected, the resistance of the relay windings and circuit resistors will probably change due to temperature rise. Also, a difference in ambient temperature from the rated level will affect circuit values. This can have considerable effect upon timing accuracy unless accounted for in the design.

The amount of chatter involved for a given design is also difficult to calculate in advance. Sensitive polar relays are likely to produce some chatter which varies at different operating current values, so that this factor should also be checked with a test model of the circuit.

Method of Design. The information given in the preceding paragraphs is adequate to permit the design of a capacitor-timed relay circuit. However, to illustrate the technique involved, a specific circuit will be designed.

A frequent situation calling for this type of circuit requires a definite minimum delay time, with some degree of flexibility as far as maximum time is concerned. The example will therefore design for the minimum time $T$, starting with a minimum value of $R_s$ (relay resistance plus a nominal $r_s = 250$ ohms to protect the contact which discharges C). The operating point will be set approximately at $T = 2R_sC$, although picking commercial values of one-per-cent capacitors may change the operating point slightly. The change in calculations necessary to design for average time instead of minimum time should be obvious from the example.
Since, in the general design problem, the object is to determine \(R_s\) or \(C\), and \(R_p\), starting with the value of \(T\), equations (17-8) and (17-9) are not the most useful. The most convenient formula is the basic equation

\[i_t = \frac{E}{R} e^{-T/RC}\]

in conjunction with equations (17-10) and (17-11), in which \(i_t\) and \(I_p\) are related. Equations (17-8) and (17-9), however, are useful in that they show where maximum and minimum values should be used to calculate maximum and minimum time. In using the exponential form of the equation, the curve of Fig. 17-3 can be used with advantage, although greater accuracy can be achieved by use of a table of exponentials.

Problem:
To design a circuit giving a minimum delay of 0.060 second.

The following are the pre-determined factors:

1. Circuit: that of Fig. 17-4.
2. Relay: that of Fig. 17-4.
3. \(E = 45\) to 50 volts; normal 48 volts.
4. \(R_s = 2600\) ohms + 250 ohms = 2850 ohms (2600 ohms is the winding resistance and 250 ohms is the external resistance).
5. Operating point approximately \(T = 2R_sC\).

The approximate value of \(C\) (from \(T = 2R_sC\)) is:

\[C = \frac{.06}{2 \times 2850} = 10.53\ \text{mf.}\]

The standard one-per-cent paper capacitors that will be used in solving this problem come in multiples of .540 mf. Therefore the combination of standard capacitors nearest to 10.53 mf should be chosen.

Let \(C = \) two capacitors at 4.32 mf

plus one capacitor at 2.16 mf = 10.80 mf.

Now, calculate the constants to give minimum time = 0.060 second.

\[\text{Minimum } R_sC = .99 \times 2850 \times .99 \times \frac{10.80}{1,000,000} = .0302 \quad \text{(both } R_s \text{ and } C \text{ assumed 1% low)}\]

\[\frac{T}{R_sC} = \frac{.06}{.0302} = 1.99\]
\[
\frac{i_t}{I_s} \text{ from curve A, Fig. 17-3, or table of exponentials } = 0.136
\]
\[
i_{t(max)} = 0.136 \frac{E_{(max)}}{R_{s(min)}} = 0.136 \times \frac{50}{2850 \times 0.99} = 0.00241 \text{ amperes.}
\]

To obtain \( I_{p(max)} \) and \( R_{p(min)} \)

\[
N_p I_{p(max)} = N_s i_{t(max)} + N_i (N.O.Tst)
\]
\[
2000 I_{p(max)} = (14000 \times 0.00241) + 1.8
\]
\[
I_{p(max)} = 0.01777
\]
\[
R_{p(min)} = \frac{E_{(max)}}{I_{p(max)}} = \frac{50}{0.01777} = 2810 \text{ ohms}
\]

Let \( R_p = 2840 \text{ ohms } \pm 1\% \)

Resultant circuit constants:

\[
r_s = 250 \text{ ohms } \pm 1\% \text{ (} R_s = 2850 \text{ ohms } \pm 1\% \text{)}
\]
\[
r_p = 2640 \text{ ohms } \pm 1\% \text{ (plus 200 ohms relay winding to give} \ R_p = 2840 \text{ ohms } \pm 1\% \text{)}
\]
\[
C = 10.80 \text{ mf } \pm 1\%
\]

To calculate maximum time:

Compute \( i_t(min) \) using equation (17-10).

First: \( I_{p(min)} = \frac{E_{(min)}}{R_{p(max)}} = \frac{45}{2840 \times 1.01} = 0.0157 \text{ ampere.} \)

Hence: \( i_{t(min)} = \frac{N_p I_{p(min)} - N_i (Opr.Tst)}{N_s} \)

or: \( i_{t(min)} = (2000 \times 0.0157) - \frac{6}{14000} = 0.001814 \text{ ampere.} \)

In order to calculate \( T_{(max)} \), equation (17-9) may be used:

\[
T_{(max)} = 2.3R_{s(max)}C_{(max)} \log_{10} \frac{E_{(min)}}{i_{t(min)}R_{s(max)}}
\]

or: \( T_{(max)} = 2.3 \times 2850 \times 1.01 \times 10.80 \times 10^{-6} \times 1.01 \times \)

\[
\log_{10} \frac{45}{0.001814 \times 2850 \times 1.01}
\]
Hence: \( T_{(\text{max})} = 0.0676 \text{ second} \).

To calculate average time:

\[
I_p(\text{avg}) = \frac{E(\text{avg})}{R_p(\text{avg})} = \frac{48}{2840} = 0.0169
\]

\[
i_t(\text{avg}) = \frac{(2000 \times 0.0169) - 4}{14000} = 0.002128
\]

\[
T(\text{avg}) = 2.3 \times 2850 \times 10.80 \times 10^{-6} \log_{10} \frac{48}{0.002128 \times 2850} = 0.0636 \text{ second}.
\]

It has been noted earlier that the effective operating current level of the relay may be shifted by the nature of the circuit. Also, there may be slight inductive and chatter effects. Therefore, the circuit should be tested in the laboratory, and the value of \( r_p \) should be adjusted, if necessary, to give the correct time.
Other Circuits. There are several variations of the basic capacitor-timed relay circuit, some of which are shown in Fig. 17-6. Fig. 17-6B takes advantage of pre-charging the capacitor in order to obtain a higher effective voltage in the secondary winding circuit.

The circuits of Figs. 17-6A, 17-6B, and 17-6C are considerably more complicated to calculate than those of Fig. 17-4 because the primary and secondary circuits of the relay are inter-related. The analysis of these circuits can be most easily handled by the application of Thevenin's theorem to the networks.

17.4 GAS-TUBE TIMING CIRCUITS

The capacitor-timed relay circuit described in Section 17.3 requires on the average about 16 mf for every 0.1 second of delay. This value cannot be greatly reduced because of the desirability of operating at the point \( T = 2R_sC \), and also because of the limits which \( R_s \) cannot exceed due to the current requirements of the relay. The result of this is that a delay time above a few tenths of a second requires excessive mounting space for capacitors and is, in general, uneconomical. There is another commonly used capacitor-timing circuit, however, which avoids certain of these limitations, although with some loss of accuracy. This circuit employs a three-element cold-cathode gas tube which operates from the exponentially rising capacitor voltage, rather than from the capacitor charge or discharge current. The gas tube acts as a trigger device which uses for the timing function two of the tube elements, and then operates the delay relay through the third element. In this way the winding of the delay relay is not involved in the timing circuit.

A typical circuit of this type is shown in Fig. 17-7. The timing action starts when the control relay operates to remove the short circuit from the capacitor, apply the high voltage \( E \) and ground the cathode \( (K) \) of the gas tube. The tube characteristics are such that the main gap will not break down at the voltage \( E \) until the control gap has broken down. The control gap, however, breaks down when the capacitor voltage, applied between the control anode \( (CA) \) and the cathode \( (K) \), reaches a critical level. For tubes normally used in such application this is about 70 volts. Current flow from anode \( (A) \) to cathode begins almost instantaneously with the breakdown of the control gap.

The voltage across the capacitor is applied to the control anode while the cathode is grounded, and it rises exponentially with time at a rate determined by \( R \) and \( C \), following curve \( B \) of Fig. 17-3. The equation for this curve is

\[ e_c = E(1 - e^{-t/RC}) \]  

(17-3)
If the control gap breakdown voltage is represented by $e_t$, the time until breakdown is

$$T = 2.3 RC \log_{10} \frac{E}{E - e_t} \quad \text{or}$$

$$T = 2.3 RC \log_{10} \frac{1}{1 - e_t/E}$$

(17-15)

In the circuit of Fig. 17-7, current flow via the anode (A) and the delay relay winding starts immediately after control-gap breakdown, and the delay relay operates. The relay can hold to the gas-tube current or, as shown in the figure, lock to a separate winding and open the gas-tube circuit. The total delay time is that given by equation (17-15), plus the relay operating time. Since the delay relay is usually a general-purpose relay, its action time may be appreciable in short-time circuits, although when the total delay time is of the order of half a second or more, the relay acting time can be neglected.

The 100,000-ohm resistance connected to the control anode does not affect the timing, but is necessary to limit the control-gap current at the moment of breakdown. The 1000-ohm resistor, whose function is to limit the current through the contact that discharges C, also has no practical effect on the timing.

Since the common cold-cathode tubes require a relatively high operating voltage, this circuit cannot be used where the supply voltage

![Fig. 17-7 Capacitor and Gas-Tube Timing Circuit](image-url)
is limited to 45 to 50 volts. However, sources of 125 to 135 volts or more are widely available, and are well-suited to the requirements of this circuit.

**Factors Affecting Accuracy.** Examination of equation (17-15) will show that the time varies directly with R and C, but decreases as E increases. For a fixed E, time varies directly with \( e_t \). Therefore, the timing equations can be written:

\[
T_{(\text{max})} = 2.3R_{(\text{max})} C_{(\text{max})} \log_{10} \frac{E_{(\text{min})}}{E_{(\text{min})} - e_t(\text{max})} \tag{17-16}
\]

\[
T_{(\text{min})} = 2.3R_{(\text{min})} C_{(\text{min})} \log_{10} \frac{E_{(\text{max})}}{E_{(\text{max})} - e_t(\text{min})} \tag{17-17}
\]

In addition, time increases with the operating time of the delay relay. This factor is negligible for long time intervals. All circuit constants should be held to as close limits as possible for accurate timing. This implies that capacitors and resistors be as accurate as is economical. Variation of the 130-volt supply by ±5 volts is equivalent to ±3.8 per cent. The control-gap breakdown voltage of the tube depends upon the type. For accurate timing, a tube type with the lowest possible variation in control-gap breakdown voltage should be used. It should be kept in mind that the characteristics of a cold-cathode gas tube tend to change with time.

The slope of the \( E_c \) curve through the operating point also affects the accuracy of the timing. Theoretically, the optimum slope for a given supply voltage occurs when the operating point is set such that \( T = RC \). However, the value of the nominal firing voltage (70 volts), when used in conjunction with a supply voltage of 130 volts, serves to give a natural operating point at about \( T = 0.75 RC \), since the capacitor is 54 per cent charged at 70 volts. The minor difference in accuracy between this point and \( T = RC \) usually makes it unnecessary to attempt to set the operating point at \( T = RC \) by means of bias voltages. The slope of the \( E_c \) curve through the operating point is also directly affected by the charging voltage, increasing as the voltage is set at higher levels. For this reason an improvement in accuracy can be obtained by using either a higher positive potential or a negative voltage supply to pre-charge the timing capacitor to oppose the +130-volt charging voltage. Care must be taken to insure that the effective supply voltage does not exceed the main-gap working voltage.

Theoretically, for a fixed value of RC, the ratio between R and C makes no difference. However, in the interests of economy it is desirable to hold the value of C as low as possible and increase R, subject to an upper limit as discussed on the following page.
A factor that affects the accuracy is the relationship of the value of \( R \) to the possible leakage resistance which may appear shunted across \( C \). Leakage resistance may drop to as little as ten megohms under service conditions, largely because of the effect of dirt between capacitor terminals when the humidity is high. Some leakage may also be contributed by the wiring and the spring pile-up of the control relay. The effect of leakage resistance, illustrated in Fig. 17-8, is to decrease the effective charging voltage and charging resistance. In general, this results in an increase in time.

One method of diminishing the effect of leakage resistance is to hold the charging resistance \( R \) to a maximum of about one megohm. At this value a leakage of ten megohms increases circuit time by about six per cent. Another method is to arrange the circuit in such a manner that time is independent of leakage resistance down to a value approximately the same as the charging resistance itself. This is feasible because, as was already noted, leakage reduces both the effective charging voltage and the charging resistance, and equation (17-15) shows that these effects are partially self-compensating. If, before the timing cycle starts, the capacitor is set to a potential opposing the charging voltage, a point can be found on the charging curve where the values of \( e_c \) and \( T \) are independent of leakage resistance, \( R_L \), down to a low value of the latter. The effectiveness of this method depends upon
making sure that the leakage is to a known potential such as ground; otherwise the correct operating point is indeterminate.

A circuit for accomplishing this is shown in Fig. 17-9, together with a sketch (not to scale) which illustrates the principle involved. The bias value in this circuit is applicable when the available supply voltages are +130 volts and -48 volts, and requires some modification for other voltages. The timing equation is derived from the exponential form given in (17-3) of the footnotes on page 384:

\[ T = 2.3 \frac{mE + e_x}{mE - e_t} \]  

(17-18)

where \( m \) is a factor \( \frac{R_L}{R + R_L} \) due to leakage. The circuit is arranged so that leakage is to ground by grounding one side of \( C \) and placing ground on a make-contact of the relay spring pile-up to which the control anode of the tube is connected. The capacitor is pre-charged to -48 volts through a low resistance (negligible compared to \( R_L \)). Under these conditions, the time at \( e_c = 30 \) volts is unaffected by the value of \( R_L \) (the factor \( m \)), as long as \( R_L \) is greater than about 2R. Therefore, the cathode bias on the tube must be set at -40 volts so that the tube fires at \( e_c \) volts.

![Circuit Diagram](image)

**Fig. 17-9** Circuit to Eliminate Effect of Leakage Resistance upon Timing Accuracy
The curves sketched in Fig. 17-9 indicate the action of the circuit. During the early part of the charging period, the effect of $R_L$ is to increase the rate of charging; during the latter part, the effect is to decrease the rate. At $e_C = 30$ volts, the two effects cancel.

**Principles of Design.** The previous paragraph discussed the essential principles involved in the design of the capacitor gas-tube timing circuit. To design a circuit for an average delay time $T$, the tube would be chosen first and the average value of the firing voltage $e_t$, obtained. Calculated from this and the circuit voltage $E$, the ratio $e_t/E$ can be used to evaluate $T/RC$, using curve B of Fig. 17-3 or a table of exponentials. From $T/RC$ can be calculated the approximate value of $C$ corresponding to the known value of $T$ and $R = 1$ megohm. The value of $C$ nearest to this figure which is obtainable in standard one-per-cent capacitors can then be obtained. The actual resistance of $R$ can then be recalculated from the value of $T/RC$. Using these values of $R$ and $C$, the minimum and maximum times can be obtained using equations (17-16) and (17-17). In calculating maximum time, it is necessary to take account of leakage resistance unless the self-compensating circuit is used.

The life of the gas tube varies inversely with the anode current, and the current values that should not be exceeded are listed under the tube characteristics. Therefore, it is necessary to insert in series with the main gap sufficient resistance to limit the anode current to a reasonable value (usually peak .030 ampere, average .010 ampere). The available voltage in the external main-gap circuit is the difference between the supply voltage and the main-gap sustaining voltage, the latter being generally of the order of 70 to 80 volts. In general, the relay resistance should be as high as possible, and in some cases additional resistance should be inserted. It is usually advisable to open the tube circuit when the relay operates in order to lengthen tube life. When choosing the relay it is necessary, of course, to make sure that its operate current requirement falls within the worst circuit margins. (minimum available relay voltage equals minimum supply voltage minus maximum sustaining voltage).

The accuracy of the timing circuit is subject to the variations encountered in voltage supply, tube constants, and resistance-capacitance values. With a voltage supply of 125 to 130 volts, accuracy of about ±20 per cent can be achieved. Variable delay can easily be obtained by making $R$ or $C$ variable in the circuit. Roughly speaking, the circuit requires about 1.33 mf per second when $R$ is set at one megohm.

Since the gas tube is a high-impedance device, the leads connected to the tube terminals are subject to induced voltages which may be high enough to fire the tube falsely unless precautions are taken.
Two types of cable transients are recognized, one a high-frequency phenomenon, the other of low frequency.

The high-frequency pick-up generally occurs on the long leads carrying battery and ground to the circuit. It occurs when the tube is in close proximity to a lead connected to an unprotected relay which releases. These transients can be protected against by a .001 mf mica condenser connected directly to the tube socket terminals. The induced low-frequency transient occurs on leads connecting the tube and a relay winding. Trouble can occur with leads as short as two feet, so that it is important to keep relay and tube very close together.

If a tube that is not fired is left with an unterminated lead connected to the tube anode or cathode, charges may accumulate on the lead that will result in false firing of the tube. A ten-megohm resistor should be connected permanently to such a lead to drain off the charges.

A so-called enabling transient high enough to fire the tube falsely may occur when the high voltage is connected across the main gap of the tube at the start of the timing period. This depends upon the circuit configuration and constants, and the design should be scrutinized carefully to insure that no trouble arises from this cause.

A voltage induced in the relay winding by magnetic interference may also fire the tube prematurely. Appropriate precautions against magnetic interference must be taken, therefore, and in addition, at least 40 volts margin should be left between supply voltage and main-gap breakdown.

Four circuit arrangements designed for particular types of application are shown in Fig. 17-10, with notes as to the conditions under which they can fail. The circuit of Fig. 17-10A, which utilizes an extra relay to check the capacitor discharge path, cannot fail by false or premature firing because of dirty contacts in the control path.

The circuit of Fig. 17-10B is arranged so that dirty contacts in the control path cannot prevent the tube from firing. It is suitable for use for certain trouble-detecting purposes where failure to give a trouble indication cannot be tolerated. The tube is extinguished after the timing period by the shunting action of the relay locking contact. The voltage surge developed at the release of the relay necessitates use of the 2500 ohms — 2 mf network across the tube to prevent false firing across the main gap.

The circuit of Fig. 17-10C is subject to failure by either false firing or nonfiring because of a dirty contact in the control path. However, it is the simplest and most economical of the several circuits and is recommended for general timing purposes. Fig. 17-10D is a modification of Fig. 17-10C for use where a less expensive tube with high
main-gap working voltage can be used instead of a closer tolerance low-voltage tube.

In specific applications of any of the various timing circuits, care must be taken that sufficient time is allowed between uses to permit the circuit to return completely to normal. If the tube has not completely de-ionized, the capacitor discharged, and the relay de-energized, the next timing interval will be in error. The restoration time required

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Fig. 17-10 Alternative Gas-Tube Timing Circuits
depends upon the particular circuit and should always be determined in relation to the requirements of the application.

**Design Example**

Problem: to design a circuit to give an average delay of ten seconds with deviation within about ±25 per cent. The circuit of Fig. 17-10C is satisfactory.

Voltage: \( E = 125 \) to 135 volts  
Control-gap breakdown: 69 to 74 volts  
Leakage resistance: 10 megohms minimum.

The value of \( R \) should be approximately one megohm. The tube breaks down at 72 volts average, and the average supply voltage is 130 volts.

Therefore:  
\[
\frac{e_t}{E} = \frac{72}{130} = .554
\]

From curve B of Fig. 17-3 the value of \( T/RC \) is seen to be:  
\[
\frac{T}{RC} = .80
\]

Substituting \( R = \) one megohm and \( T = 10 \), the value of \( C \) is 12.5 mf. The value of three \( 4.32 \) mf ± 1 per cent capacitors is \( 12.96 \) mf, and this capacitance will be used. Substituting this value in the equation for \( T/RC \) gives a value for \( R = .965 \) megohms. It will be assumed that a ±5 per cent resistor will be employed in the circuit. If the series relay is 2500 ohms, a .96-megohm resistor can be used.

The value of \( T_{(\text{min})} \) can now be calculated using equation (17-17). An alternative method would be to use equation (17-3) and curve B of Fig. 17-3, substituting minimum and maximum values as indicated in equation (17-17). Substituting appropriate values in equation (17-17) gives:

\[
T_{(\text{min})} = 2.3 \times .9625 \times .95 \times 12.96 \times .99 \times \log_{10} \frac{135}{135-69}
\]

or:  
\[
T_{(\text{min})} = 8.36 \text{ second}
\]

The value of \( T_{(\text{max})} \) is obtained from equation (17-16). However, first must be calculated the equivalent \( E \) and \( R \) which appear due to leakage. Employing the equations of Fig. 17-8:

\[
E'_{(\text{min})} = \frac{R_L}{R_{(\text{max})} + R_L} \times E_{(\text{min})}
\]

\[
E'_{(\text{min})} = \frac{10}{(.9625 \times 1.05) + 10} \times 125 = 113.5 \text{ volts}
\]

\[
R'_{(\text{max})} = \frac{R_{(\text{max})} R_L}{R_{(\text{max})} + R_L} = \frac{.9625 \times 1.05 \times 10}{(.9625 \times 1.05) + 10} = .918 \text{ megohm.}
\]
Substituting in equation (17-16):

\[ T_{(\text{max})} = 2.3 \times 0.918 \times 12.96 \times 1.01 \log \frac{113.5}{113.5 - 74} \]

\[ T_{(\text{max})} = 12.7 \text{ seconds.} \]

The relay operating time in this case would be negligible.

The circuit constants required are:

\[ R = 0.96 \text{ megohm } \pm 5 \text{ per cent} \]
\[ C = 12.96 \text{ mf } \pm 1 \text{ per cent} \]

and they give a time range of 8.36 to 12.7 seconds.

PROBLEMS FOR CHAPTER 17

17-1 Design a capacitor-timed relay circuit to give an average interval of 0.025 second with as small a variation as is practical. Use a relay of the type shown in Fig. 17-4. Assume that capacitors of ±1% accuracy are available in values which are multiples of 0.540 mf. A battery supply normally held at -48 volts, but which may range from -45 to -50 volts, is to be employed. Assume that the average ampere-turn value at which the relay operates is 4.

Determine theoretical upper and lower timing limits for the circuit.

17-2 An interrupter contact (INT) operates as shown in the diagram below. Using this contact as a timing mechanism, design a relay circuit to ground a lead TM not sooner than 60 seconds nor later than 75 seconds after a start lead ST is grounded. Lead TM shall remain grounded thereafter until ground is removed from lead ST. The removal of ground at any time from lead ST shall restore the circuit to normal.

The transfer time of the interrupter contact is less than the operate or release time of a relay. (This can be done with five relays.)

---

\[
\begin{aligned}
\text{B} & \quad \text{F} \\
\text{INT} & \quad \text{RELAY CIRCUIT} \\
\text{ST} & \\
\end{aligned}
\]
Chapter 18

CIRCUITS FOR PULSE GENERATION

As mentioned in the preceding chapter, circuits for pulse generation are closely allied to timing circuits. This similarity follows from the function of a pulsing circuit: the generation of recurring timed intervals in the form of a train of pulses. A pulse generator can be considered as a recycling timing circuit which alternately produces measured pulse intervals and inter-pulse intervals.

The need for recurring pulses arises at many points in the design of switching systems. For example, information is often passed from circuit unit to circuit unit, or from location to location, by means of trains of pulses. These pulses may consist of opens or closures of a signaling loop, or of ground or other potential condition on a single lead. Pulse trains are also utilized to time critical operations and to control time sequences.

In some applications the pulse cycle must be accurately timed; in others, considerable timing variation may be allowed. The required repetition rate may be measured in pulses per minute, pulses per second, pulses per millisecond, or, in some cases, in pulses per microsecond. Pulses can be generated by a variety of methods: by mechanical devices, by motor-driven interrupters, or by relay and electronic circuits. The form of circuit and the circuit components to be used depend upon the speed of pulsing and the accuracy of pulse and inter-pulse intervals necessitated by the circuit requirements. In this chapter, pulse generators based on relays and on tubes are considered, together with means for detecting pulse information.

18.1 SIMPLE RELAY PULSING CIRCUITS

The self-interrupting or "buzzing" relay circuit, so undesirable in most circuit applications, lends itself to pulse generation in those cases in which considerable variation in pulse length and repetition rate is permissible. Its operation is based primarily upon the principle of closing ground to a relay through the break-contact of another relay which is itself operated, directly or indirectly, by the first relay. The relays will buzz or pump as long as the initiating ground-closure is maintained.
Pulsing leads to external circuit

Fig. 18-1 Simple Relay Pulse Generators
Three examples of this type of circuit are shown in Fig. 18-1. There are many variations of these circuit forms, and the number of relays involved in each can be extended to obtain the desired intervals between pulses. The intervals obtained are, of course, directly dependent upon the operating and release times of the relays, and are accurate within the variations to which relay acting times are subject.

18.2 CAPACITOR-TIMED RELAY PULSE GENERATORS

The capacitor-timed operation of a suitable relay as a means for producing accurately timed intervals has been discussed in Chapter 17. Capacitor timing can be applied to circuits which continuously generate pulses with the same degree of accuracy as is obtainable in the equivalent timing circuits.

In the modification of the capacitor-timed relay arrangement, means internal to the pulsing circuit are provided to charge and discharge the timing capacitor alternately, and to pass the resultant cyclically-reversing current through each of the two windings of the polar relay involved. Although this could be accomplished by a supplementary relay controlled by the polar timing relay, it is usually desirable to have the contacts of the timing relay itself perform this function, thus eliminating the timing variations that would be introduced by the presence of a supplementary relay. Contacts for the control of external circuits can be obtained by inserting one or more "slave" polar relays in series with the appropriate winding of the pulsing relay. These slave relays, then, accurately follow the pulsing relay. Circuits based on this plan of operation are discussed later in this section.

Additional factors introduced by the cyclical behavior of this type of circuit require some extension of the capacitor-resistance circuit theory presented in Chapter 17. Consider a series arrangement of a capacitor and resistance which is alternately connected to battery and shorted, with the duration of each switch closure insufficient to permit full charge or discharge of the capacitor, as shown in Fig. 18-2A. Inspection of the circuit indicates that the current-voltage relations will be as shown in Fig. 18-2B. On the initial closure of the switch to battery, the capacitor is uncharged and a current of $E/R$ flows through the circuit. As the capacitor charges, the capacitor voltage $e_c$ increases, and the current $i$ decreases, exponentially. If the switch were held closed to battery for sufficient time, the current would diminish to zero and the capacitor voltage would reach a value of $E$.

However, it is assumed that when the current through resistor $R$ has decayed to a value $i_1$ and the voltage across the capacitor $C$ has reached $e_{c1}$, as indicated in Fig. 18-2, the switch is operated to ground.
Capacitor C then begins to discharge, and the current through the circuit reverses in direction. This current $i$ is again allowed to decrease, as the capacitor potential drops, until it reaches a magnitude of $i_1$. At this instant the switch is assumed to close to battery again and the capacitor starts to recharge.

If the action of the switch continues to reverse the direction of the current through $R$ each time it decays to a magnitude of $i_1$, the capacitor charges and discharges cyclically, never becoming entirely charged or discharged.

In Chapter 17, a timing circuit, shown here in Fig. 18-3A, was discussed in which a polar relay is held with its break-contact closed until the current through its secondary winding, in series with a capacitor, decreases to such a value that the primary current can operate the relay to close its make-contact. This circuit may be converted to a free-pulsing arrangement operating on the principle just considered by providing that the relay contacts produce a current reversal each time the relay acts. This may be accomplished by inserting a protective resistance in series with the battery supply $E$ and permitting the relay contact to shunt out this battery, as shown in Fig. 18-3B. Since the bias
current must also be reversed, the primary winding is connected to a voltage \( E' \) intermediate between ground and battery. Now, with the relay unoperated, the capacitor charges; and the charging current, which corresponds to the current \( i \) in Fig. 18-2, holds the relay on its back contact. When this current decays sufficiently, the current through the primary winding operates the relay to its make-contact. The currents through both windings then reverse. However, the relay does not immediately release since the capacitor discharge current in the secondary winding is initially high, holding the relay on its front contact. When the discharge current falls to the level where the primary current releases the relay, the make-contact opens, the current through both windings again reverses, and the charging current now holds the relay released. This cycle continues until the circuit is halted by external means.

![Timing Circuit A](image)

**TIMING CIRCUIT A**

![Pulsing Circuit B](image)

**PULSING CIRCUIT B**

![Symmetrical Pulsing Circuit C](image)

**SYMMETRICAL PULSING CIRCUIT C**

Fig. 18-3 Capacitor-Timed Relay Circuits
This circuit arrangement leaves a single break-contact available for external circuit use. If more contacts are necessary, slave relays can be inserted in series with the primary winding as indicated in the figure. With this circuit, the closed and open periods can each be varied in length subject to the same limitations as the relay timing circuit of Chapter 17. It may be halted or prevented from starting by an external ground placed on the make-contact of the pulsing relay.

Another form of capacitor-timed pulsing circuit is shown in Fig. 18-3C. This is a symmetrical arrangement requiring only a single voltage source, which is, in effect, connected to first one and then the other end of the circuit. This circuit is convenient for producing pulses equal in duration to the intervals between pulses, although some degree of unbalance may be obtained by making the battery protective resistances unequal. It may be halted by ground on the relay make-contact, with the break-contact path opened.

In Fig. 18-3B or 18-3C, when the circuit is at rest, the timing capacitor is either completely charged or discharged. For this reason the durations of the generated pulses, immediately after the circuit operation is initiated, are somewhat different from those obtained after the initial one or two cycles. Therefore, where it is necessary that all pulses be of the correct length, the pulsing circuit is started early and the pulse output is connected to the external circuits after the operation has stabilized.

Design of a Capacitor-Timed Pulsing Circuit. A generalized method of design for the symmetrical pulsing circuit of Fig. 18-3C is covered in this section. Somewhat the same procedure can be applied to the design of the circuit of Fig. 18-3B though with added difficulty due to the asymmetry between the two halves of the pulsing cycle. The method is based on the generation of pulses with equal on and off intervals.

In the symmetrical pulsing circuit, as redrawn in Fig. 18-4A, the protective resistances \( r \) are equal and resistances \( R_s \) and \( R_p \) include the respective secondary and primary winding resistances. The relay is adjusted to operate to either its front or back contact on \( N_i \) ampere-turns; that is, when the secondary current decreases to a value such that the primary ampere-turns exceed the secondary ampere-turns by \( N_i \), the relay armature operates. On the basis of the discussion in Chapter 17, the acting point of the relay on the exponential charging curve for the secondary current is chosen at time \( t = 2RC \). It is assumed that the numbers of turns in the primary and secondary windings are known in addition to the values of \( N_i \), \( E \), and \( r \). A value of \( C \) is also selected. In practice, it is found that \( C \) for this circuit should be close to, but no greater than, \( T/2(R_{sec} + r) \), where \( R_{sec} \) is the resistance of the relay.
winding in series with the capacitor, and T is the time of one half-cycle. A value of C greater than this may result in an impossible solution. Thus, the factors to be determined are $R_s$ and $R_p$, to give the desired pulse rate.

In Fig. 18-4B the circuit is redrawn as it appears immediately after break-contact closure has occurred. A potential $E_c$, poled as indicated, exists on the capacitor from the previous half-cycle. The small arrows labeled "OPR" indicate the direction of current flow required through primary and secondary windings to operate the relay to its make-contact. It can be seen that at the instant shown in the figure the primary current flow $i_p$ is attempting to operate the relay while the secondary current $i_s$ is holding it released.

To obtain ease of manipulation, this circuit is reduced to its equivalent series form by means of Thevenin's theorem. This equivalent form, as illustrated in Fig. 18-4B, consists of the capacitor C.
\[ R_p [(i_p)_t] + r [(i_p)_t + (i_s)_t] = E \]

or
\[ (R_p + r) [(i_p)_t] + r [(i_s)_t] - E = 0 \quad (18-8) \]

When the values of \((i_s)_t\) and \((i_p)_t\) from equation (18-6) and (18-7) are substituted in equation (18-8), a quadratic equation in \(R_p\) is obtained whose form is:
\[ aR_p^2 + bR_p + c = 0 \quad (18-9) \]

The coefficient values are:
\[ a = 0.238 N_s E + R Ni \]
\[ b = 0.238 r E(N_s + N_p) + R (2rNi - N_pE) \]
\[ c = rR(rNi - N_pE) \]

and the solution is:
\[ R_p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

This evaluation of \(R_p\) is entirely in terms of known quantities, recalling that \(R = T/2C\). The value of \(R_s\) can be obtained from \(R_p\) by use of the relation defining \(R\):
\[ R = R_s + R' \]

or
\[ R_s = R - \frac{rR_p}{r + R_p} \]

The values of \(R_p\) and \(R_s\) having been found, it is possible to calculate the timing variations as a function of variations in \(R_p\), \(R_s\), \(r\), \(C\), and \(Ni\). This involves the derivation of an expression for the pulsing period \(T\) in terms of these elements at their limits of variation in a manner similar to that used in Chapter 17.

In practice, it is found that the actual value of \(Ni\) is influenced by the rate of current change through the pulsing relay windings. For this reason, laboratory tests will often indicate that the computed values of \(R_p\) and \(R_s\) must be changed slightly to give the desired pulsing intervals.

### 18.3 GAS-TUBE PULSING CIRCUITS

Gas-tube timing circuits can also be converted to pulsing circuits that afford the same degree of accuracy as the gas-tube timing circuit. A reasonable approach to the problem of designing a tube pulse-generator is that of combining two timing circuits which operate alternately. This simplifies the design over that of the capacitor-timed
Fig. 18-5 Cold-Cathode Gas-Tube Pulse Generator
relay circuit since it is not necessary, except in special cases, to consider the effect of one half-cycle on the next. Thus, the calculation of the constants required to give the necessary time intervals for the two portions of the cycle can be made independently.

The basic plan for this type of circuit must provide:

1. Means for activating the circuit.
2. Means for timing the first half of the cycle, during which the first timing circuit is the controlling factor.
3. Means for extinguishing the first timing circuit tube and discharging the capacitor used in timing the first half-cycle.
4. Means for timing the second half of the cycle, controlled by the second timing circuit.
5. Means for extinguishing the second timing circuit tube and discharging the capacitor used in timing the second half-cycle.

A simple cold-cathode tube circuit illustrating this plan of operation is shown in Fig. 18-5A. In its operating cycle, shown in Fig. 18-5B, relay (R1) operates and locks when tube T1 fires; relay (R2) operates when tube T2 fires; relay (R2) releases (R1) which in turn releases (R2), restoring the circuit to normal. As a result of the continuity-transfer on (R1), the locking winding of (R1) is connected to a holding ground through a break-contact on (R2) before tube T1 is extinguished, and the operating path for (R1) is opened. Tube T2 extinguishes as soon as relay (R1) releases. In this circuit, the function of relay (R2) is merely to release (R1) after a timed interval. As a result, all pulsing contacts for external use should be on relay (R1). The resistors \( R_a \) serve to protect the starter gap, and resistors \( R_c \) limit the capacitor discharge currents. Relay (ST) serves to activate the circuit. As indicated in the figure, it is assumed that the tubes fires on a starter-anode potential of +70 volts.

The timing for the two parts of the cycle is determined by networks \( R_1 C_1 \) and \( R_2 C_2 \), and, to a minor extent, by the acting times of relays (R1) and (R2). The period of a single cycle \( T \) is shown in the operating diagram of Fig. 18-5B to be equal to:

\[
T = t_1 + (R1)_{OPR} + t_2 + (R2)_{OPR} + (R1)_{RLS} \tag{18-10}
\]

where the subscripts OPR and RLS refer to the relay operate and release times, respectively.

The fundamental equation for the potential across a capacitor at time \( t \), when the initial capacitor voltage is zero, is:

\[
e_c = E(1 - e^{-t/RC})
\]
Solving this for \( t \):

\[
t = 2.3 \frac{RC}{\log_{10} \left( \frac{E}{E - e_c} \right)} \tag{18-11}
\]

This expression is applicable to times \( t_1 \) and \( t_2 \) in expression (18-10) above. If \( e_c = e_t \) is the starter-anode-to-ground potential necessary to fire tubes (T1) and (T2):

\[
t_1 = 2.3 \frac{R_1 C_1}{\log_{10} \left( \frac{E}{E - e_t} \right)}; \quad R_1 \gg R_c
\]

and

\[
t_2 = 2.3 \frac{R_2 C_2}{\log_{10} \left( \frac{E}{E - e_t} \right)}; \quad R_2 \gg R_c
\]

When a tube is to be fired, the optimum operating point on the capacitor charging curve is \( t = RC \). In order to satisfy this condition exactly, it may be necessary, as discussed in Chapter 17, to apply bias voltages to the tubes, insuring the indicated relationship between \( E \) and \( e_t \). However, in practice, bias is not normally provided since inaccuracies resulting from a value of \( t/RC \) somewhat different from 1 are small. The circuit constants for each part of the cycle can be calculated independently by the methods discussed in Section 17.4, Chapter 17.

In this circuit it may be desirable to carry the control anode lead of tube (T1) through a break-contact on relay (R2), as indicated in Fig. 18-5. This modification is incorporated to prevent false breakdown of tube (T1) due to a voltage surge at the cathode caused by induction between windings of relay (R1) when the locking winding of (R1) is opened.

A cold-cathode gas-tube circuit in which the tubes are extinguished by shunting the tubes instead of by opening the anode or cathode circuits is shown in Fig. 18-6. As shown in the operating plan diagram, tube (T1) in firing: operates relay (R1) which opens the locking path of relay (R2); locks and then shunts out tube (T1); allows capacitor \( C_2 \) to start charging; and discharges capacitor \( C_1 \). Similarly, operation of relay (R2) by tube (T2) releases (R1), shunts out tube (T2) and enables capacitor \( C_1 \) to charge. This circuit takes advantage of the principle that if the main-gap potential is reduced to a value about 20 volts below the sustaining voltage, a cold-cathode tube de-ionizes more rapidly than an open circuit. The R-C networks labeled "C.P." in the figure are used to absorb the voltage surge incidental to opening the relay holding paths, not only for contact protection but also to avoid false firing of the tubes. This surge might otherwise cause ionization at the outset of a charging interval. Contacts on both relays can be used for pulsing purposes, as indicated by the sequence diagram.
Fig. 18-6 Shunt-Down Cold-Cathode Gas-Tube Pulse Generator
18.4 VACUUM-TUBE PULSING CIRCUITS

There is a wide variety of pulsing (and timing) circuits based on the fundamental vacuum tube multivibrator, shown in Fig. 18-7. The multivibrator consists of a two-stage amplifier with the output of the second stage coupled back to the input of the first stage. The circuit is such that both tubes can not remain conducting at the same time. Assuming tube (V1) to be cut off by a negative potential on grid capacitor \( C_1 \), the plate of (V1) is at the +E supply voltage. As \( C_1 \) discharges through \( R_{g1} \) and the parallel combination of RL₂ and the plate resistance \( r_{p2} \) of tube (V2), the voltage \( e_{g1} \) rises, until it reaches the cutoff value \( E_{g1} \), and the tube (V2) begins to conduct. The small drop in plate potential \( e_{p1} \) of (V1) at this instant is coupled to the grid of (V2), and amplified by (V2) to appear as a sharp rise in potential \( e_{p2} \) which in turn drives the grid of (V1) more positive. Thus tube (V2) is cut off, and the grid of tube (V1) is stabilized at zero grid potential. The grid of (V2) during this action has been driven negative from zero by an amount equal to the change in plate potential of (V1). The potential of the grid of (V2) then rises toward zero volts at a rate determined primarily by \( C_1 \) and \( R_{g2} \). When it reaches the cutoff value, tube (V2) again starts to conduct and the feedback action drives (V1) below cutoff and (V2) to a state of zero grid potential. This cycle continues as long as the proper supply potential is maintained.

The voltage overdrive of each grid as it swings positive is quickly reduced by the charge path for the capacitor through the low-resistance grid-cathode circuit which exists as long as the grid is positive with respect to cathode.

The expression for the period of one cycle of the multivibrator follows from consideration of the operation of the circuit. The period \( T \) is approximately:

\[
T = 2.3 C_1 R'_1 \log_{10} \frac{E - E_{p1}}{E_{g2}} + 2.3 C_2 R'_2 \log_{10} \frac{E - E_{p2}}{E_{g1}}
\]

where \( E_g \) represents the magnitude of grid-voltage at which conduction starts (the cutoff value); \( E_p \) is the anode-to-ground potential existing during the conducting interval of a tube; and the subscripts 1 and 2 refer to tubes (V1) and (V2), respectively. The resistances \( R'_1 \) and \( R'_2 \) are given by

\[
R'_1 = R_{g1} + \frac{RL_2 r_{p2}}{RL_2 + r_{p2}}
\]

and

\[
R'_2 = R_{g2} + \frac{RL_1 r_{p1}}{RL_1 + r_{p1}}
\]
Fig. 18-7 Basic Vacuum-Tube Multivibrator
where \( r_p \) is the plate resistance of a tube during a conducting interval, and \( RL_1, RL_2, R_g1, \) and \( R_g2 \) are as shown in Fig. 18-7A. Normally, \( r_p1 \) and \( r_p2 \) are much smaller than \( R_g2 \) and \( R_g1 \), respectively, with the result that \( R_1' \approx R_g1 \) and \( R_2' \approx R_g2 \). The circuit can be designed for periods \( T \) from the order of a microsecond or less to several seconds.

In Fig. 18-7, a relay (P) which operates as (V2) conducts, and releases when (V2) is cut off, is used to supply pulses to an external circuit. Another convenient pulse source is a tap on a cathode resistor, the potential of which alternates between zero and a positive value. This latter arrangement is applicable if the external circuit under control is composed of tube or other relatively high-impedance elements. If a relay must be driven by the circuit, the plate current capability of the driving tube must be such that the relay is operated with sufficient margin.

18.5 PULSE DETECTION

Where information is to be conveyed in the form of pulse trains, it is necessary to provide means for detecting these pulse trains and the individual pulses making up the trains. A typical example is the detection of pulse information generated by a subscriber dial mechanism in a telephone system.
In a dial pulsing system, the pulses often consist of momentary open intervals of a two-conductor loop, the number of pulses in each transmitted pulse train corresponding to a dialed digit. The loop is first closed when the loop is to be used to pass information; and it remains closed, after dialing, until it is restored to normal (open). The loop is also closed between dialed digits, this inter-digital time being considerably longer than the inter-pulse intervals.

At the switching center where the loop terminates, two devices must be incorporated to detect the pulsed information: one which follows opens and closures of the loop, in effect reporting on the instantaneous condition of the loop; and a second which ignores the individual dial pulses but operates to detect the passage of each pulse train, thus differentiating between successive digits. In addition, a third device is normally included which operates when the loop is initially closed and releases when the loop is opened at the end of usage, providing means for supervising the connecting.

A basic method of accomplishing these tasks is shown in Fig. 18-8. Relay (A) operates directly from the loop, assumed in the figure to be controlled by a telephone subscriber, and follows dial pulses. Relay (B) operates from a ground through a make-contact on relay (A). Since (B) is selected to have the required slow-release characteristics, it remains operated during dial pulses. For the convenience of the associated circuits, the input dial pulse open intervals are converted by relay (A) to ground pulses, which are cut through to the external counting circuit after relay (B) has operated. Relay (C) operates on the first break-contact closure of (A) following operation of (B), and holds over the intervals of successive pulses of a train. However, (C) releases during the period between pulse trains to give an indication of the end of each train.

PROBLEM FOR CHAPTER 18

18-1 A capacitor-timed relay pulse-generator is to be designed for laboratory use employing a polar relay of unknown characteristics. The resistances of the primary and secondary windings are found to be 200 ohms and 1000 ohms, respectively. Also, it is discovered that the relay operates from back-to-front contact either when the primary current is 2 milliamperes with the secondary open-circuited, or when the secondary current is 0.4 milliampe with the primary open-circuited.

Design a pulse generator of the type of Fig. 18-4 to operate at a rate of twenty pulses per second. A power supply of 48 volts and protective resistors of 500 ohms are available.
Chapter 19

CIRCUITS FOR CHECKING

The individual circuits of a switching system are expected to operate on an essentially unattended basis. The purpose of the system is defeated if it becomes necessary to watch over the circuits to see that they perform properly. It is not sufficient, therefore, to consider only normal conditions in the design of circuits. Abnormal conditions must be anticipated and the circuits arranged so that reasonable reactions will occur when abnormal conditions are encountered.

A switching system must avoid situations in which a trouble condition can cause serious reactions or damage to the equipment being controlled. Beyond this it is desirable for the system to give a warning when trouble is encountered and, if possible, to give an indication of the location and nature of the trouble. In some cases the system can be arranged to perform an alternative action if prevented from completing its normal action. For example, certain telephone switching systems are so designed that if they encounter trouble when attempting to establish a connection, they will make a second attempt using alternative circuit paths. Another procedure is to provide standby circuits which will automatically take over the functions of a circuit which develops trouble during its normal use. Often, specific standby circuits are not provided but several circuits will be used in rotation, the control being arranged so that a circuit developing trouble will be taken out of service. The remaining circuits then share the entire work load.

The importance of checking circuits and the extent to which they should be applied will depend upon the consequences of improper operation. Many switching systems become practically useless if there is no assurance that they perform all actions properly. In large-scale computers, for example, an occasional single fault can cause the validity of any computation performed by the machine to be in doubt unless checking methods are used.

Quite often checking features require additional apparatus or increased circuit working time or both, and must be evaluated on an economic basis. If the general excellence of circuit design is maintained on a high level, there is little need to design circuits to anticipate improbable troubles or combinations of trouble conditions. In fact, the additional circuit complexity necessary to check improbable trouble
conditions may actually increase the chance of a circuit failure. Even though every trouble is detected, a circuit which repeatedly develops trouble cannot be considered satisfactory. It is usually sufficient to check for the presence of a single trouble condition at a time. That is, if a circuit is designed to guard against any single trouble condition, it can be assumed that a second independent trouble will not occur simultaneously. Also, it is a very unusual situation which justifies designing check circuits to check check-circuits. However, check-circuits can often be made self-checking, so that a trouble which causes the checks to be invalid will be recognized.

The amount of additional apparatus required for checking can be held at a minimum by properly integrating the checking features into the operating circuit plan. Rather than provide separate relays and contacts only to perform a checking function, the functional circuit relays can be arranged to check that certain actions have been satisfactorily completed and also perform some action necessary in the next stage of the sequence of circuit operations. Thus each relay, or a number of key relays, may not only perform their particular assigned functions, but check that previous stages of the circuit action have not encountered trouble. In this way check features are often hidden in the plan of a well designed circuit, and efficient use is made of the regular circuit relays and their associated contacts.

Alarm circuits and trouble-indicating circuits are a necessary auxiliary to self-checking circuits. For example, in a control system which is capable of choosing alternative actions or of making second trials when trouble is encountered, some indication that a trouble is present should be given even though satisfactory service is provided. Otherwise it is possible that a considerable part of the system may be out of order, with no external indication until the concentration of load in the remaining working circuits overloads the system. Also, in complex systems some indication of the location of the trouble must be given; otherwise finding a trouble in a large switching network would become a very difficult task.

In the following sections, various circuit arrangements which check for or protect against particular trouble conditions will be described. For simplicity in description, many of these circuits will be developed to check for only one type of trouble condition and to ignore other conditions which are equally undesirable. For example, some circuits will detect the presence of a trouble ground while ignoring an equally troublesome open-circuit condition. As the discussion progresses it will be evident that several of the checking arrangements may be combined. The circuit engineer must determine which arrangements shall be used and how many different conditions shall be checked in a particular case. The selection of checking circuits in this chapter
is by no means comprehensive. In general, the checking circuit must be specifically designed to fit a particular situation, and therefore the examples given can only be typical.

19.1 AFFIRMATIVE AND NEGATIVE CHECK PRINCIPLES

The methods of checking a circuit condition may be divided into two broad classes. In one, "affirmative checking", action is taken only if the check finds the correct circuit condition; while in the other, "negative checking", action is taken only if a trouble condition is found.

The general philosophy of affirmative checking is to provide a checking relay which acts if the correct circuit condition is encountered during the progress of the switching action and which fails to act if an incorrect condition exists. The checking relay is then integrated into the circuit plan so that its action starts the next action in the operating sequence. Failure to check satisfactorily then results in the blocking of all subsequent circuit actions. Often the affirmative checking configurations can be so well blended with the other circuit functions that no contact or relay can be identified as being used exclusively for checking purposes. This is, of course, the most desirable form of "self-checking".

In negative checking, on the other hand, the checking relay acts only when a trouble condition occurs. Since it normally does not operate, a negative-checking relay is not easily incorporated into the circuit plan. It has the further disadvantage of acting only on the infrequent occasions when trouble develops. Therefore, the ability of the check circuit to operate is not determined on each circuit usage unless the relay is also used for some other function which occurs normally.

Since a negative check depends on the action of a relay when a trouble condition is detected, a time interval sufficient to permit the check relay to act must be included in the operating sequence at the time a check is to be made. This may increase the over-all circuit work time, since the checking interval must be sufficient to permit the relay to operate under the most adverse conditions. Positive checking methods, however, permit the circuit to proceed in its actions as soon as the check relay operates. In well planned circuits, the checking relays are often the key relays which control the acting sequence. When positive checking is employed, a signal that a trouble has occurred is usually obtained by employing timing circuits which monitor the action of the switching circuit. The acting time interval of the timing circuit is set to be slightly greater than that of the circuit action being monitored, and both circuits are started simultaneously. Under normal conditions the circuit action is completed before the timing circuit acts.
On the other hand, since positive checking circuits halt subsequent circuit actions, a trouble condition will prevent the action from being completed before the timing circuit acts. The resultant combination of the timing circuit operated and the check relay in the monitored circuit unactivated can be arranged to give an alarm or other indication of the trouble condition.

In both affirmative and negative check circuits, a highly reliable arrangement is necessary if the addition of checking apparatus is to be of practical value. The choice of the particular action to be employed

<table>
<thead>
<tr>
<th>Fault or Trouble Condition</th>
<th>Effect in Fig. 19-1A</th>
<th>Effect in Fig. 19-1B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. False ground on springs or on wiring of check network</td>
<td>False Check*</td>
<td>Failure to check</td>
</tr>
<tr>
<td>2. Locked or bridged contacts in check network</td>
<td>False Check*</td>
<td>Failure to check</td>
</tr>
<tr>
<td>3. Failure of (CK1) to release when de-energized</td>
<td>False Check</td>
<td>Failure to check</td>
</tr>
<tr>
<td>4. Dirty contact on (A) (B) . . . (N) relays</td>
<td>Failure to check</td>
<td>False Check*</td>
</tr>
<tr>
<td>5. Broken wire in check network</td>
<td>Failure to check</td>
<td>False Check*</td>
</tr>
<tr>
<td>6. Open winding of relay (CK1)</td>
<td>Failure to check</td>
<td>False Check</td>
</tr>
<tr>
<td>7. Battery missing at (CK1). (Fuse blown).</td>
<td>Failure to check</td>
<td>False Check</td>
</tr>
</tbody>
</table>

* Depending upon the location of the fault, the check circuit may not give a false check on failure of certain relays to reach the released state.
as a check and the choice of contact configuration to initiate the checking action depends to a large extent upon the designer's judgment. For example, consider a circuit in which the state of "all relays released" is to be checked in the affirmative manner. Two methods are shown in Fig. 19-1. In Fig. 19-1A, the checking action is the operation of relay (CK1) by a series chain of break-contacts. In Fig. 19-1B, the checking action is the release of relay (CK1) by the opening of paralleled make-contacts. A satisfactory check is indicated by ground on lead CK.

An analysis of Fig. 19-1 brings out the fact that a number of faults may occur in the checking circuit itself which may cause a false check indication to be given. The more common trouble conditions and their effects are listed in the table on the facing page.

It will be noticed that the contact configurations of the A and B figures are negatives of each other, and that on a given type of internal fault one gives a false check while the other indicates trouble. Under practical circumstances the circuit of Fig. 19-1A is the more reliable since the type of faults which result in false checks are somewhat less probable than with the circuit of Fig. 19-1B. On the other hand, if the circuits of Fig. 19-1 are used to give a check that at least one relay is operated (that is, all relays released represents a trouble condition), internal faults in the checking arrangement will have effects opposite to those described. For this application, therefore, the circuit of Fig. 19-1B appears more reliable.

19.2 PROBABILITY-TYPE CHECKING

One very satisfactory self-checking technique depends on transmitting information or control signals in such a manner that at least two simultaneous trouble conditions are necessary to cause a false circuit action. A single trouble will cause the signals to be incomplete, and the circuits will recognize this as a trouble condition. With reliable apparatus and circuits, the probability of two simultaneous troubles is so small that additional checking is usually unnecessary. Examples of this are the uses of "self-checking" and "self-correcting" codes which are discussed in Chapter 12. The "two-out-of-five" code, used to represent the values of numerical digits, is perhaps the most widely used self-checking scheme. These codes can be checked by means of symmetric networks. For example, Fig. 19-2A shows a check network for the relays of a register which stores a digit in the two-out-of-five additive code. Closure of the path through the network gives an affirmative check that a valid combination of relays is operated. Fig. 19-2B and 19-2C show the symmetric networks for checking the operation of relays in the decimal code (one-out-of-ten) and biquinary code (one-out-of-two and one-out-of-five).
When a number of digits or other items of information are recorded, the checking networks for each digit may be connected in series to give a single check indication. It is not necessary that all of these digits be recorded in the same code, provided that all the codes are of the self-checking type. When several checking networks are connected

![Diagram of 2 Out of 5 Check Circuit](image)

**2 OUT OF 5 CHECK CIRCUIT**

![Diagram of 1 Out of 10 Check Circuit](image)

**1 OUT OF 10 CHECK CIRCUIT**

![Diagram of Bi-Quinary Check Circuit](image)

**BI-QUINARY CHECK CIRCUIT**

Fig. 19-2 Probability Check Circuits
in series, the number of contacts and interconnecting leads becomes large, and the possibility of a trouble cross to ground developing in the network increases. In order to detect this trouble condition, two checking relays can be used. The arrangement is shown in Fig. 19-3, where four items of information are recorded in a variety of self-checking codes and the checking relays, (CK1) and (CK2), are placed at opposite ends of the series circuit. If the check is satisfactory, both (CK-) relays operate. A trouble ground in any of the checking networks will prevent the operation of the (CK2) relay. Contacts on the (CK-) relays can be arranged to indicate the check and trouble conditions. Thus, the circuit becomes self-checking since it checks not only that the coded information is indicated properly, but also indicates the occurrence of a trouble in the check circuit itself.

A negative type of check to determine that not more than one of a group of relays is operated involves the use of a marginal relay. This check, related functionally to the circuit of Fig. 19-2B, might be used when insufficient contacts for a symmetric network are available. Two forms of the circuit are shown on Fig. 19-4. The marginal relay is selected to non-operate on current through a single numerical relay, but to operate on current through two or more. The principle can be extended to give trouble indications only for three or more relays operated, etc., although the marginal requirements become increasingly difficult to meet. A sensitive relay can also be added to indicate that no
The Design of Switching Circuits

Fig. 19-5 Combined Network for Up-Check and Down-Check of A Single Digit

less than the proper number of relays have operated. This in effect converts the circuit to give an affirmative check.

It is often desirable to check that all of the relays of a circuit unit, such as a register, are released before they are re-operated in a new combination. A down-check, obtained by a series chain of break-contacts as indicated in Fig. 19-1A, may be combined with the symmetric circuit of the probability check (up-check) as shown in Fig. 19-5. When making both the up- and down-checks on a number of digits, the network of Fig. 19-5 can be considered equivalent to a transfer, with the closure of the back contact representing the down-check and closure of the front contact the up-check. These networks can be connected to check any number of digits, as shown in Fig. 19-6 where the network for each digit is indicated as a transfer.

Another less frequently used probability-checking method is to transmit information or control signals over two separate paths and arrange the circuit so that the proper signals must be received over both paths for the circuit to progress. Examples to illustrate this principle are shown in Fig. 19-7. Figs. 19-7A and 19-7B each contain two relays (A) and (B) which ground a check lead CK in response to the operation of relay (X). If trouble conditions consist of grounds or opens on the interconnecting leads A and B, at least two troubles are necessary to cause the CK lead to be grounded when (X) is not operated.

Fig. 19-6 Combined Up-Checks and Down-Checks for any Number of Digits
Certain single troubles that do not interfere with correct action of the circuit, however, do not affect the action of the CK lead. As an added precaution, a network as shown on the springs of (A) and (B) can be used to ground an alarm lead if a single trouble occurs.

The above schemes seldom stand out directly in circuit applications but are generally integrated into the circuit plan. The connecting leads A and B may, for instance, take different paths through the contacts of other relays. The relays (A) and (B) may perform other functions and need not act at the same time if other contacts in the A and B leads are provided, but must both be operated together before the circuit sequence can finally proceed. In this type of usage the alarm feature must be modified.

A related probability method for checking the transmission and reception of control information such as numerical digits is to transmit the information twice, recording it in each case on separate registers. The two registers are then checked one against the other to see that the same relays are operated in both. The method can be extended to provide a means for correcting errors by transmitting the information three times and recording on separate registers. In case of an error, the information on the two registers which agree is used. These methods would be used only in exceptional cases. When accuracy of this order is required, self-checking or self-correcting codes can be used.

The above methods can be applied to control circuits where extreme reliability is necessary. Two (or three) identical circuits are provided to perform the same functions, and the circuits are allowed to act simultaneously. The circuits are then checked against each other, either at a final stage or at several intermediate stages. The reliability of well designed switching circuits is such that checking to this extent is practically never required.
19.3 CHECKING SINGLE RELAYS AND CONTACTS

When it is desired to check that all relays of a circuit are in the desired operated or unoperated state at some stage in the sequence of actions, a series chain of contacts - makes on operated relays and breaks on unoperated relays - can be used. These paths may often be employed to control some essential action so that the circuit becomes self-checking and will not advance beyond this stage of its operation unless the relays are in the proper positions. This often requires more contacts than are essential to provide the necessary control. For example, consider a case where an event is to be controlled by a path which closes when two relays, (A) and (B), are operated, and the normal sequence of actions is such that (A) operates before (B) and remains operated until after (B) releases. A make contact on (B) alone is sufficient to provide the required control. However, if a make on (A) is placed in series with the make on (B), the circuit path will not close in the event (A) fails to operate. Thus, the action of relay (A) is checked. To take the most advantage of this type of checking in the design of control paths, only those combinations which actually occur in the normal acting sequence should be used. The use of invalid combinations for circuit simplification (as discussed in Chapter 6) should not be resorted to. Invalid combinations which occur as the result of a trouble condition then will not close circuit paths falsely.

![Diagram](image)

Fig. 19-8 Checking a Single Contact

Occasionally in circuit design a single contact is so important that a check of the closure of this contact is desired before proceeding with the circuit action. A contact checking scheme is shown in two forms in Fig. 19-8. The operation of relay (CK) is an indication that the lead X is grounded and (A) is operated. The second contact on (A) isolates (CK) from X so that it will not operate if X is grounded by other means when (A) is not operated.

It is often desirable to check the locking paths and contacts of relays to insure that they will hold when their direct operating path is
Circuits for Checking

A scheme for checking a hold path is shown in Fig. 19-9A, where the operation of the (CK) relay indicates that relay (A) has operated and closed its locking contact. The double-winding relay is used to prevent (CK) from operating directly from the operating ground. At first glance it may appear that, since (A) and (CK) always operate together, contacts on (A) might be used for a check and that (CK) serves no useful purpose. However, (CK) indicates not only that (A) has operated but also that its locking contact is good, and that (A) can be expected to hold when its operating ground is removed. Contacts on (A) could indicate that the relay was subject to holding failure, but this indication would be available only after the failure had actually occurred. Fig. 19-9B shows a similar arrangement where a single (CK) relay indicates that at least one of a group of relays has operated and closed its locking contacts. Two contacts on each relay are necessary to prevent the locking ground of one relay from operating the others.

Other elaborations of these basic schemes are shown in Fig. 19-10. In Fig. 19-10A a locking chain operates the single (CK) relay to indicate that all relays have operated and closed a path to locking ground. A method using single-winding relays is shown in Fig. 19-10B. This gives a negative-type check, trouble being indicated by a failure of (CK) to hold after the operating grounds are removed. If it is desired to check that at least one relay in each of several groups A, B, . . . . N has operated and closed its locking contacts, the circuit of Fig. 19-10C may be used. The relays of a group are multiplied as indicated.

The above schemes employing double-winding relays check only that the relay operates and closes its locking contact. An open locking

Fig. 19-9 Checking Locking Contacts

opened. For example, such a check may be necessary in storing information on relays in a register to insure that the stored information is not garbled or lost when the input leads are opened.
winding would be indicated by the premature release of the (CK) relay later in the sequence when operating grounds are removed. To make this additional check requires additional circuit complication or acting time. One such scheme is shown in Fig. 19-11, where the action of

![Diagram A](image1.png)

**AFFIRMATIVE CHECK OF LOCKING CONTACTS; NEGATIVE CHECK OF LOCKING WINDINGS**

![Diagram B](image2.png)

**NEGATIVE CHECK OF LOCKING CONTACTS**

![Diagram C](image3.png)

**CHECKING THAT AT LEAST ONE RELAY IN EACH OF N GROUPS HAS OPERATED AND CLOSED ITS LOCKING CONTACT**

Fig. 19-10 Checking that a Number of Relays Have Locked
Circuits for Checking

Fig. 19-11 Check for Locking Ground

relay (A) is checked. The operation of the control relay (C) causes (A) to operate and lock. When (C) releases, removing the operating ground, relay (CK) will operate from the locking ground of (A), indicating that (A) is operated and locked. This test, however, is made only after the operating ground is removed by the release of (C).

Fig. 19-12 Polar Relay Check for Locking Ground

The scheme of Fig. 19-12 applies the operating ground through a winding of the check relay which gives an affirmative indication by operating when the locking contact closes. The check relay (CK) is a differential polar relay. One winding of (CK), P1, is poled to oppose the second winding, P2, and hold (CK) in an unoperated position until (A) operates. The locking ground at (A) shunts winding P1, and causes (CK) to operate on current through P2. Relay (CK) may require mechanical or electrical bias to hold it in the unoperated position in the open-circuit condition. The operating circuit for (A) may be closed as shown in the figure, or this ground may be connected to winding P1 permanently or through previously operated contacts, and the final circuit closure made in the connecting lead at a point such as X.

When a number of similar relays are controlled in combinations over individual control leads, it is often desired to check that all relays are capable of operating. One arrangement is shown in Fig. 19-13 where the relay (A1), (A2), ... (AN) are checked. Here the individual control leads are connected together through back contacts of a "splitting" relay (SP). Ground on any one or more control leads causes all of the (A-) relays to operate. If all (A-) relays succeed in operating, the make-contact chain on these relays is closed, causing the up-check relay (CU) to operate and in turn operate the splitting relay (SP), which splits the control leads so that the (A-) relays on ungrounded leads may release. Fig. 19-14 shows a similar arrangement which checks that all the relays release as well as operate. Here ground from the control
leads grouped at the (SP) relay checks that all (A-) relays are released by passing through a back contact chain to operate the down-check (CD) relay. Relay (CD) locks and operates (CD1), which connects the control leads to the windings of the (A-) relays. The remainder of the action is as in the previous figure.

These two figures, 19-13 and 19-14 also illustrate an application of the principle shown in Fig. 19-9A. The operation of (SP) checks the locking path of (CU). Without this check, failure of (CU) to lock would result in a "pumping" condition, since the operation of (SP) releases one or more than one of the (A-) relays, in turn opening the operating path of (CU).
19.4 CHECKS FOR OPENS, GROUNDS, AND CROSSES

When relays at a distance are operated over an interconnecting lead, advantages are sometimes obtained by using battery signals rather than ground over the interconnecting lead. This is particularly true when the relay must not be permitted to operate falsely. A trouble condition is more likely to cause the lead to be crossed to ground than to battery, since the points at which battery appear are usually kept at a minimum to prevent hazards. Ground connections on the other hand are widely used in control networks, and the apparatus mounting frames themselves are usually grounded. Therefore use of battery to operate the relay as shown in Fig. 19-15 will obviate any possibility of false operation from a trouble ground. Furthermore, the back-contact ground, known as a "covering ground", prevents false operation from an inadvertent cross to battery. A protective resistor is placed in the battery lead, to prevent excessive current flow in case the connecting lead does become grounded.

It is often desirable to check that normal conditions exist on a control lead before proceeding with the circuit action. When preparing to close a ground to a control lead, for example, it may be desired to check that the lead is neither open nor prematurely grounded, while in other cases it is sufficient to check that it is not grounded. One method of checking for trouble ground on a connecting lead is known as a "standing test" and is shown in principle in Fig. 19-16, where ground is placed on the lead during active intervals to control a relay at the distant end. During idle intervals the lead is not merely left open, but is

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**Circuits for Checking**

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connected through the back contact of a relay such as (C) to battery through the winding of a checking relay (CK). The circuit stands in this condition during idle intervals, and in case a trouble ground occurs, (CK) operates to give an alarm signal. This negative-checking arrangement may be integrated in a circuit plan with the affirmative-checking arrangement of Fig. 19-11, so that the operation of (CK) during idle intervals is an alarm condition and during active intervals is a check on the locking ground of the distant relay.

Fig. 19-17 Check for Grounded and Open Leads by a Series Relay

Another arrangement which checks affirmatively for both opens and grounds on a lead at the time it is used is shown in Fig. 19-17A. Ground is applied through the winding of a check relay (CK) which will operate if the connecting lead is neither open nor grounded. The distant relay (A) will operate in series with (CK) if (CK) has low resistance. In some instances the (CK) relay may have a high resistance so that the distant relay (A) will not operate until after a preliminary check of the connecting lead is made. The distant relay may be operated later by applying a direct ground to the lead, shunting the (CK) relay. In the arrangement of Fig. 19-17B, the (CK) relay is used first to check the connecting lead. Upon operating, it places a direct ground on the lead and holds itself through a locking winding. Relay (A) does not operate in series with the winding of (CK). The direct ground may pass through contacts which are closed at a later stage in the circuit action.

The scheme of Fig. 19-18 makes an affirmative check for false ground on a control lead before applying ground to operate a distant relay (A). The check relay (CK) is operated in a local circuit through a break contact of a control relay (C) prior to the time the final closure
to the distant relay (A) is to be made by the operation of relay (C). Relay (CK) prepares a locking path to its winding; and, at the time (C) operates, the operating ground of (CK) is removed. If then a false ground is encountered on the control lead, relay (CK) will hold to this ground; but if the lead is clear (that is, if the far end is open or connected to battery), relay (CK) releases to apply ground through its back contact to the lead. This is an incomplete test, since a failure in the operating path of (CK) voids the test. However, the action of (CK) may be checked by causing it to perform some necessary function in the circuit sequence at the time it operates.

The scheme of Fig. 19-19, employing a polar relay (CK), checks for opens and grounds on the connecting lead. The two windings of (CK) are differential, the winding (P2) connected to battery producing more ampere turns than (P1) when the control contact (C) is open, so that (CK) is normally held on its back contact. When (C) is closed, an open connecting lead does not disturb the differential balance and a grounded connecting lead shunts the operating winding, causing an increase in bias through (P2) to hold the relay on its back contact. Thus, neither of these trouble conditions will cause the (CK) relay to operate. The normal condition of battery on the distant end of the control lead, however, causes (CK) to operate, since the current through the biasing winding (P2) is decreased and that through (P1) is increased.

The scheme of Fig. 19-20 uses a sensitive relay with differential windings to check for trouble on a connecting lead. The resistance R is adjusted to match the winding resistance of relay (A), so that under normal conditions the currents through the two windings of (CK) are
equal. Almost any trouble conditions such as an open or ground or a cross to a similar circuit will upset the balance and cause (CK) to operate. This is a negative type of check since (CK) operates only when trouble is encountered.

A scheme using a differential relay which checks that two leads are always opened or grounded at the same time is shown in Fig. 19-21. This again is a negative check since relay (CK) operates only in case of trouble. This particular circuit is easily adapted to check the operation of the double-transfer lockout circuit* by connecting the relay to the ends of the two transfer chains. The check relay can be arranged to control associated transfer equipment which automatically activates an alarm and shifts all control leads to an alternative group of lockout relays in case of trouble. Relay (CK) checks the continuity of the output chain of the lockout circuit with one winding, and the continuity of the operating chain with the other winding. When the circuit is idle, both windings of (CK) are energized through the corresponding chain of break-contacts, but (CK) cannot operate because its windings oppose. If for some reason, such as a dirty contact or an open lead, either chain is open, (CK) is operated through the other. False ground on the operating chain will have the same effect. When the circuit is busy, on the other hand, both windings are normally de-energized. Relay (CK) now checks for false ground on the output chain or false battery on the operating chain.

When a group of leads is used to transmit information between two circuits, it may be desirable to check these leads for opens, grounds, and crosses to insure proper operation. An example of this is when the group of leads passes through several connector relays where troubles may develop. A basic circuit for making this check is shown in Fig. 19-22. Here the leads A1, A2, A4, and A5 are grounded by crosspoints

* See Chapter 15.
Circuits for Checking

on a crossbar register switch in the transmitting circuit to represent numerical digits in a 1-2-4-5 additive code (0 represented by 1 and 4). The levels of the crossbar register switch correspond to digits on the decimal basis. All leads not required for a particular digit value are connected by the switch to a common check lead CK1. For example, if the digit 3 is being transmitted, the A1 and A2 leads are grounded and

Fig. 19-22 Check for Grounds, Opens, and Crosses on a Group of Leads
the A4 and A5 leads are connected to the CK1 lead. Miscellaneous leads such as TP1 and TR for other information are handled in a similar manner. For example, the TR lead is grounded if the (TR) relay is operated. Otherwise it is connected through a back contact to the CK1 lead. All information leads terminate on register relays in the receiving circuit.

The circuit action is as follows: All register relays in the receiving circuit operate initially either from direct ground from the register switch or from ground on the CK1 lead supplied from a back contact of (CK4) in the receiving circuit. A make-contact chain on the register relays closes to operate (CK1), indicating that all register relays have operated. Relay (CK1) operates (CK4), which locks and removes ground from the CK1 lead. This permits all receiving register relays to release except those grounded in the transmitting circuit. The release of at least one register relay permits (CK1) to release, and this in turn causes (CK6) to operate, indicating that the check has been completed satisfactorily.

If any lead is open, the corresponding register relay in the receiving circuit will not operate and consequently (CK1) will not operate. A false ground or a cross is detected, however, only when it would cause trouble. Note that for any particular operation the connecting leads may be divided into two groups, those grounded in the transmitting circuit and those connected to the CK1 lead. If one of the leads connected to the CK1 lead has a trouble ground, this ground will be passed to all leads connected to the CK1 lead and will hold the corresponding receiving register relays after (CK4) operates. Thus all register relays will hold either to trouble ground or to legitimate ground from the transmitting circuit. This will prevent the release of relay (CK1), thus

![Diagram](image-url)

Fig. 19-23 General Circuit for Checking Output Leads
blocking the circuit. Trouble ground on one of the leads which is also grounded in the transmitting circuit, for example lead A1 or A2 when digit 3 is being transmitted, will not be recognized nor will it cause false information to be registered in the receiving circuit. However, different combinations will be transmitted on subsequent operations, and eventually the grounded lead will appear in the group on the CK1 lead and be detected.

In a similar manner, crosses between leads which are grouped on the CK1 lead at any given time are not detected and do not cause trouble. On some succeeding operation, however, the two crossed leads will fall in the two different groups. Ground applied on one lead by the transmitting circuit is then crossed to the CK1 lead and the trouble is detected. Note that an open, ground, or cross on the CK1 lead is also detected.

The same procedure may be used when the original information is registered on relays in the transmitting circuit. When the registration is in a different code than that to be transmitted over the connecting leads, the translating network on the transmitting register relays must ground the leads in the required combinations and connect all ungrounded leads to the CK1 lead. One procedure for designing the translating network is shown in Fig. 19-23. A network is developed to ground a particular lead for the required operation combinations of transmitting register relays. The negative of this network is arranged to connect the lead to the CK1 lead. Thus the lead is either grounded or connected to the CK1 lead. This procedure is repeated for each of the output leads.

PROBLEMS FOR CHAPTER 19

19-1 In order to provide single-error correction for digital information, identical information is transmitted independently to three registers. The three registers are compared and, in case of a discrepancy, the information on the two registers which agree is taken as correct. The information is recorded on relays in the four-element binary (1-2-4-8) code. Design a circuit which will:

1. Deliver correct output information in the binary code if no error occurs, or if a single error is present.
2. When a single error occurs, light a common alarm lamp.
3. When a single error occurs, indicate, by lighting one of three lamps, the register in which the error occurred; and indicate, by lighting one of four lamps, the designation of the relay which failed.

(This can be done with less than an average of 13 contact springs per relay.)
19-2 Design a checking arrangement for two relays which will indicate that one and only one of the two relays has operated and closed its locking contact. The circuit must give an affirmative indication by placing ground on an output CK lead before the operating ground for the relay being checked is removed.

19-3 A relay located at a distant point B, and having a local locking circuit, is to be controlled from a point A by means of a single lead (and ground return). A check circuit is required which will perform the following actions:

1. Upon closure of a path to ground at A, the circuit will check that the lead is not grounded and is continuous to battery supplied through the winding of the relay at B.
2. After making the above checks, the circuit will cause the relay at B to operate; and will then check that the lead is grounded, indicating that the relay has closed its locking contact.
3. A CK lead at A is to be grounded if and only if the above actions are completed satisfactorily.

(This can be done with two check relays.)

19-4 A 32-relay circuit is provided to register a set of digits. The circuit consists of six 5-relay digit registers, designated the A, B, C, D, E, F registers, and two relays designated 3D and 6D. Digits are registered in the two-out-of-five code. Legitimate operating conditions are a six-digit registration with the 6D relay operated; or a three-digit registration on the A, B, C registers (D, E, F registers normal) with the 3D relay operated.

Design a checking network to ground a DC (down check) lead when all 32 relays are released, and to ground an UC (up check) lead under the following two sets of conditions:

(a) A correct 2/5 registration in registers A, B, C; registers D, E, F released; relay 3D operated; relay 6D released.

(b) A correct 2/5 registration in registers A, B, C, D, E, F; relay 6D operated; relay 3D released.

(This network can be designed with no more than two transfers and one break contact per relay.)

19-5 A pair of relays, (X) and (Y), are to be controlled from a distance over a two-conductor loop, as illustrated in the figure on the facing page. When the loop is closed, relay (X) operates, operating relay (Y). Relay (Y), in operating, locks to off-normal ground and reverses the loop conductors to relay (X).

The circuit to be designed is to operate as follows: When the single make-contact on key K is closed momentarily, the circuit checks the loop conductors for correct polarity and absence of an open or crossed condition. If this check is satisfactory, the circuit closes the loop to operate relays (X) and (Y) at the distant end. The
check-and-control circuit then checks that relay \((Y)\) has operated by again investigating loop polarity and integrity. At the satisfactory completion of this latter check, the CK lamp should be lighted. The CK lamp is to remain lighted until the RLS key is operated, restoring the check-and-control circuit to normal. (This can be done with two additional relays).

![Circuit Diagram]
Chapter 20

CIRCUITS FOR REGISTRATION

In order to perform its actions properly, a switching system must receive instructions and collect information. Since any single item of information, in many cases, is not used immediately when it is obtained, it is necessary in switching systems to "remember," "record," "store," or "register" information. In this way, information obtained in the early stages of system action may be used in controlling later actions; and also, bits of information received at different times may be correlated for appropriate circuit actions.

This ability to store information is of primary importance in switching systems. For example, in dial telephone systems the digits of a dialed number may be received one at a time and stored until sufficient information has been received to start the actions of common-control equipment to set up a connection. Another obvious example of the need for information storage is in large-scale computing machinery, which requires both long- and short-time storage of numbers or other information. In these machines, semipermanent storage means are used for such data as input values, routine control instructions, tables of values which are occasionally required, and the final output results. Short-time storage means are provided to record temporarily intermediate results in a series of computations.

Circuit and apparatus elements used primarily to record items of information are called "register" elements. This chapter discusses register elements and various methods of placing information into, and subsequently taking it out of, registers.

20.1 TYPES OF REGISTER ELEMENTS

Information is usually recorded on register elements in the form of a code. The apparatus elements used for registration are similar to those used to represent elements of a code as discussed in Chapter 12. Register elements, however, have an additional requirement not imposed upon simple code-representing elements. This requirement is that the elements must be capable of remaining in an active condition (or "off normal" position) after the external activating signal or action has terminated.
A number of methods used for long-time storage of information produce records in a permanent or semipermanent form. For instance, there are systems that record information in the form of punched holes in paper tapes or cards. The information is read either by means of mechanical fingers which feel for the presence of holes and activate electrical contacts, or by light beams which control photoelectric cells through the holes. Other systems make use of magnetic recordings of electrical pulse signals on magnetic tape, wire, or rotating drums. Locating a particular item of information among a number of similar items with either of the above two methods requires that the reading machinery be able to scan through a number of items and have a means of recognizing the desired item when it is reached.

The teletype printer also may be mentioned as an information-storage device. These instruments are often used to make a printed record of the output information of a computing or accounting system.

All long-time storage systems in use at present require intricate mechanical apparatus and multifunctional relay or electronic control circuits. These will not be discussed further in this chapter. Short-time storage systems, however, use conventional types of switching apparatus in circuit arrangements similar in nature to other unifunctional circuits. The remainder of this chapter discusses registers of this type.

The general-purpose relay is a register element commonly used in switching systems. It may be operated from an external source and

<table>
<thead>
<tr>
<th>Register Element</th>
<th>Number of Positions</th>
<th>Typical Use</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Relay</td>
<td>Two</td>
<td>Operate</td>
<td>Locking</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Lock to ground</td>
</tr>
<tr>
<td>Rotary Switch</td>
<td>Multi-position</td>
<td>Step or run to particular terminal</td>
<td>Mechanical latch</td>
</tr>
<tr>
<td>Crossbar Switch</td>
<td>Ten</td>
<td>Close crosspoint</td>
<td>Lock the hold magnet</td>
</tr>
<tr>
<td>Vertical Unit</td>
<td></td>
<td></td>
<td>Remain fired on sustaining voltage</td>
</tr>
<tr>
<td>Gas Tube</td>
<td>Two</td>
<td>Fire</td>
<td></td>
</tr>
</tbody>
</table>

Table 20-1 Typical Register Elements
locked through its own contacts in a local circuit so that it will remain operated when the original activating signal is removed. It can indicate the registered information to other circuits by means of contacts, and it can be restored to normal and made ready for re-use by momentarily opening the locking path. Since a relay is a two-position device, a number of them must be used in combinations to represent extensive information. The relay and several other typical switching elements usable for registration are summarized in Table 20-1.

![Diagram of Register Circuit](image)

**Fig. 20-1 Use of Functional Register Circuits**

### 20.2 APPLICATIONS OF REGISTER CIRCUITS IN SWITCHING SYSTEMS

The block diagram of Fig. 20-1 illustrates the use in switching systems of register circuits whose only function is to store information temporarily. The register block consists of a number of register elements capable of storing a suitable amount of information. An information source obtains access to the input leads of a register, and places the information in the register where it is stored. At a later time a circuit, indicated by the block labeled "Reader" in Fig. 20-1, connects to the output leads of the register and obtains the stored information. In most cases the reader is a part of a multifunctional circuit which is able to make use of the registered information. The register will hold the information until the holding control circuit passes a signal over the hold lead to restore the register to normal. (This signal is usually the removal of holding ground.) The holding control is often an integral part of the over-all system control which is able to determine when the information is no longer needed.

In some systems, several information sources may supply information through connectors to a single register. Conversely, a single information source may supply information to several registers. In the most general case, there may be several information sources and several registers with a means for connecting them together in all combinations. The multiple symbols on the input connector leads in Fig. 20-1 indicate that these arrangements may be used.
A similar situation may occur on the output leads where either several registers or several readers or both may be used with suitable connecting means.

In many applications of the registration principle, the register relays are an integral part of a multifunctional circuit, and separate output connectors and readers are not required. In these cases, the register relays receive and record input information, while contacts on these relays direct the multifunctional circuit to perform the actions indicated by the stored information.

20.3 RELAYS AS REGISTER ELEMENTS

The general-purpose relay, when used as a register element, is arranged in some form of locking circuit. A simple arrangement is shown in Fig. 20-2A where the relay is provided with two contacts: one for locking and the other for delivering output information. The action of this circuit is obvious. Removal of the locking ground restores the relay to normal. When the output signal consists of ground on a single lead, a single make-make spring arrangement may be used for both locking and output, as shown in Fig. 20-2B.

Register relays are often used in large quantities, and consequently it is desirable to simplify the circuit as much as possible. The arrangement shown in Fig. 20-3A uses a single make contact both for locking and for output indication. This arrangement can be used where
the permanent connection between input and output leads does not cause undesirable circuit actions. When inexpensive types of relays not having twin contacts are employed as registers, a make-make spring arrangement with two independent contacts connected in parallel, as shown in Fig. 20-3B, is sometimes used for greater reliability.

The circuits of Fig. 20-2 and 20-3 permit ground from the holding path to be fed back on the input lead. This may be unsatisfactory in some cases. The locking ground back-up is eliminated by the circuits of Fig. 20-4. A continuity-transfer is used in Fig. 20-4A, while Fig. 20-4B employs a separate holding winding on the relay. In either of these arrangements, output information can be taken from the locking contact as indicated in the figure; or, if this is unsatisfactory, separate contacts can be provided.

For speedy operation of register circuits, the relays will often be chosen with low-resistance operating windings and correspondingly heavy operating currents. Where the stored information must be held for an extended period of time, this arrangement may lead to excessive current drain or overheating. Such a disadvantage can be overcome by the double-wound relay arrangement of Fig. 20-5A. A low-resistance operating winding is used, and the locking winding has a high resistance. The two windings may be used in series as indicated in Fig. 20-5A. A similar effect is accomplished by inserting a series resistance in the locking path as in Fig. 20-5B.
A final relay register circuit is shown in Fig. 20-6. This employs a marginal relay with more critical adjustment requirements than the relays of the previous circuits, but requires no contacts in the holding circuit. In this circuit the value of the resistance is chosen so that the normal current through the relay winding is below the non-operate value, but is sufficient to hold the relay in an operated position once it has operated. Application of ground to the IN lead causes the relay to operate and then hold until the circuit through the RLS key is opened. This arrangement is not as reliable as others, but it may be used with certain types of relays where only one make contact is provided and the circuit arrangement requires that the output must not be crossed with the input or locking grounds.

20.4 REGISTER INPUT CONTROL

Several precautions must be observed in the circuit arrangements for placing information into a relay register circuit. Most of these are concerned with the sequence of actions within the information source and the coordination of control between the input connector and locking circuits. The basic arrangement is indicated in Fig. 20-7. The information source grounds a combination of its output leads. These are connected to the register to cause corresponding relays to operate and lock. There are no circuit complications where speed requirements are not important and where the complete final information is available in the information source when the connector operates. However, the connecting and locking controls must be carefully coordinated in circuits where the information source may produce a sequence of output combinations before arriving at the final one, as in the case of a pulse-counting circuit, or where information from a previous use may still be
present in the information source at the time the circuits prepare to record new information. If the locking ground is present, wrong information must not be momentarily presented to the register relays, since falsely operated register relays will lock to the common locking ground. Either the connector must not be closed, or the locking ground must not be applied, until after the final combination of grounded output leads has been established in the information source.

If the connector is closed and the locking ground held open so that the register relays are free to follow the preliminary actions of the information source, a fast-acting circuit is possible as long as the control arrangement can insure that the locking ground is applied before the connector is released. This usually can be accomplished by means of a sequence spring arrangement on some key relay in the associated control circuits. The general scheme is indicated in Fig. 20-8. In the circuit action, the control relay operates when the final information is available and, by means of a make-before-break transfer, applies the register locking ground before releasing the connector. Simultaneously with the release of the connector, the information source may be restored to normal and prepared for use with another connector and register circuit.

With the arrangement of Fig. 20-8, the use of double-wound register relays is usually necessary to eliminate crosses between the operating and locking circuits. Suppose that the sequence of action within an information source is such that after the removal of ground from an output lead L1, ground is immediately applied to a second output lead L2. If single-winding register relays as in Fig. 20-7 are connected to the respective leads, they would be expected to follow the action on

Fig. 20-8 Control of Connector and Register Locking Ground
the leads if the locking ground path is held open. However, if the first register relay is slightly slow in releasing and opening its locking contact, the second may operate and connect the information ground on lead L2 to the common locking lead, thus falsely holding the first relay operated. This trouble is eliminated by using double-wound relays as in Fig. 20-8.

If the connection between the information source and the register is not made until the information source is in a stable condition, locking ground may be allowed to stand on the register locking lead, and single-wound relays may be used. When the information source is to be re-used, the connection to the register must be released. In general, some means must be provided to insure that the register relays have sufficient time to operate between the operation and subsequent release of the connector. This is often done by a timing arrangement in the connector control circuits. When information is registered in a self-checking code, variations of the scheme shown in Fig. 20-9 may be used. Information in this figure is delivered in a two-out-of-five code. When a digit is ready to be registered, the REG lead in the connector control is grounded, operating the connector relay. When the register relays have operated, a check-circuit ground causes the (CK) relay to operate, in turn releasing the connector. The (CK) relay may also start the next action in the circuit operation.

As an alternative to delaying the closure of connector relays, the information leads to the register may be disabled until the information source has stabilized. The information source often consists of a translation network on a group of relays; and in this case it is a simple matter to hold open the common ground lead by means of a single contact until the information relays are properly operated.
20.5 REGISTER OUTPUT CONTROL

Where register circuits are provided only for the purpose of storing information, some means must be provided for transferring information from the register to those circuits which will make use of the information. Often these circuits re-register the information on relays which control subsequent circuit actions. The process of obtaining information from a register is usually the simple act of closing a connector relay on the output leads of the register. In many applications, items of information such as numerical digits which have been stored in several registers on a digit-by-digit basis are read in a single action on the output leads. From the input side, the register relays form a number of single-digit registers, while from the output side they are equivalent to a single multidigit register.

In other applications, the information stored in a number of single-digit registers must be transmitted out, one digit at a time, on a common set of code leads. One method of accomplishing this is shown in Fig. 20-10. The connectors $C_1$, $C_2$, ... $C_n$ are closed one at a time in sequence to place the registered digit information on a group of common output leads.

An alternative method of transmitting registered digits in sequence is illustrated in Fig. 20-11. This does not require individual digit output connectors but, instead, uses translating networks on the register relays and a means for applying grounds to these networks in

![Diagram](image-url)
sequence. In the figure, the reading ground is advanced from one translator to the next by operating the reading-sequence control relays.

A particular design of translator is necessary for use in the arrangement of Fig. 20-11. If the output code requires the grounding of only one output lead at a time, a single input reading ground is sufficient. However, if two or more leads are grounded by the output code combinations, the number of reading grounds must be made equal to the maximum number of leads grounded in any one code combination. A translating network arranged to connect a single input lead to two or more output leads would connect these leads together, and this would cause one digit register to interfere with another through the multiplied output leads. For example, if a two-out-of-five code is used, two input leads are required; and, for each code combination, the translator must connect the two leads through separate paths to the two correct output leads. The reading-sequence control relays, then, must carry two transfer chains to advance two grounds from one register to the next. This arrangement with a suitable translating network is shown in Fig. 20-12.

An intermediate translator of the type discussed in Chapter 12 may be used to reduce the number of ground leads through the reading-sequence control relays. A single translator is located in the multiplied register output leads. An arrangement of this kind was shown as Fig. 12-14 in Chapter 12. The digits are registered on two contacts of cross-bar switch vertical units. Only two input grounds are provided from a
digit-sequence circuit, but intermediate translating relays permit three output leads to be grounded at one time.

20.6 ELECTRONIC REGISTRATION METHODS

Cold-cathode gas tubes are by nature "self-locking" devices and thus are adaptable as register elements. The general principle of using a gas diode tube as a register is illustrated in Fig. 20-13. A voltage which is below the firing voltage of the tube but which is sufficient, when acting through the load resistor, \( R_L \), to sustain the tube in a fired condition, is supplied between the cathode and anode. In order to register information, a positive input pulse of sufficient amplitude to fire the tube is applied to the anode. Fig. 20-13 shows the input circuit as a capacitor in series with a resistor, but the exact arrangement will depend on the nature of the input information source and the characteristics of the gas tube being used. Output information is obtained by observing the voltage at the anode. This can be done by other gas tubes suitably biased to fire if the register tube is in a fired (or unfired) condition. A single- or multi-tube register circuit can be restored to normal by opening the sustaining circuit path as indicated by the (RLS) relay in Fig. 20-13.

The various techniques of tube circuit design suggest many variations in the basic gas-tube register circuit. The load resistor may be
placed in the cathode circuit instead of the anode circuit so that negative input pulses applied to the cathode will fire the tube. The output voltage is then positive above ground for a fired tube, and zero for an unfired tube. In some applications the load resistance may be split to place resistance in both the cathode and anode circuits. The tube may be fired by either positive or negative pulses, and the output of a fired tube may be either a positive or negative change in voltage depending on whether connection is made to the cathode or anode.

Various types of multi-element gas tubes may be used as registers. For many applications a useful arrangement employs a gas triode. When the tube is fired by means of the starter anode, conduction is transferred to the main gap, and the output may be taken from a local resistor in the cathode circuit. This provides practical isolation between input and output circuits. An alternative output arrangement is to use the starter anode as a probe to obtain an output voltage from the main gap.

Vacuum tubes may be used as registers in switching systems when critical voltage or speed requirements are to be met. In general, two tubes are required to obtain a locking effect. The basic arrangement of the well-known Eccles-Jordan circuit, adapted to registration purposes, is shown on Fig. 20-14. The condition of (A) conducting and (B) nonconducting may be defined as a "normal" condition, and the opposite condition, with (B) conducting, as the "operated" or active condition. The circuit can be made to transfer from the normal to the operated condition by applying a positive pulse to the grid of tube (B) as indicated in Fig. 20-14. The output may be obtained from the plate circuit of either tube.

Since the circuit is stable with either tube (A) or tube (B) conducting, some means must be provided to insure that a particular tube conducts when power is first applied. The same scheme may be used to
restore the registers to normal after use. Tube (A) can be caused to
counter by momentarily opening the plate circuit of the (B) tube or by
applying a positive "reset" pulse to the grid of the (A) tube. Where
several Eccles-Jordan register circuits are used as register elements,
they may all be set by the same means. This is indicated in Fig. 20-14.

Under suitable conditions, capacitors storing an electrical charge
may be used as inexpensive register elements. They may be used as a
three-valued device by placing either a positive or a negative charge on
them and using the discharged condition as the normal condition. In-
creased reliability may be obtained, however, by using them in a two-
valued system (positive or negative charge), with the discharged condi-
tion being interpreted as an error or trouble condition. The chief
problems in using capacitors as registers is leakage resistance, which
causes charged capacitors to lose their charge slowly. In ordinary ar-
rangements, this means that relatively large capacitors must be used,
and extreme precautions must be taken to keep the leakage resistance
high. In general, this method of recording information is applicable
only to short-time storage.

Fig. 20-14 Basic Eccles-Jordan Circuit
PROBLEMS FOR CHAPTER 20

20-1 A circuit is required to register coded digits on two alternate sets of five relays. The digits are sent from a ten-button keyset (each key comprising two make-contacts) which grounds two-out-of-five leads per digit in the standard code. The first digit is registered and held in register #1. The second digit is registered and held in register #2. When the third digit is entered, register #1 is released and the third digit is registered therein. This procedure is followed with succeeding digits, registration alternating between registers #1 and #2. Beginning with the second digit, both registers are always occupied.

Each digit as entered must be checked for a legitimate code; and if the check fails, an alarm lamp must light and the circuit block. (This can be done with six relays in addition to the registers.)

20-2 Two registers, A and B, are each capable of storing any number from 0000 to 9999 on a biquinary basis. Each register contains 28 relays. A contact network controlled by the register relays is to be designed to ground one of three output leads as follows:

1. Lead GR is to be grounded when the number stored in register A is greater than that stored in register B.
2. Lead EQ is to be grounded when the numbers stored in the two registers are equal.
3. Lead LE is to be grounded when the number stored in register A is less than that stored in register B.

In designing the required network, assume that no errors will occur in the registration of the two numbers. (This can be done with an average of less than two transfers per relay)
Chapter 21

CIRCUITS FOR CALCULATING

Automatic digital computing systems incorporate some of the most complex and highly-developed forms of switching circuits. In deriving the desired mathematical results from data introduced into the mechanism, a computing system must proceed through an involved sequence of discrete operations. The exact sequence followed in the solution of a particular problem may depend not only upon the initial instructions set into the system, but also upon numerical conclusions obtained at various stages in the computation. As an illustration, it may be necessary for the computer, at some stage, to "decide" which of two subsequent routines to follow, depending upon whether a numerical result at this stage is positive or negative in sign. This situation appears in the determination of dividends and square roots, as discussed later in this chapter.

Although the design of a switching system which operates in this manner is necessarily complicated and requires a comparatively large amount of equipment, many of the circuit blocks which the system comprises are composed of simple circuits which have been considered in previous chapters. Even to one unfamiliar with automatic computing arrangements, it is evident that such unifunctional circuits as counting, translating, selecting, connecting, and registration should be applicable to a system for carrying out calculations.

In addition to these functional circuits, it is necessary to include in a computing system means for performing the basic arithmetic processes of addition, subtraction, multiplication, and division. Such circuits - circuits for calculating - are considered in the present chapter. In this chapter, the term "calculator" is applied to the relatively simple devices which perform the basic arithmetic operations. A "computer" comprises one or more calculators, information storage facilities, etc., and the control necessary to carry out, automatically, sequences of the basic operations. The term "computing system" refers to the large and complex mechanisms which may include several computers and their over-all control.

For the most part, the treatment in this chapter is confined to relay circuits. However, at the end of the chapter is a discussion of the design of adding circuits based on the use of electronic gate elements.
of the type considered in Section 10.2 of Chapter 10. This discussion is intended to illustrate the practical application of logical design methods to electronic circuits.

21.1 CODES

An extremely important consideration in the design of relay calculating circuits is the choice of the code to be used for representation of the numbers. It will shortly become clear that the most straightforward approach to a method of calculating with relay circuits is to treat the basic arithmetical operations as translation processes. Therefore the code to be used should easily lend itself to the types of translation that are required for addition, subtraction, etc. Related to this aspect of the situation in that it also involves translation is the facility with which input information can be converted to the circuit code, and the circuit code can be converted to output information. In general, input information and output information are most easily handled in decimal form; therefore the mutual translatability of the circuit code with respect to the decimal code (or whatever input-output codes are used) must be taken into account. A final consideration is the necessity for a high level of accuracy. To whatever extent is feasible and economical, circuit troubles must be prevented from producing output errors. A major step in this direction can be taken by requiring the use of one of the self-checking codes. The expense of self-correcting codes can probably only be justified when there is an extreme necessity to obtain continuous operation even in the presence of circuit troubles.*

When available codes are examined in the light of these factors, the choice becomes very much restricted. Straight binary-type codes are the most efficient when calculation is to be performed by two-valued devices, but translation of multi-digit input-output decimal numbers to and from a complete binary system is quite difficult. This can be remedied by use of the modified binary code in which each decimal digit is represented by an independent binary group of four elements. This arrangement, however, is not self-checking unless two elements are assigned to each binary place; when this is done, eight elements are required per digit and the economy of the code is lost. The straight decimal code requires ten elements per digit and,

<table>
<thead>
<tr>
<th>Decimal Digit</th>
<th>Biquinary Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>00—0</td>
</tr>
<tr>
<td>1</td>
<td>00—1</td>
</tr>
<tr>
<td>2</td>
<td>00—2</td>
</tr>
<tr>
<td>3</td>
<td>00—3</td>
</tr>
<tr>
<td>4</td>
<td>00—4</td>
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<tr>
<td>5</td>
<td>5—0</td>
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<tr>
<td>6</td>
<td>5—1</td>
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<tr>
<td>7</td>
<td>5—2</td>
</tr>
<tr>
<td>8</td>
<td>5—3</td>
</tr>
<tr>
<td>9</td>
<td>5—4</td>
</tr>
</tbody>
</table>

Table 21-1
The Biquinary Code

* There are other, more subtle, considerations which sometimes affect the choice of a code in particular circumstances. These will not be discussed in this chapter.
although self-checking, is inefficient in its use of apparatus. The two-
out-of-five code appears to be a likely candidate from several points of
view but it introduces considerable difficulty in setting up the translating
networks for calculating. A final code choice which meets necessary
objectives with a reasonable quantity of apparatus is the seven-element
biquinary code discussed in Chapter 12 and listed in Table 21-1 on the
preceding page.

This code converts easily with respect to the decimal system; it
permits relatively simple calculating networks; and it is self-checking,
since only one element in each of the bi- and the qui- portions of the
digit code is legitimately operated. Therefore the biquinary code will
be used in this chapter for the design of calculating circuits.

21.2 ADDITION

As an aid to the examination of the process by which the sum of
two numbers is determined, consider as an example the mental addition
of two three-digit numerals: 352 and 296. The following steps are per-
formed by the human mind, consciously or unconsciously, in reaching a
solution:

1. By referring to a permanently stored or memorized table of
addition, we find the sum of the digits in the "units" place, 2 and
6, to be 8. We memorize or otherwise store the sum, 8. Since 8
is less than 10 (the numerical base, or radix), there is no "carry-
over" to the addition of the digits in the "tens" place.

2. Again employing the stored or memorized table of addition, we
add the "tens" place digits, 5 and 9. The sum is 14, a number
larger than 10. We store the digit 4 and carry forward the digit 1
to be added with the digits in the "hundreds" place.

3. Finally, we sum the "hundreds" place digits, 3 and 2, plus the
1 carried from the addition of the "tens" place digits. We store
the summation digit, 6.

4. Now, the complete sum of 352 and 296 is found in the storage
facility as 648.

It is to be noted that, in each digital position, the summation digit
and the digit (0 or 1) to be "carried out" to the next position depends
upon the two digits to be added and the digit "carried in" from the
immediately preceding position. There are, in effect, three inputs (two
digits to be added and a carry-in digit); and two outputs (the summation
digit and a carry-out digit). This is illustrated in Fig. 21-1A.

The method of addition as described above is essentially a se-
quential process. In effect, no pair of digits can be added until carry
information has been obtained from the addition of the preceding pair.
However, it is possible to devise a method of relay circuit addition whereby all digits are added simultaneously and all output digits appear at the same time. This method is based, first, upon the use of translating networks on the input digit register relays to obtain the output digit and the carry-out information; and second, upon the provision of two carry-in leads, one of which is grounded for a carry-in of 0 and the other for a carry-in of 1. Each output digit lead and each of the two output carry leads is in effect connected to the two carry-in leads through separate translating networks. This is illustrated in Fig. 21-1B. Thus the correct carry output and the correct digit output appear immediately when the relays have operated and the carry-in signal has appeared. Since the carry signals depend only upon contact networks on the digital relays, the carry-in signal for each position is available as soon as the relays have operated. This is the method that will be utilized in the design of adding circuits in this chapter.

Fig. 21-2 Block Diagram of Biquinary Adder
Addition of biquinary numbers is essentially the same as decimal addition. The two two-part digits in each position are added together and to the carry-in digit, to give the summation digit and the carry-out digit. The two quiet parts of the digits in a position and the two bi-parts must, however, be added independently, making use of internal carry signals. The process of biquinary addition is illustrated by the block diagram of Fig. 21-2. Here again the two decimal numerals 352 and 296 are added together. In biquinary form, the A decimal number 352 is expressed as (00—3) (5—0) (00—2), and the B decimal number 296 appears as (00—2) (5—4) (5—1), where the sum of a biquinary pair is equal to the corresponding decimal digit. In the diagram, a carry of 1 or of 0 between decimal digit positions is indicated by a heavy C1 line or a heavy NC line, respectively.

The carry digit between decimal digits or biquinary pairs in the summation is the same as in decimal addition. However, inspection shows that the internal carry is 0 if the sum of the quinary digits is less than 5 (L5), and the carry is 5 if the sum of the quinary digits is 5 or more (5M). The carry between members of the biquinary pair is indicated in Fig. 21-2 by either an L5 line or a 5M line drawn heavy.

The block diagram of Fig. 21-2 can be considered as a scheme for mechanized addition. The two numbers to be added are placed in an A register and a B register, respectively. These two registers are divided into sections corresponding to the units, tens, hundreds, and so on, decimal digits, and these sections are joined by the carry leads NC and C1. The operation of addition is performed by contact networks on the register relays.

The basic design problem then reduces itself to the synthesis of a circuit for a typical digit block, for example, the T-digit block outlined on the figure. After this circuit is designed, the complete adder of Fig. 21-2 can be constructed from several of the basic circuits suitably connected by the carry leads. In order to achieve maximum possible speed of operation, the requirement will be introduced that only make-contacts are to be used in the adding networks*. In order to carry this requirement to its logical conclusion, the checking networks must also comprise only make-contacts. This can be done by the use of a double output check instead of the usual one-out-of-n check, whereby one output lead is grounded if at least one relay in a group is operated, and the second output lead is grounded (negative check) if more than one relay in the group is operated. The checking method will not be discussed further.

* Use of break-contacts on many types of relay increases the unoperated armature air-gap over what it would be with nothing but make-contacts, and thereby increases the operate time of the relay.
Circuits for Calculating

For the purposes of this chapter, the basic adder circuit is divided into four parts: the quinary-digit adder, the internal-carry network, the binary-digit adder, and the carry-out network.

The Quinary-Digit Adder. The input to the quinary adder must consist of two quinary digits, to be known as \( A_Q \) and \( B_Q \), and one of two leads grounded, NC or C1, depending upon the digit, 0 or 1, carried in from the preceding basic adder circuit. The adder output is the sum, the quinary digit \( C_Q \); and in addition, one of two leads, L5 or 5M, grounded, depending on the magnitude of \( A_Q + B_Q + C_{IN} \). Assuming that the digits \( A_Q \) and \( B_Q \) are denoted in standard form by the conditions of relays \( A_0 \), \( A_1 \), \( A_2 \), \( A_3 \), \( A_4 \), and relays \( B_0 \), \( B_1 \), \( B_2 \), \( B_3 \), \( B_4 \), respectively, the design of the quinary-digit adder involves the determination of a contact network on the (A-) and (B-) relays which will ground one of leads \( C_0 \), \( C_1 \), \( C_2 \), \( C_3 \), and \( C_4 \), indicating the sum of the \( A_Q \) and \( B_Q \) digits. The synthesis of a network to ground the L5 and 5M carry leads will follow in the next section of this chapter.

A table of possible operate combinations of the (A-) and (B-) relays, together with the desired C-lead grounded for each combination, appears in Fig. 21-3. The table represented by the upper left corners of the central blocks, that for \( NC_{IN} \) grounded, is simply the table of addition for \( A_Q + B_Q + 0 = C_Q \); and the table represented by the lower right corners, for \( Cl_{IN} \) grounded, is the table for \( A_Q + B_Q + 1 = C_Q \). For those combinations which result in a \( C_Q \) digit followed by a plus sign (+), as shown in the tables, it will be necessary to ground the 5M internal-carry lead since, in those cases, the sum of \( A_Q + B_Q \) plus the carry-in digit is 5 or more. For all other combinations, this sum is less than 5, so that lead L5 will be grounded.

---

**Fig. 21-3 Table for Quinary Addition**
The desired make-contact networks for the C-leads can be determined from inspection of the table of addition. For example, the table indicates that ground should appear on lead C3 for the following combinations: (A3 + B0), (A2 + B1), (A1 + B2), (A0 + B3), and (A4 + B4) when lead NCIN is grounded; and (A3 + B4), (A2 + B0), (A1 + B1), (A0 + B2), and (A4 + B3) when lead C1IN is grounded. The network controlling the condition of lead C3, then, may be as illustrated in Fig. 21-4A. Fig. 21-4B shows the same network in an abbreviated symbolism commonly used by calculator circuit designers. In this symbolism, a circled intersection indicates a make-contact on the relay whose designation appears near that intersection. A "dotted" intersection indicates a permanent connection, as in conventional diagrams. The complete contact network for quinary addition is shown in symbolic form in Fig. 21-5.

Carry-out Network for Quinary Adder. Depending upon the decimal sum of the two quinary digits, Aq and Bq, and the carry-in value, either lead L5 or 5M is grounded, as discussed in connection with the addition table of Fig. 21-3. The contact networks controlling L5 and 5M may be derived by inspection of the table.

Examination of Fig. 21-3 shows that, except for those sums lying in the squares on the diagonal from lower left to upper right of the
Circuits for Calculating

Fig. 21-5 Complete Contact Network for Quinary Addition

Fig. 21-6 Networks for "Less than 5" or "5 or More"
table, the grounding of L5 and 5M depends only on the values of $A_Q$ and $B_Q$, without regard for the conditions of leads NC$^\text{IN}$ and C$^\text{IN}$. The L5 and 5M contact networks corresponding to these combinations, then, need pass only through contacts on the (A-) and (B-) relays. On the other hand, contact networks corresponding to sums lying on the diagonal require control by the NC$^\text{IN}$ and C$^\text{IN}$ leads. For example, with ($A_3$) and ($B_1$) relays operated, L5 is grounded when NC$^\text{IN}$ is grounded, but 5M is grounded when ground is supplied to C$^\text{IN}$.

Suitable relay contact networks to control the internal-carry leads may be drawn by inspection of the table of Fig. 21-3. These contact networks are shown in Figs. 21-6A and 21-6B.

The Binary-Digit Adder. The function of the binary digit adder is to determine the sum of the binary digits, $A_B + B_B$, taking into account the carry information from the quinary adder, and also to ground one of two leads, NC$^\text{OUT}$ and C$^\text{OUT}$. The digits are stored in the A and B registers by the operation of relay ($A_{00}$) or ($A_5$) and relay ($B_{00}$) or ($B_5$), the subscripts 00 and 5 corresponding to the binary digits. The binary sum, similarly, appears on either lead C$^\text{00}$ or lead C$^5$. The carry-in digit from the quinary adder is either 0 or 5 depending upon the condition of leads L5 and 5M, respectively.

A table for binary addition, similar to that used for quinary addi-
tion, is first written down as shown on Fig. 21-7. The table is then used

---

**Fig. 21-7** Table for Binary Addition

**Fig. 21-8** Networks for Binary Addition and Carry-Out
in the same manner as the earlier table to develop the binary addition circuit sketched on Fig. 21-8A.

Carry-out Network for Binary Adder. The plus signs in the blocks of Fig. 21-7 indicate the combinations which must ground the $C_{1_{\text{OUT}}}$ lead, while the blocks without plus signs represent the combinations for grounding $NC_{\text{OUT}}$. Two of the digital combinations, that for $A_{00} + B_{00}$ and for $A_{5} + B_{5}$, ground $NC_{\text{OUT}}$ and $C_{1_{\text{OUT}}}$ respectively without regard to the value of the input carry leads, and therefore can be developed with local grounds. The remaining digital combinations must be taken in conjunction with the $L5$ and $5M$ carry inputs to produce the output carry signals. The complete carry-out network is shown on Fig. 21-8B.

Complete Adder Unit. To arrive at the complete contact network for a single stage of biquinary addition, it is merely necessary to combine the four component networks derived in the preceding paragraphs. This is accomplished as shown in Fig. 21-9. Recalling the over-all circuit action originally specified, the two numbers to be added are

---

Fig. 21-9 Complete Adder Network Unit for Single Biquinary Pair Summation
placed in biquinary registers, a single register unit for each decimal digit. The addition is performed by summing each corresponding pair of decimal digits. The contact network of Fig. 21-9, controlled by the register relays, carries out the summation of two biquinary pairs. A necessary supplement to this network is, of course, a contact arrangement to check that one and only one of the binary relays and one and only one of the quinary relays in each register unit are operated.

An adding circuit may be arranged to operate on numbers of any desired number of digits by connecting these single decimal-digit-adder
circuit units together, as indicated in block form in Fig. 21-2. The digit capacity of such an adder is limited by the number of connected adder units. The adder unit on each end of the complete adder circuit, of course, must be modified since there is a carry-in of 0, except in special cases discussed later in this chapter, to the unit farthest to the right (shown in Fig. 21-2 as a grounded NC lead), and no carry-out from the final, or highest place, unit.

The output C- leads grounded in accordance with the sum information A + B, effect the registration of the sum in some storage device as, for example, a relay register. However, this storage register may in itself be part of an adder circuit, so that the sum C may conveniently be added to another number D which has been recorded on the (D-) relays of a fourth register. The sum of C + D (actually, the sum of A + B + D) can then be made to operate the proper (B-) relays of the original adder. To this can be added a new number placed in the now cleared A register, and so on for multistage adding.

The block diagram of an adder system constructed on this plan appears in Fig. 21-10A. In this figure, control leads are shown dashed and leads carrying numerical information are solid. To simplify the figure, a single solid lead is used to symbolize the group of leads necessary to indicate each number.

As an aid to visualizing the principle of operation of this system, assume that the circuit is to perform the following summation: 352 + 296 + 75 + 116 + 127. The solution is carried out as shown in tabular form in Fig. 21-10B. It is evident from the figure that the process is composed of sequentially-controlled two-number additions. Numbers to be added are placed only in registers A and D, and sums appear only in registers B and C. Therefore, if the first number of the group of numbers to be added is placed in register A, it is necessary to prime, or prepare, register B so that a first summation number can appear in register C. This priming may be accomplished by automatically setting the number 00...0 into register B.

A second system for successive addition, among the many methods possible, is illustrated by the block diagram of Fig. 21-11A, and an example indicating the sequence of circuit actions appears in Fig. 21-11B. In this system, numbers to be added are always placed in register A, except for a priming number placed in register B at the outset of the addition process. In the example in Fig. 21-11B it is assumed that the circuit is primed with the second number to be added, rather than with a zero setting. The final sum is found in either register B or register C, depending on how many numbers are added.

Obviously, a method incorporating priming by a zero setting requires more steps in adding; and, as a result, the use of this method
The Design of Switching Circuits

produces a somewhat slower over-all circuit. However, there is a compensating simplification of the detailed circuit design and a saving in the total number of relay contacts necessary in the circuit.

21.3 SUBTRACTION

Subtraction, as normally carried out by human beings, involves the unconscious use of a memorized subtraction table similar in form to the table of addition discussed in the preceding section. Likewise, a relay circuit designed to perform subtraction in similar fashion could include a wired-in subtraction table. Such a circuit would directly translate two input digits $A$ and $B$ into a third digit $C$, where $C = A - B$. However, to conserve the number of circuits necessary in a calculator, the adder circuit can be made to carry out a subtraction process by the principle of "complementary addition", as discussed in the following paragraphs.

![Functional Block Diagram of a Second Representative Adder System and Adding Procedure](image)

**Fig. 21-11** Functional Block Diagram of a Second Representative Adder System and Adding Procedure
Consider the expression \( A - B = C \), assuming that \( A, B, \) and \( C \) are limited to, say, a usable maximum of six digits.* Adding \( 10^6 \) to each side of the expression:

\[
A + 1,000,000 - B = C + 1,000,000
\]
or

\[
A + (999,999 - B) + 1 = C + 1,000,000
\]

Therefore, \( C \) can be determined by adding \((999,999 - B)\) to \( A \), increasing this sum by unity, and finally discarding from the result the digit 1 in the seventh place to the left, if that digit appears. The quantity \((999,999 - B)\) is the so-called "nines" complement of \( B \), thus giving this method of subtraction its name.

It is immediately apparent that complementary addition should be easily applicable to the type of adder circuit already discussed. The only supplementary device which must be incorporated is a circuit to determine \((999,999 - B)\). Assuming for the moment that this "nines" complement of \( B \) can be set into one of the adder registers, the remainder of the process is straightforward. Unity can be added to the sum of \( A \) and \((999,999 - B)\) by grounding the \( C_1 \) carry lead to the units-place adder unit. The number \( C \), then, is found in the lower six digit places of the summation register.

In actuality, as indicated above, the number appearing in the summation register is \( C + 1,000,000 \), although the quantity \((+1,000,000)\) is discarded by neglecting the indication of the seventh-place adder unit. It is important, nevertheless, to provide a unit register in this seventh place. As seen, the seventh-place digit will be 1 if \( A > B \). However, if \( A < B \) and, as a result, \( C \) is negative, the seventh-place digit will be 0 since \((-C) + 1,000,000\) is less than \(1,000,000\). For this latter condition, the number recorded in the summation register is \((-C) + 1,000,000\) which is the "nines" complement of \( C \) plus unity.

Recapitulating then, if the seventh-place digit (or, in the general case, the highest place available in the system) appearing in the summation register is 1, the number \( C \) held by the remaining summation digit registers is positive. Similarly, if the seventh-place (or highest place) digit is 0, the number \( C \) is negative, and the number appearing in the summation register is the "nines" complement of this number plus 1.

The conversion from a number to its "nines" complement may be carried out by a simple cross-connection scheme on a per-digit basis. For example, the "nines" complement of 478263 is 521736; each digit of the complement is obtained by subtracting the corresponding digit of

* The same reasoning is applicable to whatever maximum number of digits is established as the calculator limit.
the number from 9, or, in effect, by cross-connecting 4 to 5, 7 to 2, 8 to 1, 2 to 7, 6 to 3, and 3 to 6. Since the biquinary, rather than the decimal, notation is used by the adder circuit, the cross-connection scheme must be on a biquinary basis. This cross-connection, for a single biquinary pair conversion, is shown in Fig. 21-12A; each biquinary pair in the number to be converted is passed through a similar network.

Referring now to the adder block diagrams of Figs. 21-10A and 21-11A, it is evident that nines complement conversion networks should be available to some, if not all, number-carrying leads. As has been seen, it may be necessary to convert a "difference" number, appearing in a register as a complement, from the complementary to the true form, as well as converting the "subtrahend" number to its complement before placing it in an adder register. So that a number may either be converted or not, as determined by the control mechanism, it may be convenient to provide two relays (K) and (T); relay (K) when operated performs the conversion; and relay (T), operated, serves to connect straight through. The final circuit, for a single biquinary pair, appears in Fig. 21-12B in symbolic form.

21.4 MULTIPLICATION

As in the case of subtraction, there are two methods of multiplication, either of which may be used as a basis for mechanized calculation. The first of these corresponds to mental multiplication, in which a multiplication table is consulted and partial products are added to obtain the final product. The alternative method is that of "repeated addition"; for example, to multiply 6 by 3, the digit 6 is added 3 times:
6 + 6 + 6 = 18. Although a circuit based on this latter method might seem to be unduly slow in obtaining a final solution, it is, on the average, fully as rapid as a circuit based on the first method. Moreover, the second method may be adapted more conveniently to modifications of the basic adder circuits.

Multiplication Employing a Relay Multiplication Table. To provide a framework on which to devise a system of mechanized multiplication, consider the procedure followed in solving a simple problem. For example, we shall multiply the multiplicand 847 by the multiplier 3. If our memory is poor, we first multiply 3 x 7 by consulting a table, and find the product, 21, which we write down. We then find 3 x 4 = 12, which product we write, shifted one place to the left, below the first product, 21. We determine the product of 3 x 8 from the table, and write this product, 24, again shifted to the left, below the earlier products. The sum of all products is the solution originally desired. This sequence of actions is indicated by Fig. 21-13A.

This plan may be followed almost exactly by a relay circuit, as shown by the block diagram of Fig. 21-13B, assuming that a relay multiplication table is realizable. This multiplication table may operate as a translator which functions according to the rule \( X \cdot Y = Z \), where \( X \) and \( Y \) are decimal digits and \( Z \) is a number composed of a tens and a units decimal digit.
Employing this relay multiplication table, then, the product of $3 \times 7$ is obtained, and is placed in the A register of the adder. The product of $3 \times 4$, shifted one place to the left,* is placed in the B register, and the product of $3 \times 8$, doubly shifted, is set into the D register. The summation of the contents of the A and B registers appears in register C. As a final circuit action, the number in the D register is added to the number now in the C register; the sum obtained in register B is the complete product of $847 \times 3$.

Although the plan just discussed is a workable scheme, it is somewhat slow, requiring three sequential operations of the relay multiplication table and two steps of addition for the problem treated. A more rapid system can be devised at the cost of additional relay multiplication tables which operate simultaneously. A block of this second system appears in Fig. 21-14B. Here two adder registers are provided; in one is placed a number composed of the units digits of the outputs of

* When any number is shifted to the left, the vacated digital positions to the right must be filled with zeros.
the multiplication tables, and in the second is placed a number composed of the output tens digits. The corresponding mathematical procedure is shown in Fig. 21-14A. Note that the digits placed in the B register correspond to "carry" digits.

As stated above, the relay multiplication table circuit appearing as a block in Fig. 21-13 and 21-14 is a translator. Although a circuit could be designed to multiply biquinary pairs directly, a less complicated arrangement results from the design of a relay table on a decimal basis. If the latter table is employed, the two decimal digits at the output of the table are translated to pairs of biquinary digits which, in turn, are placed in the conventional adder registers.*

The design of the multiplication translator is straightforward. The final circuit obtained may be of the form shown in Fig. 21-15, in which the input multiplicand digit is introduced by operation of one of the (R-) relays, and the (M-) relay operated corresponds to the multiplier digit. The two digits of the product are transferred to the adder registers, the units digit to the A register and the tens digit to the B register, as indicated by the diagram of Fig. 21-14.

The (M-) relays may be of a multicontact type so that several multiplication tables can be constructed on a single set, each table having its own group of (R-) relays. Thus, in the example 847 x 3, the multiplier relay (M3) and the multiplicand relays (R8), (R4), and (R7), in tables 1, 2, and 3, respectively, are operated, the (M3) relay controlling contacts in all three tables.

The block diagram of Fig. 21-14 may be expanded into an arrangement which can handle multipliers of any desired number of digits, as shown in Fig. 21-16A. This circuit obtains partial products by multiplying the multiplicand by each digit in the multiplier (as in the preceding paragraphs), and then adding these partial products to arrive at the final product. It is, of course, necessary to shift the partial products depending on the position of the corresponding multiplier digit.

As an illustration of the operation of this circuit, the steps involved are shown graphically in Fig. 21-16B, where 435 is multiplied by 279. The first operation is that of obtaining the product of 435 x 2 = 0870, accomplished exactly as in the basic block diagram of Fig. 21-14. This partial product, 0870, appears in the C register. Then, the partial product 435 x 7 = 3045 is similarly obtained in the D register. The next step directly transfers the partial product, 0870, from the C register to the A register, and transfers the partial product, 3045, from the D register to the B register, shifting the former product one place to the left. The sum of these products, 11745, appears in the C register. The last partial product, 435 x 9 = 3915, is set into the D register. The

* An alternative might involve adder registers arranged for decimal addition.
The contents of the C and D registers are again transferred to adder registers A and B (shifting the sum of the products, 11745, one place to the left) and the total sum, the desired product $435 \times 279 = 121365$, appears in the C register. It is evident that this system is applicable to numbers of any limited number of digits.

In the method of multiplication already discussed, it has been necessary to shift certain numbers one place to the left before placing them into adder registers. This can, of course, be accomplished by connecting input leads carrying the units digit to the tens-digit relays of the register, the tens-digit leads to the hundreds-digit relays, and so on. However, in those cases where it is desired to carry out a multiplication of the form $A \times 100 \ldots 0$ it would be convenient to pass the
number A through a variable-shift circuit, rather than proceeding sequentially as in the example above.

The variable-shift circuit, illustrated in Fig. 21-17, is a logical solution to the problem. In this figure, single leads are employed to represent groups of digit-carrying leads. When relay (SO) is operated, no shift takes place; the input leads carrying digit X are connected to the output digit X leads. The operation of relay (S1) causes the input number to be shifted one place to the left; the input (X-1) digit leads being connected to the output X digit leads, and so on. Similarly, operation of relay (S2) shifts the input number two places to the left.

This variable-shift circuit may compose the block labeled "shift" in Fig. 21-16A. Thus, each time a number is transferred from the C register to the A register in the illustration of Fig. 21-16A, the (S1) relay in the shift circuit is operated to produce a one-place shift to the left. Now, assume that the multiplier is, for example, 2074, and the multiplicand is 4356. The first partial product, 4356 x 2, can be shifted two places to the left when it is transferred from the C register to the A register, thus, in effect, obtaining the partial product of the double multiplication, 4356 x 20, in a single step. It would be desirable to insert a variable-shift circuit in the transfer path from the D register to the B register, also, to provide a means for automatically multiplying by 10 the products appearing in the D register.

Fig. 21-17 Variable-Shift Circuit
Multiplication by Repeated Addition. The second general method of multiplication, that of repeated addition, lends itself readily to relay circuitry. On referring to the adder block diagrams of Figs. 21-10 and 21-11, it is obvious that, to obtain the product of, say, 7 x 4, it is only necessary to pass the column of digits 7 + 7 + 7 into the adder registers and take out the sum. In particular, for the system of Fig. 21-11, the digit 7 may be placed in register A and left there until the four additive steps have been completed.

The control of the process is equally simple — at least theoretically. The multiplier digit 4 is placed in the control circuit where it may set a "count-down" counting circuit.* This counting circuit, then, can limit the number of adding steps taken by the adder, and halt its operation when the last stage of the counting circuit has acted.

* See Chapter 11.
Numbers of more than one digit can be multiplied in similar fashion. For example, consider the process of obtaining the product of 37 x 64. First, the partial product of 37 x 4 is obtained by summing 37 + 37 + 37 + 37 = 148. To this result, 148, is added the partial product of 37 x 6 = 222, similarly procured, remembering that the partial product, 222, must be shifted one place to the left. The detailed sequence of operations is indicated by Fig. 21-18A, assuming the use of the basic adder system of Fig. 21-11. Note that the second partial product, 222, is shifted by shifting the multiplicand 37 in the A register after the first partial product of 148 has been determined. Also note that this second partial product never appears as such.

Evidently, this plan of operation requires more steps for multiplication by a digit equal to or greater than 5 than for a digit less than 5. However, a so-called short-cut method of multiplication can be used to reduce the number of steps required for multipliers of 5 or larger. Referring again to the multiplication of 37 x 64, it can be seen that the partial product of 37 x 6 can be determined in a different manner from that used above, as indicated by the expressions to follow:

\[ 37 \times 6 = 37 \times (10 - 4) = 37 \times 10 - 37 - 37 - 37 \]

Stated verbally, the partial product 37 x 6 can be obtained by subtracting multiplicand 37 a number of times equal to the "tens" complement of the multiplier digit 6 and adding 1 to the multiplier digit next to the left of the digit 6. Since, in the present case, the multiplier digit to the left of 6 is 0, the 0 becomes 1, and it is necessary to determine the partial product of 37 x 1 as a final step. Fig. 21-18B shows the complete procedure using the short-cut method. For clarity, indications of the particular registers involved are omitted. Steps involving subtraction are, of course, accomplished by complementary addition, already discussed. Two of these steps occur in the illustrative problem: one, 148-370, immediately after the first partial product has been obtained; and one at the conclusion where 1232 is subtracted from 3700. All the remaining steps are simple addition in conjunction with position shifts of the multiplicand after each partial product is obtained.

Since the adder registers operate on a biquinary basis, it is easy for the circuit to determine when a particular multiplying digit requires the short-cut (or subtraction) method. When the (−00) binary relay for a multiplier digit is operated, straight repeated addition is indicated since that digit is less than 5; when the (−5) binary relay is operated, the short-cut method is used since the digit is 5 or greater. With the short-cut method the average number of sequential steps is about 2.5, or about half the average number of steps necessary if only repeated addition were employed.
21.5 DIVISION

In accordance with the close relationship between multiplication and division, there are two procedures for division corresponding to the two multiplication procedures already discussed. The first makes use of a relay multiplication table, and the second employs the principle of repeated addition or, in actuality, repeated subtraction.

Division Employing a Relay Multiplication Table. In ordinary long division the two processes employed are multiplication and subtraction, in correct sequence. For an illustration, consider the division \( \frac{238}{7} \). This is commonly carried out as follows:

\[
\begin{array}{c}
34 \\
7 \overline{238} \\
-21 \\
\hline
28 \\
-28 \\
\hline
00 \\
\end{array}
\]

As indicated, the divisor 7 is multiplied by 3, and the product (shifted one position to the left in this case) is subtracted from the dividend 238. Then, from this remainder, 28, is subtracted the product of the divisor multiplied by 4. The remainder, or difference, of this second subtraction is zero, and thus the quotient is complete. The quotient is given by the multiplier digits used in the process.

The determination of the proper multipliers, 3 and 4, in the example above, is accomplished mentally by an almost intuitive sense. This intuition is gained by experience in the use of the multiplication table. However, since it is not convenient to build intuition into a relay circuit, a circuit for division must find the correct multiplier, in each individual case, by some systematic method.

One system of multiplier determination is to try the decimal digits sequentially until the correct one is found. In particular, to find the first digit of the quotient, the divisor may first be multiplied by 5 and the product subtracted from the dividend. If the difference obtained is negative, successively smaller digits are tried until a positive difference appears. If, on the other hand, the difference after the multiplication by 5 is positive, the digits 9, 8, 7, and 6 are tried in sequence until a positive difference again results. This process is repeated for the determination of each digit of the quotient, as indicated by the example of Fig. 21-19, where 874 is divided by 23. If the quotient is non-terminating,* the process may be repeated as far as desired, limited only by the storage capacity of the system.

* As an example, the quotient of \( \frac{30}{14} \) is non-terminating.
Since the control of a circuit constructed on this plan of operation is somewhat involved, the corresponding block diagram is not illustrated in this chapter. Obviously, however, the functional block diagram of such a circuit would include the multiplication circuit, adder registers (set for complementary addition), and storage registers in which to place the quotient digits.

\[
\begin{array}{c}
874 \div 23 \\
23 \times 5 = 115 \quad 874 \\
\underline{-115} \\
-92 \\
23 \times 4 = 92 \quad 874 \\
\underline{-92} \\
3 \\
23 \div 874 \\
-69 \\
+184 \\
23 \times 3 = 69 \quad 874 \\
\underline{-69} \\
+184 \\
23 \times 5 = 115 \quad 184 \\
\underline{-115} \\
+ \\
23 \times 9 = 207 \quad 184 \\
\underline{-207} \\
- \\
23 \div 874 \\
-69 \\
184 \\
-184 \\
000
\end{array}
\]

Fig. 21-19 Division by Method of Trials

**Division by Repeated Subtraction.** Division by repeated subtraction is similar to the method just described except for the process of determining quotient digits. Here, rather than employing a trial-and-error procedure, the divisor (suitably shifted) is repeatedly subtracted from the dividend, or the remainder, until the difference obtained is zero or negative. The number of subtractions resulting in positive differences is the desired quotient digit.

Suppose, for example, that the quotient of 1504 \( \div \) 47 is to be found by the repeated subtraction method. As shown in Fig. 21-20, the divisor 47, suitably shifted to the left, is subtracted from the dividend three times before a negative difference is found. The least positive difference, that resulting from the third subtraction, is the remainder. The divisor, shifted one place to the right of its previous position, is repeatedly subtracted from this remainder, in this example two times, until a zero or negative difference is found. Since in Fig. 21-20 the second subtraction results in a difference of 0, no further quotient digit exists. As in the previous method of division, the process can be carried out as far as desired for lengthy or non-terminating quotients.
Circuits for Calculating

21.6 SQUARE ROOTS

It is possible to conceive numerous methods of obtaining square roots by relay circuitry; specifically, one obvious approach would involve the use of a relay table of square roots which could be "consulted" as desired. Another method is, however, found to be preferable in practice.

As a preliminary to the discussion of this method, consider the table of squares in Table 21-2. This table of squares indicates that the square root of the square of an integer may be found by a form of repeated subtraction. To find \( n \), given \( n^2 \), the odd integers 1, 3, 5, and so on, are subtracted in order from \( n^2 \) until the over-all difference is zero. The number of subtractive steps carried out is

<table>
<thead>
<tr>
<th>( n )</th>
<th>( n^2 )</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>1 + 3</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>1 + 3 + 5</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>1 + 3 + 5 + 7</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>1 + 3 + 5 + 7 + 9</td>
</tr>
</tbody>
</table>

Table 21-2
Table of Squares
equal to \( n \). As an illustration, this process is carried out in Fig. 21-21A where the square root of 25 is determined. A mathematical justification of this method is given in Appendix I, Section 21.8 of this Chapter.

In order to reduce the total number of repeated subtractions necessary for the determination of the square roots of large numbers, the system just described can be modified so that the root may be found by a succession of comparatively short sequences of repeated subtractions. This procedure is illustrated by Fig. 21-21B in which the square root of 1169.64 is found.

The first sequence of repeated subtractions is performed with the subtrahend digits (1, 3, 5, 7, ... ) shifted to the left an odd number of places, this number of places being equal to, or one less than, the number of digits to the left of the decimal point in the number of which the root is desired. The repeated subtractions are carried out until one
additional subtraction would give a negative difference. In this example, three steps of subtraction are necessary, resulting in a remainder of 269.64; the first root digit is 3, the number of subtractive steps.

The last used subtrahend, in this case 5, is then increased by unity and shifted one place to the right. This number, 6, in conjunction with the digits 1, 3, 5, ... in the place immediately to the right of this number, is subtracted sequentially from the remainder 269.64, as shown in Fig. 21-21B. The subtractions are performed until, as before, the smallest positive difference is obtained. In the example, this remainder is 13.64, reached after four subtractive steps. Thus, the second root digit is 4.

The process continues as already discussed. Unity is added to the last used subtrahend, 67, and the result is shifted one place to the right. The digits 1, 3, 5, ..., each combined with this result, are subtracted in order from the remainder 13.64. Here, after two steps, a remainder difference of zero is arrived at, concluding the process. The complete root, then, has been found to be 34.2.

A justification for this method of determining square roots is found in Appendix II, Section 21.9 of this Chapter. Although the example employed above concludes in a final remainder of zero, and hence is a terminating root, the same process is applicable to non-terminating roots. In the latter case, the determination of root digits may be continued as far as desired (within the capabilities of the registers), the total number of subtractive sequences being equal to the number of root digits.

21.7 DESIGN OF AN ELECTRONIC BINARY ADDER

Calculating circuits to perform the functions discussed in the preceding sections can, of course, be designed using electronic elements. In general, the principles applicable to relay circuit design are also applicable to electronic circuit design. In order to illustrate this, and, in addition, to provide an example of the application of switching algebra to electronic circuit design, a circuit for the addition of two binary digits will be constructed of electronic "and" - "or" gates of the types discussed in Section 10.2 of Chapter 10. In applying switching algebra, it will be remembered that "and" and "or" gates correspond respectively to the operations of addition and multiplication, and the negative relationship is achieved by driving a gate opposite to its indicated electrical polarity.

The required circuit is to handle the addition of the digits in one position of two multi-digit numbers. Multi-digit numbers may be added by providing a number of adders equal to the number of digits in the larger of the two numbers. Each circuit will have two inputs, A and B, for the digits to be added, and a third input, C<sub>IN</sub>, for the carry-in digit
from the next less significant place. Two outputs will be provided: one, designated \( C \), for the sum; and one, designated \( C_{OUT} \), for the carry-out to the next higher significant place. An active input signal has + polarity, and the same polarity will be required for an active output.

The gate elements that can be used in the solution are restricted to vacuum tube and varistor or rectifier devices, where the former type provides gain for the signal and the latter introduces signal loss. It will be presumed that the input signals to the circuit appear at full level and that the output signals must be delivered at comparable strength. This implies the requirement that the output gate elements must be vacuum-tube devices.

Algebraic manipulations of the initial mathematical statement of the circuit permit converting the circuit into a wide variety of equivalent forms. The particular form chosen depends on several considerations, including the physical realizability of the gate elements that may be implied by the final equations, the power required at the output, the types of gates preferred, etc. The following can be taken as a set of guiding rules for the solution of the present problem.

1. The relationships among small groups of variables should be made as uniform as possible throughout the equation in order to permit re-using the same gate for each of several appearances of the same related groups of variables. For example, if the expression can be manipulated into a form where the term \( A + B \) appears repeatedly, the same \( A + B \) gate element can be used for each appearance of the term. In this regard, if the term \( A + B \) appears in one factor and the term \( A'B' \) in another, either term, as convenient, can be written in its negative form to make combination possible. Thus \( A'B' \) can be expressed as \( (A + B)' \) and the same gate element can be used in both cases. If the sign of the normal output of the gate represents \( A + B \), the negative signal \( (A + B)' \) or \( A'B' \) can be obtained by connecting the gate to an inverter or, more simply, by changing sign at the input to the succeeding gate. This was discussed in Chapter 10.

2. The appearances of operational symbols (addition and multiplication, or + and \( \cdot \)) should be minimized as much as possible since each symbol represents a gate. However, this is modified by the results obtained from the manipulations suggested by the first rule.

3. Where possible, gates whose inputs and outputs are all of the same polarity should be utilized, since this form can be realized with inexpensive varistor elements. However, since this type of gate introduces signal loss, no more than two varistor gates in series should be used before use of a vacuum-tube gate.
4. Gates with inputs of opposite polarity should be avoided where possible since they are difficult to realize physically. However, if they do occur, use of a signal inverter in one of the input leads can convert the gate form to a more desirable type. Where necessary in the solution of the present problem, signal inverters will be used.

5. In the case of any gate, the output polarity can be assigned as desired, either to make the gate a particular type or to obtain a particular polarity of input for a succeeding gate.

The first step in developing a circuit is to tabulate the input-output combinations as follows:

<table>
<thead>
<tr>
<th>INPUTS</th>
<th>OUTPUTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Digits</td>
<td>Carry-In</td>
</tr>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 21-3
Input-Output Combinations for Binary Adder

Since the table shows numerical values, the interpretations of 0 and 1 are opposite to those used in switching algebra; 0 represents the passive condition and 1 represents the condition when a signal is present. The four input combinations producing an output sum of 1 may be written from the table as:

\[ f(C) = (A + B' + C'_{IN})(A' + B + C_{IN})(A' + B' + C'_{IN})(A + B + C_{IN}) \]

As a first simplifying step, this can be converted to:

\[ f(C) = \left[ C_{IN} + (A + B)(A' + B') \right] \left[ C'_{IN} + (A + B')(A' + B) \right] \]

Each bracketed factor contains summation terms of A and B. The terms in either factor can be put into a form comparable to the terms in the
other factor so as to permit use of common gates. Since the second bracketed factor contains the terms in the form which implies gates with inputs of opposite polarity, it will be manipulated into the general form of the first bracketed factor. Hence,

\[
f(C) = \left[ C_{IN} + (A + B)(A' + B') \right] \left[ C_{IN} + AB + A'B' \right]
\]

\[
f(C) = \left[ C_{IN} + (A + B)(A' + B') \right] \left[ C_{IN} + (A' + B')' + (A + B)' \right]
\]

The A-B terms within the parentheses in the second bracketed factor are now identical to the similar terms in the first factor, and all four terms can be realized with just two gates, one for \((A + B)\) and one for \((A' + B')\). However, the relationship between these terms differs in the two factors. This can be taken care of by rewriting \(f(C)\) as follows:

\[
f(C) = \left\{ C_{IN} + (A + B)(A' + B') \right\} \left\{ C_{IN} + [(A + B)(A' + B')]' \right\}
\]

This equation is the final algebraic form of the summing circuit of the binary digit adder. Inspection shows that the circuit can be realized with six gates interconnected as shown on Fig. 21-22. In this figure, the gate input-output polarities that are known at the start are shown without parentheses. If the attempt is made to assign the remaining polarities so that as many gates as possible can be of the varistor type (same polarity inputs and output), the signal values shown in parentheses result. This establishes the two "or" gates as the undesirable "unlike-input" type. This difficulty can be remedied by changing the polarity of one input lead to each of these gates either by use of a different type of preceding gate or by use of signal inverters. The results achieved by both methods are indicated in Fig. 21-23, where (VT) and (VAR) within a gate symbol represent vacuum-tube type and varistor type respectively, and (INV) represents inverter. The inverter may be a transformer if pulse signals are employed, or a triode amplifier for d-c or pulse signals. Which of the two circuits is preferable depends upon the conditions of use of the circuit, and the costs of the

![Fig. 21-22 Basic Summing Circuit for Electronic Binary Adder](image-url)
specific apparatus. In this case, it will be assumed that the circuit of Fig. 21-23B will be used. It is interesting to note that in either case the circuit is composed of two identical sections.

The carry-out network must now be designed and integrated with the summing circuit. From Table 21-3, it is seen that the circuit equation is:

\[ f(C_{OUT}) = (A + B + C'_{IN})(A + B' + C_{IN})(A' + B + C_{IN})(A + B + C_{IN}) \]

This reduces to:

\[ f(C_{OUT}) = (A + B)(C_{IN} + AB) \]

or \[ (B + C_{IN})(A + BC_{IN}) \]

or \[ (A + C_{IN})(B + AC_{IN}) \]

Of these three equivalent circuits, the first is closest in form to the summing network \( f(C) \). The similarity can be improved by an additional manipulation which will permit re-use of gates in Fig. 21-23. The final form is:

\[ f(C_{OUT}) = (A + B) \left[ C_{IN} + (A' + B')' \right] \]

This can be combined with the summing network to give the complete
The Design of Switching Circuits

circuit of Fig. 21-24. This circuit can be constructed from five varistor "and" gates and three pentode "or" gates, together with three signal inverters which may be transformers or triodes depending upon circumstances. It will be recognized that the circuit of Fig. 21-24 is but one of a variety of equivalent circuits, any of which can be developed by algebraic manipulations similar to those used in this section.

In high-speed electronic computers, binary numbers are often represented in serial form by a sequence of pulses. The time interval for each digit is accurately controlled by a series of "clock" pulses generated by a master oscillator. If the value of a digit is 1, a pulse is allowed to pass into a lead carrying the number of which this digit is a part; if the digit is 0, the pulse is suppressed. The least significant digit is transmitted first in time sequence. A single binary adder unit, of the type already designed, may be used in conjunction with a time-delay unit to add binary numbers of any number of digits in this serial form. The arrangement is indicated in Fig. 21-25. The carry-out signal is passed through a delay network or other device which causes it to be delayed a time equal to the interval between successive clock pulses. Thus the carry from one digit arrives at the input at the same time as the signals representing the following digit of the numbers being added. The sum of the two numbers applied to the A and B inputs then appears as a sequence of pulses on the sum output lead.
21.8 APPENDIX I: \( n^2 \) AS A SUM OF FACTORS

By a simple manipulation it may be shown that the square of the number \( n \) can be written as: \( n^2 = (n - 1)^2 + 2n - 1 \)
and, therefore, \( n^2 - (n - 1)^2 = 2n - 1 \)

Stated verbally, this latter expression indicates that the square of an integer \( n \) minus the square of the next lower integer* equals the difference (2n - 1). Thus the square of \( n \) may be expressed by a summation of all these differences taken from 1\(^2\) to \( n^2 \):

\[
 n^2 = \sum_{1,2,3,\ldots}^n (2m - 1)
\]

But \( \sum_{1,2,3,\ldots}^n (2m - 1) \) is merely the sum of all odd integers from 1 to (2n - 1), and therefore \( n^2 = 1 + 3 + 5 + 7 + \ldots + (2n - 1) \).

Note that the number of terms in \( \sum_{1,2,3,\ldots}^n (2m - 1) \) is equal to \( n \).

21.9 APPENDIX II: DETERMINATION OF SQUARE ROOTS
BY REPEATED SUBTRACTION†

Assume that the desired square root is a three-digit decimal number, \( 10^2A + 10B + C \). The square of this number is

\[
 10^4A^2 + 2 \cdot 10^3AB + 10^2B^2 + 2 \cdot 10^2AC + 2 \cdot 10BC + C^2.
\]

The first sequence of subtractive steps involves the subtraction of \( 10^4 \cdot 1, 10^4 \cdot 3, 10^4 \cdot 5 \ldots \) in order until the smallest positive difference is obtained. Carrying out this sequence:

\[
 10^4A^2 + 2 \cdot 10^3AB + 10^2B^2 + 2 \cdot 10^2AC + 2 \cdot 10BC + C^2
\]

\[
\begin{align*}
-10^4 \cdot 1 \\
-10^4 \cdot 3 \\
-10^4 \cdot 5 \\
\ldots \ldots \ldots \ldots \ldots \\
-10^4 \cdot (2A - 1)
\end{align*}
\]

First Remainder = \( + 2 \cdot 10^3AB + 10^2B^2 + 2 \cdot 10^2AC + 2 \cdot 10BC + C^2 \)

* The expression is not limited to integers, but the method discussed here involves only integral values of \( n \).
† Discussion of this method is included in Section 21.6 of this chapter.
This is the smallest positive difference, since the next subtrahend in order would be \(10^4(2A + 1)\); and \(10^4(2A + 1)\), as may be shown, is larger than the remainder above. The digit \(A\) is now known, since it is equal to the number of subtractive steps in the sequence.

The last-used subtrahend, \(10^4(2A - 1)\), is modified by adding \(10^4 \cdot 1\) and dividing by 10 (the equivalent of a shift of one place to the right). This constant factor, plus \(10^2 \cdot 1, 10^2 \cdot 3, 10^2 \cdot 5, \ldots\) taken in order, is subtracted from the first remainder:

\[
2 \cdot 10^3 AB + 10^2 B^2 + 2 \cdot 10^2 AC + 2 \cdot 10BC + C^2
- \left[ 2 \cdot 10^3 A + 10^2 \cdot 1 \right]
- \left[ 2 \cdot 10^3 A + 10^2 \cdot 3 \right]
- \left[ 2 \cdot 10^3 A + 10^2 \cdot 5 \right] = - \left[ (2 \cdot 10^3 A)B + 10^2 B^2 \right]

\]

Second Remainder = \(+ 2 \cdot 10^2 AC + 2 \cdot 10BC + C^2\)

The number of steps in the second sequence is \(B\), and the second remainder is the smallest positive difference, since the next subtractive factor in order would be \([2 \cdot 10^3 A + 10^2 (2B + 1)]\), a factor which may be shown to be larger than

\([2 \cdot 10^2 AC + 2 \cdot 10BC + C^2]\).

The final group of subtrahends is obtained by adding \(10^2 \cdot 1\) to the last-used subtrahend in the second sequence, \([2 \cdot 10^3 A + 10^2 (2B - 1)]\) and dividing by 10. This constant factor plus 1, 3, 5, \ldots taken in order, is subtracted from the second remainder:

\[
2 \cdot 10^2 AC + 2 \cdot 10BC + C^2
- \left[ 2 \cdot 10^2 A + 2 \cdot 10B + 1 \right]
- \left[ 2 \cdot 10^2 A + 2 \cdot 10B + 3 \right]
- \left[ 2 \cdot 10^2 A + 2 \cdot 10B + 5 \right] = - \left[ (2 \cdot 10^2 A)C + (2 \cdot 10B)C + C^2 \right]

\]

Third Remainder = 0

Since the number of subtractive steps in this final sequence is \(C\) and the
remainder is 0, the complete root, \(10^2A + 10B + C\), has now been determined.

By extension it is apparent that the above process holds for numbers with any number of digits.

**PROBLEMS FOR CHAPTER 21**

21-1 An adding circuit is to be designed to determine the sum of two five-digit binary numbers, A and B, as shown in block form below. Each of these numbers is represented by ground or absence of ground on five input leads, ground on a lead representing the binary digit "1" and absence of ground representing the binary digit "0". The sum, C, of A + B is to be indicated in binary form on a group of six output leads in the same fashion. (This can be done with ten relays.)

![Adder Circuit Diagram](image)

21-2 Design a circuit to meet requirements similar to those of Problem 21-1, except that the output number C should be the difference of numbers A and B rather than their sum. It may be assumed that A is normally greater than or equal to B. However, if B should be greater than A, an alarm lead should be grounded.

In devising a circuit plan, do not use the method of complementary addition.

21-3 Design a check network for a biquinary register to indicate that one and only one relay in each of the bi- and qui- parts of the register is operated. Use only make-contacts for this circuit, and distribute them so that all qui- relays have the same number of contacts. This circuit was discussed on page 464 of this chapter. (This can be done with 24 make-contacts.)
Chapter 22

GENERAL PRINCIPLES OF MULTIFUNCTIONAL CIRCUIT DESIGN

A multifunctional switching circuit, as the name implies, comprises any circuit entity that performs more than one circuit function. This broad definition covers almost all circuits that are designed for practical application, since in general the single-function circuits discussed in the preceding chapters are too restricted in nature to be used except in combination. In size and complexity, multifunctional circuits cover the range from large, immensely complicated organisms, such as telephone switching systems or computing systems, down to relatively simple arrangements for automatic control of a machine. However, no matter at what point in the scale a multifunctional circuit falls, it is clear that design techniques enter a new phase over and above what has already been discussed. The question that immediately arises is, what design techniques can be formulated that will be applicable to multifunctional circuits in general.

In view of the diversity of functions and applications of switching circuits as a class, there is undoubtedly no specific design technique that is valid for all types of circuit. The nature of the subject is such that almost every design project is unique to some degree and requires creative effort in its solution. The natural result of this is that what is a straightforward and useful method in one case is completely unproductive in another. However, a valuable and universally applicable approach to the design of multifunctional circuits can be enunciated and, if followed intelligently, will yield highly useful results. This approach can best be understood by first analyzing critically the general nature of well-designed circuits, where the term "well-designed" implies an orderly, logical, and straightforward solution.

Thus, any switching system can be analyzed circuitwise in terms of interrelated groups of functional blocks, where each block represents a single or closely associated set of single-function switching circuits. There may be intermediate breakdown levels between the system and the circuit blocks, the levels depending, to a great extent, upon the nature and size of the system. A large system, for equipment or utilization reasons, may consist of several major units, each of which is a complex multifunctional circuit. An analysis of a system of this type would show not only the relationship of the major units with each other
and their environment, but also the functional block co-ordination within each unit. A small system, on the other hand, might comprise no more than a few functional blocks related to each other and to the environment. Viewed in this light, the apparatus units such as relays, taken individually, are very subsidiary components of the system. The primary unit of importance is the single-function circuit block.

The correct approach to multifunctional circuit design is to some extent the inverse of the analytical operation. From a statement of circuit requirements, a functional plan is developed in terms of known or conceptually evident circuit blocks, representing simple circuits similar to the single-function circuits discussed in the preceding chapters. As the design proceeds, the functional blocks are coordinated and integrated to the point where a comprehensive block diagram of the proposed circuit exists. Only at this late stage, with certain reservations, is the detailed design in terms of apparatus components started. It is this method of approach that will be discussed in the present chapter.

It is beyond the scope of this volume to pursue the study of multifunctional circuit or system design exhaustively or in all its ramifications. For example, the subjects of apparatus design and equipment design, which are intimately related to circuit design, will not be touched upon at all. The many practical aspects which enter into commercial circuit design will only be hinted at. The heart of the matter, from the circuit designer's point of view, will necessarily be treated in very broad and general terms in this chapter. However, in order to illustrate the method concretely, a specific example of the design of a typical multifunctional switching circuit will be presented in the next two chapters. Beyond this point, the designer must rely on experience, common sense, and imagination.

22.1 CIRCUIT REQUIREMENTS

The design of any circuit is built upon a statement of requirements. The initially available requirements may range from a broad general account of the purpose of the circuit to a detailed listing of input conditions and the specific actions that must be performed. If the circuit is an entity in itself or belongs to a system which is in the later stages of development, precise requirements are usually available and can be applied directly to the design of the circuit. On the other hand, if the circuit is part of a system whose development has not fully crystallized, it is rarely possible to determine detailed requirements which relate a particular circuit to the others with which it must work. This is particularly true when the design of system components is distributed among several people. The most that can be expected in this situation is a general statement of the job that must be performed by the circuit
and the nature of the input information that will be available. As the designs of the several circuits of the system advance concurrently, the output conditions of one circuit become the input conditions of another, and specific requirements develop as each designer determines his needs and what he can furnish. If the job falls in this category, the designer may have considerable freedom of choice in specifying his own requirements, but suffers from the penalty that his circuit must often be changed to meet new conditions in associated circuits. On the other hand, when requirements can be rigidly stated, there may be a disadvantage of inflexibility in designing a circuit to meet the fixed conditions. Although new and improved methods occur to the designer, they may involve changes in associated circuits that cannot be accepted.

The status of requirements, then, varies from job to job. In many cases, the only explicit requirements given to the designer concern the general functions of the circuit. Thereafter the designer must ferret out for himself the mass of detailed requirements dealing with input conditions, limits, output conditions, time relations, necessary precautions, etc. These requirements fall into two categories: those imposed by external conditions over which the designer has little control; and those that develop from the circuit plan, from the form of the circuit itself, and from apparatus and equipment considerations. The first category of requirements must be collected and co-ordinated as completely as possible as the initial step in circuit design. The second category usually appears only during the course of circuit design.

When the requirements are known, the job of developing a multi-functional circuit divides into four principle stages, two of which constitute planning and the other two the detailed design. The first stage is the devising of a scheme or method of performing the required functions; the second stage is the embodiment of the scheme in a block diagram form in which the blocks represent interrelated unifunctional units that are known or are realizable; the third stage is the detailed design of the component circuit blocks and their co-ordinating circuit paths to give a schematic representation of the circuit plan. In the final stage, certain practical and commercial considerations must be taken into account involving the addition of apparatus codes, contact numbers, contact protection, etc., to the circuit.

22.2 UNDERLYING FACTORS

Before discussing these four stages of design in more detail, several underlying factors which the designer must always keep in mind should be mentioned. Stated in brief, circuits should be designed in such a way that they: (1), meet the functional requirements; (2), are reliable; (3), are economical with respect to manufacture, installation, and life;
and (4), are easy to maintain. The emphasis placed upon the latter three items is, of course, dependent upon the application of the circuit, the conditions of use, and the number of units to be manufactured.

The first of these items requires no discussion. The second, reliability, may have a profound effect upon the circuit design. It implies immediately a good basic design with no built-in hazards. Beyond this, the degree of reliability required depends on the job that the circuit has to do and its relative importance in the over-all system. The reliability of a circuit, particularly as it involves checking features added to the basic circuit, must take into account factors of economy and maintenance. A circuit can be rejected from a cost standpoint because of an excess of checking features just as surely as it can be discarded for unreliability. As in so many things, the designer must strike a happy balance. The relationship with maintenance arises from the fact that checking features should be arranged to give adequate indications of trouble conditions in addition to guarding against false operation. This can be taken care of by designing the check circuits so that their manifestations are clear to a maintenance man, and by tying them into appropriate alarm devices and trouble indicators or recorders.

This whole matter of reliability is also associated very closely with apparatus. For example, different types of relays are in current use that have single contacts exposed to air, twin contacts exposed to air, and mercury contacts sealed in glass. The probability of an open contact due to dirt decreases rapidly with each successive type. It is obvious that factors such as this should react strongly upon the manner in which a circuit is designed.

There are many ways in which the designer can influence the economy of his circuit. The first is in the efficiency with which he makes use of the apparatus. It is axiomatic, of course, that good design is not wasteful of relays, contacts, extra windings, or any of the component apparatus from which circuits are built. However, an attempt for economy should not drive the engineer to such tight design that maintenance expenses in the field may be unduly increased.

Choice of types of apparatus is another factor in the cost of circuits. Consistent with reliability and adequate life, the design should be based on the least expensive apparatus. In relay circuits, for example, the designer should attempt to use high-production relays and avoid special types of relays. In this same respect, apparatus that requires large maintenance effort to keep in adjustment should be used only where absolutely necessary.

In addition to first cost, the factors of service life and maintenance affect the over-all cost of a circuit. It is worthwhile to make original provision for such items as contact-protection networks and
more expensive contact metal, entirely aside from the question of reliability, if the service life can be extended appreciably. However, efforts along these lines must be keyed in with similar accomplishments in the rest of the system. A circuit designed for fifty-year life in a system built for thirty-year life is certainly questionable.

A more subtle aspect of circuit design economy has to do with the manufacturability of circuits which are to be produced in quantity. For a given large number of relays, manufacturing costs are less if the over-all circuit can be broken up into small independent units, capable of mass production, than if the circuit must be handled as one big unit. Small units are adapted to assembly line construction, wiring, and testing techniques, whereas large cumbersome frames of relays definitely are not. This implies segregated functional design as opposed to design in which all functions are so inextricably interrelated that it is impossible to divide the circuit into units.

From the standpoint of maintainability, if a circuit is designed in a straightforward and logical manner, is reliable, utilizes apparatus simple to adjust and replace, and gives adequate indications of trouble that may occur, it will offer little difficulty in service.

22.3 THE FOUR STAGES OF DESIGN

After this discussion of the background factors of design, the four stages in the development of a multifunctional circuit can be covered more intelligently. The four stages, to repeat, comprise: (1), devising a scheme or method; (2), planning in terms of a functional block diagram; (3), designing the detailed circuit paths and configurations; and (4), applying the practical adjuncts such as contact protection, apparatus codes, and designations. The four stages will be discussed as consecutive steps, although in practice the designer may find it necessary to jump back and forth during the course of development. For example, the first or second stage sometimes depends upon a new circuit concept which may or may not be valid. There is no point in blindly continuing the circuit planning before the new concept has been demonstrated to be feasible by working out at least part of the circuit in some detail. Also it will very frequently happen that new or modified ideas will occur, during the later stages of design, that require changes in the basic plan. Therefore, the following outline should not be construed as a fixed and immutable mode of attack, but rather as a general procedure which can be modified by circumstances.

The first stage of design, the devising of a scheme, offers the same difficulties to description as any other creative act. Proficiency depends upon knowledge of the fundamentals of circuit design, familiarity with many so-called unifunctional circuits and their practical
applications, and understanding of apparatus capabilities and shortcomings. Superimposed upon all these characteristics, the designer must be able to utilize this knowledge in translating the requirements of a job into circuit terms. Often the basis of a solution will be a simple but new circuit configuration around which can be woven a multiplicity of well known circuit concepts to build it out to complete form. Sometimes the scheme may involve nothing more than a new relationship among standard functional circuits. Perhaps the designer can merely adapt a previously used circuit to his new requirements. In this case, of course, most of the design phase of circuit development is eliminated.

The second stage of circuit design, developing a functional plan or block diagram, can be described in more concrete fashion. The process consists primarily of elaborating the basic scheme, in terms of blocks representing circuit functions, to the point where all requirements excepting those that affect only circuit details are satisfied. As far as possible, no detailed design should be attempted at this stage. The designer should draw heavily upon his knowledge of what can be accomplished by simple aggregations of basic circuit elements, such as relays. In particular, he should attempt to think in terms of unifunctional circuits of the type discussed in the preceding chapters. Individual circuit elements such as relays are in themselves too limited in performance to permit broad planning of a circuit.

The ultimate objective of this planning stage is a block diagram in which each block represents a single-function circuit, and on which all important interrelationships between blocks are shown. When this objective has been achieved, there exists a framework which vastly simplifies the detailed job of designing the circuit paths and adapting known unifunctional circuits to meet the stated requirements. The alternative, which consists of building the circuit relay by relay, is much too inefficient to be practical.

The most satisfactory approach to developing a block diagram is to start with a few main subdivisions of the over-all circuit and successively break these down until each block represents a unifunctional circuit. The chief guide during this process is the set of known requirements which must be kept constantly in mind. In a surprisingly large number of cases in the planning of switching circuits, familiar functional circuits are found to be applicable. When a new circuit concept is encountered, the designer can usually recognize whether an appropriate circuit can readily be designed. If this is so, the circuit can be designated on the diagram and the design deferred until later. However, in some cases it may be necessary to perform a certain amount of detailed design immediately to insure that the concept is feasible. In some cases, the planning may have to be re-oriented to permit a practical solution.
becomes difficult to understand the circuit, or to recognize what functions a given part of the circuit is performing. This is of particular importance when people other than the designer must build and maintain the circuits, since extra time spent in testing and maintenance may cost more over a period of time than the original price of a few additional relays. However, these considerations are modified by the intended use of the circuit. Circuits which are to be manufactured in large quantity are very sensitive to cost. Furthermore, circuits of this type are usually relatively simple in nature, and the doubling up of functions does not overcomplicate their understanding. On the other hand, with larger and more intricate circuits such as the major components of computer or telephone switching systems, more emphasis should be placed on straightforward design. This should, of course, not be construed as an excuse for wasteful design in these large circuits.

During the detailed design, the designer must continually check his circuit against all the original requirements plus all the additional requirements that inevitably develop from the particular circuit method he is using. All conceivable combinations of circumstances which might affect the circuit operation must be considered. Such matters as release under all normal and trouble conditions, guarding against attempted use when in trouble, reaction to trouble in other circuits, etc., are important. The degree and type of checking and trouble-indicating features, as discussed previously, should be carefully worked out. Finally, all limiting, marginal, and race conditions must be investigated to insure that adequate working margins are available.

When the circuit has reached the stage on the drawing board where it theoretically meets the requirements, there are usually many final engineering details to be worked out. Although it is frequently necessary during the detailed design stage to choose particular apparatus for parts of the circuit, much apparatus will be of a general-purpose nature where the code selection can be deferred. A major part of the final stage of circuit design, then, consists of making the choice of apparatus to fit the particular needs of the circuit. Among the factors to be considered are speed of operation, contact metal on relays, voltage and wattage limits, tolerance ratings, and magnetic, electrostatic, and vibration interference with certain types of apparatus. The designer is often required to engineer circuits for some minimum life, perhaps ten, twenty, or forty years. This necessitates a calculation of the number of operations of such apparatus as relays, and the provision of adequate contact protection.

Another item that is normally deferred until the final stage of design is the fusing of the circuit. This should be arranged so that there is adequate protection against the dangerous effects of trouble-crosses and short-circuits, and yet unnecessary complications should not be
introduced into the circuit wiring. Where the circuit is part of a large system, the fusing should be arranged, if possible, to minimize the disruptive effects of a single fuse failure upon the functioning of other parts of the system.

An associated aspect of the general design problem is laboratory testing. When commercial production is involved, it is usually desirable to set up a test model of a large, complicated, or critically important circuit. Rigorous testing of the model under the worst probable operating conditions often discloses defects or errors in the original circuit which should be changed before production. Testing may also indicate the need for improvements in operating conditions or desirable changes in requirements.

The discussion in this chapter has been intended to highlight the general principles of multifunctional circuit design. It is obvious that the major portion of circuit design involves creative effort, for which it is always difficult to establish specific rules and procedures. For this reason, the present discussion will be implemented by an illustrative example of the correct approach to a design problem and a typical circuit solution, to be given in the next two chapters. The primary emphasis is upon the creative aspects of design. That is, the text will deal with the development of theoretical circuits in the sense that particular apparatus will not be specified, and little or no mention will be made of what has been designated the fourth stage of design.
Chapter 23

PLANNING A MULTIFUNCTIONAL CIRCUIT

In this chapter and the next, the methods outlined in Chapter 22 will be applied to a specific design problem to illustrate both the method and the underlying principles. The treatment of the problem is divided into two main parts: the planning of the circuit, covered in this chapter; and the detailed design work, discussed in the next chapter. The planning phase of the job will include: determining all the requirements of the circuit; laying out general and tentative block diagram solutions to the problem; and finally breaking down the general block diagram to specific functional form which satisfies the general and detailed requirements. In the final form, each block will represent as far as possible a clearly definable circuit function which can be treated with reasonable independence during the detailed design stage.

As was mentioned in the preceding chapter, the design method is very much subject to the individuality and experience of the designer. Given the same design problem, two designers will often plan along completely different lines and produce equally good functional block diagrams. Thus, in the attack on the problem as presented in this chapter, the various steps taken are not necessarily the only logical ones. The particular approach, rather, is one which will best illustrate the method in general.

23.1 STATEMENT OF THE PROBLEM

The example chosen for illustration is part of an automatic telephone switching system and is called a "Dial Pulse Register Circuit*. This circuit is well defined and, except for a simple input connection and one set of output conditions, does not depend upon other circuits of the system for its operation.

The first step in the approach to the problem is to express the purpose of the circuit. In this case, the purpose can be broadly summarized as follows: to receive pulse signals representing four digits from a telephone dial or similar calling device; to keep a record of the digits; and, when the fourth digit has been received, to make the numerical information available in a form suitable for use by an external circuit.
Before an exact statement of the requirements to be met by the
circuit can be made, the conditions under which it must operate and the
actions which it must perform must be determined. First consider the
input conditions which convey information to the circuit. The input to
the register circuit is delivered over a pair of wires which is con-
nected to a telephone line. In the telephone system this is not a perma-
nent connection, but is established through connector circuits at the
time the subscriber wishes to place a call. The design of these circuits
is not a part of the present problem, and their exact nature is unim-
portant since the register circuit does not function until after this con-
nection is made. At the subscriber's end of the telephone line is a tele-
phone set which contains a dial. The telephone set is arranged so that,
when the handset is in the cradle, an open circuit exists between the
pair of line wires. When the handset is lifted, contacts operate to place
the voice transmission equipment in the circuit in such a manner as to
form a d-c current path between the two line wires. Contacts on the
dial, which are normally closed, are included in this path. When the
dial is operated, these contacts produce pulses equal in number to the
number being dialed. Each pulse consists of an open interval in the d-c
path between the line conductors. The rate of these pulses is nominally
ten per second, and the open interval of each pulse is approximately
one-twentieth of a second.

The input signals transmitted from the telephone set to the reg-
ister will consist of an initial d-c circuit closure between the pair of
input leads at the time the circuit is connected to the telephone line.
This will be followed by trains of dial pulses corresponding to the
dialed digits. The pulses for one digit are separated from the next by a
closed interval which is longer than the interval between successive
pulses. In the present case it will be assumed that the dialed number
will consist of four digits. When dialing is completed, and after the
registered information is no longer needed, the input circuit will be
opened for an extended interval. The circuit must then restore to nor-
mal. Also, in case the subscriber abandons a call at any stage of the
dialing by replacing the handset, thus opening the input circuit, the reg-
ister circuit must restore to normal.

One further input signal condition must be recognized. This is the
"preliminary pulse" which is sometimes produced if the handset is
lifted from its cradle in an irregular manner. This may cause the con-
tacts in the telephone set to first close and then momentarily open,
producing a signal similar to that produced when dialing the digit "1".
To guard against this effect producing a wrong number, the digit "1"
will not be used as the initial digit of a four-digit number, and the cir-
cuit must be arranged to "absorb" preliminary pulses by ignoring all
single pulses which may be received before a digit of two or more
pulses has been dialed.
In response to the input control signals described above, the circuits must perform certain definite actions and produce certain output signals. A tone signal (dial tone) must be applied to the line conductors at the time the circuit is first connected to the line, as a signal that the circuit is ready and dialing may be started. To avoid confusion to the subscriber, this signal must be removed during the time the first digit is being dialed. The circuit must count the pulses as they are received and record each digit in a suitable code. After the fourth digit is received and recorded, a start signal must be produced which will indicate to associated circuits that all expected information has been received. The entire four-digit number must then be available in coded form on output leads which can be connected to some other circuit as needed.

23.2 DETAILED REQUIREMENTS

Before the circuit planning can be started it is necessary to state in some detail the exact requirements which must be met. These will include the electrical nature and the time characteristics of the input signals to which the circuit must respond, and will give the actions which must be performed and the nature of output signals which must be produced. Many of the requirements can be deduced from the general discussion which has been given. For others it will be necessary to examine the environment in which the circuit is to work. In the present case it will be assumed that the necessary investigations have been made and that the following requirements have been determined:

R1. The circuit shall respond to signals transmitted from a calling device (dial) in the following manner:

R1.1 The input path extending to the calling device will consist of a pair of wires or a "loop" which may vary in length over a considerable range. The resistance of the maximum length, however, will be low enough to insure that a suitable relay can operate in series with the loop.

R1.2 The calling device, when not in use, will hold the loop open.

R1.3 The calling device will close the loop to indicate a request for the use of the register circuit.

R1.4 The calling device will open the loop for an extended period to indicate that release may take place.

R1.5 Each train of pulses from the calling device will consist of from one to ten regularly spaced open or break periods of equal length in the closed loop, with the intermediate closed periods equal in length to the open periods.
R1.6 The speed of pulsing may vary between ten and fifteen pulses per second.

R1.7 The closed loop intervals between trains of pulses will be sufficiently long to be clearly distinguishable (from a circuit standpoint) from the closed loop periods during the pulsing of a digit.

R2. The circuit shall have a capacity to receive and store a four-digit dialed number.

R3. A circuit-ready signal (dial tone) must be transmitted to the calling telephone and must be removed from the loop during or at the end of the pulsing of the first digit.

R4. The circuit must disregard a single pulse for the first digit (preliminary-pulse absorption).

R5. The signal to an associated circuit when four digits have been received shall consist of a ground placed on a "start" lead.

R6. After four digits have been received and registered, the dialing of a fifth or any subsequent digits must not disturb the previously registered digits.

R7. The circuit must be under continuous supervision from the calling device so that, if a call is abandoned at any stage of the dialing, the circuit will restore to normal.

R8. The recorded digits must be made available to an associated circuit on four groups of leads, one group for each of the four digits. The digits must be indicated by grounding the leads of a group in the 1-2-4-6 additive code given in Table 23-1.

In addition to the above requirements, there are certain implied requirements such as that the circuit shall be as economical as possible (consistent with reliability), operate on available power supply, and so on.

### Table 23-1

<table>
<thead>
<tr>
<th>Digit</th>
<th>1-2-4-6 Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1,2</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>1,4</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
<tr>
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<td>1,6</td>
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<td>2,6</td>
</tr>
<tr>
<td>9</td>
<td>1,2,6</td>
</tr>
<tr>
<td>0</td>
<td>4,6</td>
</tr>
</tbody>
</table>

23.3 PRELIMINARY PLANNING

After a study of the purpose and requirements of the circuit a general plan can be developed. The ultimate objective of this planning is to develop the block layout to the point where the individual blocks represent circuit actions for which an applicable circuit is known or can readily be conceived. As the first step in visualizing the circuit, the block of Fig. 23-1, can be drawn representing the circuit as a whole with input and output connections indicated.
From the statement of the problem and the requirements it is recognized that the digit information, coming in over a loop as four sets of pulse signals, must be received and registered in time sequence. These digits must be transmitted out simultaneously, as ground signals on four separate groups of output leads. This immediately suggests that four separate circuit blocks, one per digit, may be used together with a sequence or steering circuit. Therefore the circuit block of Fig. 23-1 can be elaborated to that of Fig. 23-2. Each individual internal circuit must be capable of counting a train of pulses and storing the digit.

This is a familiar circuit function (counting, Chapter 11) and can be indicated on the diagram immediately. A counting circuit can count a train of pulses and, at the conclusion of the pulse train, hold its relays operated in a combination which indicates the number of pulses in the train. Presumably all four circuits are identical. However, this raises the question of how the circuit can tell when the pulses for one digit end and those for the following digit start. From requirements R1.5 and
R1.7 it is evident that the input condition is the same during the intervals between pulses of a train as during the intervals between digits (closed loop) with the only difference one of time, the interdigital time being longer. It seems obvious, then, that some time-measuring scheme must be used to distinguish one digit from another.

Before proceeding with this study, it might be wise to examine the input loop condition both during the dialing of a single digit and during the progress of the entire call. The situation during the dialing of a digit "5" is shown in the sequence diagram of Fig. 23-3A. The loop is closed prior to dialing, and it opens and closes intermittently during dialing until five opens have appeared; whereupon the loop closes until the next digit starts. In Fig. 23-3B is shown the loop condition during a complete call. The loop is open until the calling device requests service by closing the loop. Four pulse trains follow, each similar to that of Fig. 23-3A, and eventually the loop opens to indicate termination of the call.

The "counting-storing circuit" block of Fig. 23-2 can now be broken down into smaller functional units. In the first place the calling device or dial is located at the far end of a subscriber line which will have a varying loop resistance from one line to another. It does not seem advisable to depend on the dial contacts over the loop to control directly the relays of a counting circuit, particularly since counting circuits often involve some type of series or shunt relay action. Therefore it appears that a separate pulse-repeating relay is desirable whose sole function is to detect and follow dial pulses. This is shown as a block in Fig. 23-4A, and is labelled "pulse repeater".

This arrangement can detect and count the pulses in a single train, but cannot determine when the train has ended. A method of accomplishing this has been discussed (Chapter 18) and consists essentially of a slow-release relay which operates on the first pulse of a train and holds between succeeding pulses until the longer interdigital
interval occurs. This is shown as a block labelled "pulse-train detector" in Fig. 23-4B.

Another detail of the counting-storing block can be recognized and included in the block diagram at this time. This is a translating network which will be required on the counting relays to translate to the specified 1-2-4-6 output code. This is included in Fig. 23-4B.

Having now determined in more detail the nature of a counting-storing circuit block, the block diagram of Fig. 23-4 may be modified for further development of the circuit. The steering or sequence circuit must be provided to connect the input path to each counting-storing circuit at the proper time. A sequence circuit of the type discussed in Chapter 11 is suitable for this purpose and can be controlled to advance the input from one counting-storing circuit to the next by the pulse-train detector output signals of each counting-storing block. This is shown in Fig. 23-5A.

Another method of handling this problem might be to arrange each counting-storing circuit to transfer the input path to the next succeeding

Fig. 23-4 Expansion of Counting-Storing Block
Planning a Multifunctional Circuit

circuit at the end of each digit as shown in Fig. 23-5B. However, this does not appear as clear-cut as Fig. 23-5A, and will not be considered further.

Returning to Fig. 23-5A, a circuit must now be supplied to provide the "supervision" implied by requirements R1.3, R1.4, and R7. This supervisory circuit will respond when service is requested, maintain

![Diagram of Digit Steering with Sequence Circuit](image1)

**DIGIT STEERING WITH A SEQUENCE CIRCUIT**

![Diagram of Digit Steering with Transfer Circuits](image2)

**DIGIT STEERING WITH TRANSFER CIRCUITS**

*Fig. 23-5 Methods for Digit Steering*
supervision over the call, and hold operated each counting-storing circuit after its digit has been received and until disconnection takes place. This obviously must be common to all the counting-storing circuits and must be under control of the line or loop condition.

If the supervisory circuit, designated "supervisory holding circuit," is included, and the block of Fig. 23-4B is visualized within each of the counting-storing blocks of Fig. 23-5A, the diagram of Fig. 23-6 can be drawn, representing a workable plan (neglecting some details such as dial tone and the output start signal). Probably this arrangement can be considerably simplified and improved, but the circuit plan can now be checked against the requirements to determine its practicability.

The input signal conditions of requirement R1 have been considered where they affect the circuit planning. Several of them are important only in the detailed circuit design and will be considered at that stage of the circuit development.

The capacity for four digits, as stated in R2, has been taken care of. Requirements R3, R4, R5, and R6, concerning dial tone, preliminary
pulse absorption, the output start signal, and protection against subsequent digits, have not been considered specifically. They can probably be taken care of in the detailed design or can be performed as supplementary functions by the relays of other functional blocks. The supervisory requirement, R7, and the translation of the output information to the proper code, R8, have been taken care of in the plan.

It can be seen that all the requirements have either been accounted for, are in mind but deferred for later consideration, or affect only the detailed design work. Therefore, since the plan seems adequate, the next stage is to study the plan in detail for possible improvements or simplifications.

23.4 DEVELOPING THE BLOCK DIAGRAM

In a circuit that develops along the present lines, with a plurality of identical blocks (the counting-storing circuits), it is highly probable that there are single-function blocks which can be extracted from the identical larger blocks and made common to all blocks. If this can be done, there are several advantages, including reduction in number of relays and over-all circuit simplicity. Under certain conditions this procedure may have a disadvantage in that failure of the common circuit results in total failure of the whole circuit in circumstances where partial functioning of the circuit would otherwise be satisfactory. However, this is not a valid objection in this case, since failure of a block would be just as serious on an individual as a common basis.

When the details of the counting-storing block in Fig. 23-6, are examined, then, certain points immediately become evident. The block which retains the digit information must remain on an individual basis since four separate digits must be stored. The pulse repeater, however, is used only during the reception of the particular digit, and at other times remains idle. There is an identical block for this purpose in each of the counting-storing blocks, all with the same function and with the same characteristics. There seems to be no reason why the pulse repeater cannot be made common and connected to the line ahead of the sequence circuit block.

The same reasoning applies to the pulse-train detector block. It is operated by the pulse repeater, and its primary function is to control the sequence circuit. This function, in fact, can probably be performed more efficiently by a single pulse-train detector than by four in multiple.

Up to now the means of controlling the supervisory block and the method of circuit action within this block have been left as an open question. When the pulse repeater is made a single common component
connected directly to the input, it seems evident that it may now control the supervisory block. The nature of the supervisory block can also be visualized. From requirements R1.3, R1.4, and R1.5 it is evident that the chief problem of supervision is that of recognizing the difference between the short open period of a dial pulse and the larger open period of a disconnect signal. It is probable that in the detailed design this can be accomplished by a slow-acting relay which does not respond to the open period of individual pulses.

![Diagram](image)

Fig. 23-7 First Modification of Block Diagram

The block diagram of Fig. 23-7 indicates the above changes. The pulse repeater and the pulse-train detector are made common and placed ahead of the sequence circuit. The control of the supervisory holding circuit is obtained from the common pulse repeater.

The next step is not so clear-cut and requires some degree of judgment and, perhaps, experience. Note the counting-storing circuit block. This, as indicated by the title, is a double-function block. It is not inconsistent with a single circuit block in this case because a counting circuit stores the information received merely by locking-in its last operated condition. However, an examination of the requirements imposed by the counting function and the storing function may indicate the desirability of providing separate circuits for each function. Counting circuits of the types discussed in Chapter 11 require about eight relays to count ten, while it is apparent from Chapter 20
that the same amount of information can be registered by a circuit requiring only four relays. Since only one counting circuit in Fig. 23-7 is in the process of counting at any one time, and the remainder of the time it is acting as a register, it should be possible to use only one common counting circuit and four register circuits. If a common counting circuit requires eight relays and each register circuit four relays, the total required by the split scheme would be twenty-four relays; whereas four counting-storing circuits would require thirty-two relays.

Another factor that enters into this decision is that at least some of the counting relays must be sensitive and fast-acting to follow dial pulsing. This may increase costs. No such restriction is placed upon a register circuit. An additional possible factor to be considered is that the locking winding of the register relays can be of high resistance to give low current-drain, whereas the counting relays may have to hold through lower resistance windings. This change to provide a common counting circuit and four separate register circuits is indicated on the block diagram of Fig. 23-8.

A point that becomes important when this change is made is that the common counting circuit cannot hold to the supervisory-holding circuit since it must completely release between digits. However, since the pulse-train detector must be responsive to the individual pulse-train periods (indicated in Fig. 23-3B), it can furnish the holding ground for the counting circuit. The holding ground will then be removed at the

![Fig. 23-8 Second Modification of Block Diagram](image-url)
end of each digit (at the same time that the sequence circuit is advanced), and the counting circuit will release in preparation for the next digit. This is indicated in Fig. 23-8. A method of holding the register circuits must also be provided.

The sequence of actions when registers are made into separate blocks and obtain their information from a common counter is more complex than the previous scheme and must be considered carefully. The sequence diagram of Fig. 23-9 shows the assumed action of the pulse repeater and the pulse-train detector during one complete digit and part of the following digit. The pulse-train detector operates soon after the first pulse of a digit starts, and holds after the last pulse for a time which must be greater than the interval between pulses. The time at which the pulse-train detector releases, indicated (t) in Fig. 23-9, is the time at which the circuit first recognizes that a digit is actually completed. This is a critical point in the sequence since no other control signal occurs until the next digit starts. Before the next digit starts, all the actions necessary for registering one digit and preparing to receive the next must be completed. These actions are:

1. Register the digit which has just been counted.
2. Advance the sequence circuit to prepare for the next digit.
3. Release the counting circuit.

The sequence of actions is critical. The digit must be registered in such a manner that the register relays lock before either the counting circuit releases or the sequence circuit advances to disconnect the counter from the digit register. Several schemes for accomplishing this can be devised. Three are outlined below.

Scheme (A): At the end of a train of pulses at time (t), digit information is transferred from the counting circuit to the register circuit through the sequence circuit. When the digit has been properly registered, signals are sent back from the register to permit the counter to release and the sequence circuit to advance. With a two-out-of-five self-checking register code it is a simple
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matter to obtain a signal indicating that a digit has been satisfactorily registered, but this is not the case with the specified four-element code. Of course the digits can be registered on a two-out-of-five basis and retranslated in each register block to the 1-2-4-6 code, but this seems unduly complicated.

Scheme (B): At the end of a train of pulses at time (t), digit information is transferred from the counting circuit to the register as in Scheme (A). A time-delay circuit (probably a slow-release relay controlled from the pulse-train detector) is provided which holds the counter and the sequence circuit for sufficient time to permit the register relays to operate and lock to the supervisory-holding circuit. The release of this time delay circuit a short time after (t) then permits the counter to release and the sequence circuit to advance, disconnecting the counter output leads from the digit register and leaving them open until the action is repeated in registering the next digit.

Scheme (C): Prior to the first digit (probably when the circuit is first alerted) the output leads of the counter are connected through the sequence circuit to the relays of the first digit register. However, locking ground is not supplied to the register relays at this time so that they are free to operate and release in response to the actions of the counting relays during the pulsing of the first digit. Thus, at the end of the digit, the register relays are operated in their proper combination before the pulse-train detector acts. At time (t), when the pulse-train detector acts, the circuit acts to apply locking ground to the first digit register and to advance the counter output leads from the first to the second digit register. At the same time, holding ground is removed from the counting circuit to allow it to restore to normal. This will probably require the use of continuity-transfer springs on certain relays to insure that the register locks before the counter releases. This must be given careful consideration in the detailed design.

Any of the three schemes probably can be developed into a satisfactory circuit.* However, Scheme (A) may be discarded immediately due to the code difficulties mentioned. Decision between Schemes (B) and (C) is a case of "designer's choice", since either scheme appears to be satisfactory. However, since Scheme (C) requires less interdigital time for registering, it will be followed in the detailed design.†

* In fact, they are all found in commercial switching systems, particularly Schemes (B) and (C) which are widely used.

† Offsetting the speed advantage of this scheme is the fact that the register relays perform a number of useless actions in following the counting relays. This may result in shorter total life of these relays and the translating contacts on the counter relays.
Before constructing the final block diagram, the possibility of further modifications or additions to the circuit plan can be considered. Two requirements that have not yet been met are provision of dial tone and an output start signal. Dial tone will be provided from an external tone source, but must be removed from the line immediately after the start of dialing. It is apparent that this can be done by some relay of the sequence circuit which acts at the conclusion of the first digit when the sequence advances from the first to second digit. The start signal also can probably be obtained from a relay in the sequence circuit since relay actions occur at the time the last digit is registered. The exact details in both cases must be determined during the detailed design.

A review now of the earlier stages of the circuit planning indicates one further simplification that may be possible. It was recognized at the start that the counting circuit could be used to store a digit as well as to count it, but it appeared to be more economical to store the digits in separate register circuits and permit the counting circuit to be re-used on succeeding digits. However, no re-use of the counting circuit is required after the last or units* digit has been received. Therefore, following the same line of reasoning, it appears that the units register

* Hereafter the first, second, third, and fourth digits, and the corresponding register circuits, will be referred to respectively by the terms "thousands" (TH), "hundreds" (H), "tens" (T), and "units" (U) as derived from their positions in a four-digit number.
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is superfluous; and that, on this last digit, the counting circuit itself can perform the storing function. This means only that a holding ground must pass from the sequence circuit back to the counter to prevent its release following the last digit. The sequence circuit must also hold the counting circuit output leads connected through as units output leads after the last digit is dialed.

Another requirement which has not been specifically considered is requirement R6, which states that any dial pulsing after the fourth digit must not interfere with the circuit action. This can be done by disabling the counting circuit input after the fourth digit is dialed. This action, together with the permanent locking of the counting circuit for registering the units digits and the production of the output start signal, occurs at the time, the pulse-train detector releases and the sequence circuit advances at the end of the fourth train of pulses. Since there are evidently sufficient control actions taking place at this time, further consideration of an exact means of accomplishing these actions can be left for the detailed design work.

A final requirement that affects the circuit is R4, specifying that preliminary single pulses shall not be recorded. This, also, can probably be taken care of with existing relays (the number "2" counting relay, for example, can be used to differentiate between an initial "1" and other digit values) and does not need a new block to be shown.

The final block diagram for the circuit is shown in Fig. 23-10. Working from this block diagram and the list of requirements, it is now possible to draw up a sequence chart for the circuit which will be a valuable guide in the final design stages. The block diagram and the sequence chart, in effect, sum up all the results of the planning stage.

The sequence diagram is illustrated in Fig. 23-11. The top line shows a typical input signal (instantaneous line conditions). The pulse-repeater circuit must follow the input signal, and it is illustrated by line 2. The counting circuit, in turn, is active only during the digit intervals; it is indicated by line 3 as active during the four pulsing periods. The pulse-train detector, which determines the end of each pulsing period, operates during pulsing as shown by line 4.

Line 5 in the sequence diagram indicates how the sequence circuit must connect the counting circuit output leads through to the (TH) register before the first digit is dialed, and must transfer to subsequent registers between digits. After the units digit the sequence circuit must hold these leads connected through as the units output leads. Line 6 indicated the operation of each register circuit at the time of the corresponding digit.

Dial tone is shown by line 7, starting immediately after circuit seizure and terminating during the first digit. As shown by line 8, the
start signal, indicating the end of dialing, is cut through immediately after registration of the last digit. Finally, the supervisory and holding circuit must operate for the entire duration of the call, and is so shown with line 10.

The final block diagram and its associated sequence chart should now be carefully rechecked against the requirements. In this case all requirements have been met except those which specifically apply to the detailed design.

Assuming that this procedure has been followed and that the final plan checks and rechecks, the detailed design work can begin. This is covered in the next chapter.
Chapter 24

DETAILED DESIGN OF A MULTIFUNCTIONAL CIRCUIT

The preceding chapter has illustrated the principles and techniques applicable to the planning stage of designing a typical switching circuit. The example used was a dial-pulse register which accepts and records four dialed digits, and prepares them in coded form for use by another circuit. In this chapter, the design principles are advanced to include the design of the detailed circuits to meet the requirements in accordance with the plans developed in Chapter 23.

The final block diagram of the dial-pulse register in Chapter 23 co-ordinates a group of functional blocks which represent familiar relay circuits. For this reason the circuit design aspects of the problem reduce to adapting well-known single-function circuit configurations in order that they may fit together into an integrated whole which meets the over-all register requirements. Although this represents an exceptional circuit design problem, since in the usual case some elements of the problem are new or at least unfamiliar, the approach to the detailed design phase is typical.

When the circuit plan in the form of a block diagram and a sequence chart has been completed, theoretically the detailed design can be started at any block. The specific requirements for each block and the major interconnections with associated blocks are known, so that to a large extent each block is an individual design problem. It is generally good practice to start designing the key blocks or those that offer the most difficulty, since these are most likely to require modification of planned input or output signals involving other blocks. The design can then proceed through the other blocks in order of descending difficulty, or as experience in the earlier design stages indicates to be the most profitable. If there is no reason to choose a starting point on this basis, the designer can start at the first block in time order of use and follow the sequence diagram or his own preference thereafter.

When the circuits representing all the principal functions of the problem have been designed and co-ordinated, there often remain several minor functions which must be incorporated into the circuit. Examples from the dial-pulse register are the means for removal of dial tone and for disregarding a preliminary pulse. It is often possible to overlay these minor functions, in the form of a contact network, upon
the relays of the major functional units without disturbing or re-arranging their method of operation. If this can be done, it is worth-while in that it holds down the expense and space requirements of the circuit. However, if this process modifies the operation or increases the complexity of the original circuit to the point where the primary functions are obscured, it is better to introduce new functional units. The dividing line between these two methods of attack is largely a matter of judgment; the only rule that can be given is that, in the final circuit, individual functions should be reasonably clear-cut and recognizable.

During the entire process of design, the circuit should be continually checked against the over-all requirements of the problem. Each circuit block is designed to specific and limited requirements formulated during the planning stage. As the blocks in circuit form are integrated, higher orders of requirements come into the picture, culminating in the original set of requirements that specified the problem. If these latter requirements are not kept in mind during the detailed design and the integration stages, undesired reactions may creep into the circuit which later may require patchwork methods to eliminate.

With this brief exposition of pertinent design principles, the circuit design of the dial-pulse register can now begin. Since, in this problem, no particular functional block is of key importance, the design will start with the associated group of pulse-repeater, pulse-train detector, and supervisory holding circuits, and progress through the counting, register, and sequence circuits. The minor circuits for removing dial tone, disregarding a preliminary pulse, locking the counting circuit on the units digit, rejecting a fifth digit, and giving an output start signal, will be incorporated in the design at whatever stage seems most appropriate.

24.1 PULSE REPEATING, PULSE-TRAIN DETECTING, AND SUPERVISORY HOLDING CIRCUITS

This group of affiliated functions is required wherever successive trains of dial pulses are used in the transmission of information. The general problem of pulse detection over a loop has been discussed in Chapter 18, and a basic circuit arrangement for performing the functions of pulse repeating, pulse-train detecting, and holding was illustrated. This circuit together with a sequence diagram is reproduced in Fig. 24-1. The pulse repeating relay (A) operates when the switchhook contact of the calling device is closed and thereafter follows the opens and closures of the loop during dialing. The supervisory or holding relay (B) operates from the front contact of (A) and remains operated until (A) has been released sufficiently long to indicate disconnection. The back contact of (A) closes the ground pulses to the counting circuit.
and to the pulse-train detecting relay (C) through a front contact of (B) to insure that in the normal register condition this path is open. When this circuit is connected to the counting circuit, the combination will have to be scrutinized closely to make sure that there is no interference between relay (C) and the counting relays. If there is interference, the operating path for (C) can be taken from a separate contact on (A). Another possible difficulty with this circuit is the false pulse that occurs at the time of disconnection. When the associated circuits have been designed, the effect of this will have to be carefully examined.

The requirements for relay (A) are: (1), that it should repeat the dial pulses with as little distortion as possible; and (2), that it should not unbalance the line to the point where noise or crosstalk becomes objectionable. The first requirement implies that the operate and release times of the relay should be the same. This ideal has not yet been achieved with practical relays under the varying loop conditions that are met. In the choice of a particular relay, the possible distortion introduced by (A) must be compared with the operate and release time of the (B) and (C) relays and the counting relays, and then all these
factors must be carefully co-ordinated. These relationships set restrictions on the loop conditions which, in this problem, have not been explicitly stated. Also, because of the intimate relationship of loop conditions and pulsing limits imposed by the (A) relay, loop limits can be relaxed by tightening pulsing limits and vice versa. In the design of practical dial-pulse circuits this is a major point of investigation, but further exploration of the subject is beyond the scope of this chapter.

The requirement of loop balance is achieved by making equal the a-c impedance to ground of both sides of the loop. This is complicated by the requirement that dial tone must be connected to the line when the circuit is prepared to receive pulses. Although there are several methods of achieving balance and connecting dial tone, with the particular choice dependent upon transmission as well as switching factors, in the present case it will be assumed that the circuit of Fig. 24-2 is satisfactory.

![Fig. 24-2 Method of Connecting Dial Tone to Loop](image)

The principal requirement for the (B), or holding relay, of Fig. 24-1 is that it must not release during the open periods of a train of dial pulses [during which the front contact of the (A) relay is open]. This interval can be calculated from the known dial-pulse limits and the worst distortion produced by the (A) relay, and is well within the capabilities of general-purpose slow-release relays. Make-contacts on the (B) relay must be provided to furnish off-normal holding grounds for the sequence and register circuits. Also, the dial-tone path should pass through a front contact of (B) so that the path may be closed when the circuit is prepared to receive pulsing.

As can be seen from Fig. 24-1, relay (C) must operate during the first back-contact closure of (A) at the onset of a train of pulses and must hold over the succeeding operated periods of (A). In addition, the release time of (C) must not be so long that it introduces the possibility of covering the interdigital period. This limits the permissible variation in release time for the (C) relay, but again is within the capabilities of general-purpose slow-release relays. The (C) relay must provide a front-contact path from ground for locking the counting circuit during each train of dial pulses. This contact will open at the end of a pulse train to release the counting circuit in preparation for the next train.

It was decided during the planning of the circuit to let the register relays for a particular digit follow the counting circuit without locking as the pulses are received, and to provide a register-locking ground at the termination of the digit. The implication of this is that the locking of the register circuit must occur simultaneously with or prior to the
release of the counting circuit. The most practical way to achieve this is to provide a continuity-spring combination on (C), of which the front contact holds the counting circuit and the back contact establishes a temporary register-locking ground. The latter path, which must be distributed to the register circuits by the sequence circuit to be effective, will subsequently be supplemented by a permanent locking ground.

A final output required from the (C) relay is a ground closure to advance the sequence circuit between digits. Since the sequence circuit must establish connecting paths between the counting circuit and the register circuits before the first digit and between succeeding digits, a back contact on (C) is probably required.

The pulsing circuit incorporating the paths described above and with new functional designations is shown in Fig. 24-3. The circuit paths are tentative; they are subject to modification when the associated functional circuits are designed. However, at the moment they appear to be the best method of accomplishing the block diagram plan.

24.2 THE COUNTING CIRCUIT

There is a wide variety of counting circuits from which a choice may be made for the dial-pulse register. Many of the more important of these circuits have been discussed in Chapter 11, and the most applicable selection will be made from that source.

The most important qualification for the counting circuit is that it must be fast-acting in order to follow dial pulsing. The original requirement concerning pulsing speed stated that it might vary from ten

![Fig. 24-3 A Modified Pulsing Circuit](image-url)
to fifteen pulses per second with equal make and break periods per pulse cycle. This corresponds to a time variation of 0.033 to 0.050 seconds for a make or break period. The output pulses of the (P) relay, using the new relay designation, will exceed these limits to some extent, although the operate and release times of (P) are partially compensating. It is desirable, therefore, that the counting circuit should require very few relay actions within a pulse make or break interval. In addition, the counting circuit should use relays as efficiently as possible, and also should permit the use of a simple network for translating to the 1-2-4-6 output. This latter qualification is imposed primarily because the contact springs necessary for a complex network increase the operating time of the counting relays.

Since ten pulses is the maximum that need be counted for each digit, one of the partially recycling counters is probably the most efficient for the dial pulse register. Circuits of this type discussed in Chapter 11 are listed below:

**COUNTING CIRCUITS**

<table>
<thead>
<tr>
<th>Counter Type</th>
<th>Relays</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recycling prime counter*</td>
<td>12 relays</td>
</tr>
<tr>
<td>Recycling odd-even counter†</td>
<td>6 relays plus pulse divider</td>
</tr>
<tr>
<td>Odd-even single-decade counter</td>
<td>6 relays plus pulse divider</td>
</tr>
</tbody>
</table>

* Modified so that relays (6) and (6') lock to off-normal ground and pulse lead enters at relay (5'). See the discussion in Chapter 11.
† Modified so that relay (6) locks to off-normal ground.

On the basis of number of relays, the first of the above circuits loses out although it is a reasonably fast circuit. The other two circuits are identical in number of relays, but differ in their method of operation. The first of these two circuits leaves two or four of the odd-even counting relays operated at the end of each pulse of a digit, whereas the second circuit leaves only one or two. Therefore the second circuit will permit a simpler translating circuit to the 1-2-4-6 output code.

Before this circuit can definitely be chosen, its operating sequence must be examined to make sure that it meets the time requirements. A pulse-dividing circuit is necessary to convert the (P) relay pulses to odd and even pulses. An analysis will indicate that the standard two-relay pulse-dividing circuit is unsatisfactory from the time point of view since it depends upon shunt-down methods for operation. The counting circuit, incorporating a faster three-relay pulse-dividing circuit,* is shown on Fig. 24-4. The sequence diagram for the circuit appears on Fig. 24-5.

* Discussed in Chapter 11.
Analyzing first the action of the pulse-dividing circuit, when the loop opens for the initial pulse, the (P) relay releases and, in sequence, the (W) relay operates. Reclosure of the loop re-operates (P) and permits (Z) to operate. The second pulse releases (P), operates (Z1), and releases (W) in sequence. This involves the sequential action of three relays within the break period of a dial pulse and, repeating on every even pulse, is the worst situation encountered during a series of ten pulses. However, as mentioned before, the re-operation time of (P) on the next pulse lengthens the available time for (Z1) and (W) to act, and the latter two relays can be chosen for fast action. Standard relays are available to meet these requirements, although, in the ultimate, the characteristics of (P), (W), (Z), and (Z1) set a limit to the loop over which dialing can take place.

The diagram of Fig. 24-5 indicates that if the odd-even pulses to the counting relays (1), (2), and so on are generated directly by the (W)
relay, a time equal to a complete pulse cycle is available for operation of each counting relay plus the release of the preceding relay. This imposes only moderate time requirements on the counting relays. An advantage of the circuit is that the relay configuration corresponding to each successive pulse is set up toward the beginning of each pulse cycle. This speeds up the registration of the digit as the pulses spill in, and relaxes the time requirements of the register relays. This analysis indicates that the odd-even single-decade counting circuit is satisfactory for use in the dial-pulse register.

The translating network for the counting circuit can be developed from the table at the left, which relates the configuration of the counting relays to the condition of the corresponding 1-2-4-6 output leads.

<table>
<thead>
<tr>
<th>Pulse Number</th>
<th>Operated Relays of Fig. 24-4</th>
<th>Output Leads Grounded</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1)</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>(2)</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>(3)</td>
<td>1,2</td>
</tr>
<tr>
<td>4</td>
<td>(4)</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>(5)</td>
<td>1,4</td>
</tr>
<tr>
<td>6</td>
<td>(5),(6)</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>(6),(1)</td>
<td>1,6</td>
</tr>
<tr>
<td>8</td>
<td>(6),(2)</td>
<td>2,6</td>
</tr>
<tr>
<td>9</td>
<td>(6),(3)</td>
<td>1,2,6</td>
</tr>
<tr>
<td>10</td>
<td>(6),(4)</td>
<td>4,6</td>
</tr>
</tbody>
</table>

when it is operated alone, and does not ground them when it is operated together with relay (6). A ground path through a back contact of (6) and front contacts of (5) to leads 1 and 4 takes care of this. The resulting circuit is shown in Fig. 24-6.

The design of the counting circuit (or, more strictly, the adaptation of the counting circuit to the dial-pulse register) is now completed to the extent that requirements are presently known. Analysis of the operation of the combined pulsing and counting circuits discloses no

![Fig. 24-6 Translating Network for Counting Circuit](image-url)
deviation from the original requirements and no hazards except, per-
haps, interaction between the (PTD) relay and the pulse-dividing circuit.
These two circuits operated from a common P lead as shown in Fig.
24-4. In addition to mutual effects upon relay acting times, there is a
possible self-locking path for (PTD) via the CL lead, a front contact of
(W), a winding of (Z), a back contact of (Z), and the P lead to (PTD).
However, the condition of (W) operated and (Z) released never legiti-
mately exists at the end of a pulse. Therefore there should be no circuit
reaction from this condition except, perhaps, a slight additional delay
in the release of (PTD) at the end of a digit. The circuit should eventu-
ally be checked in the laboratory for these possible hazards. If there is
trouble, the operating path for (PTD) can be separated from the P lead
by an additional make-contact on relay (P). The penalty for this change
may be increased difficulty in making the (P) relay meet the pulsing
requirements.

24.3 THE REGISTER CIRCUIT

The design of the sequence circuit should, perhaps, come after
the counting circuit in accordance with the development pattern followed
up till now, since the design of the sequence circuit depends to some
extent upon the form of the digit-register circuit. However, the digit-
register circuit will be considered first in this instance. The obvious
requirements for the register circuit, one of which must be provided
for each digit except units, are that it should accept and hold the ap-
propriate digit recorded by the counting circuit and make available the
digit on 1-2-4-6 output leads for external use. It was decided during
the planning phase that a four-relay circuit, should be used, with each
relay corresponding to one element of the 1-2-4-6 code. To this end,
the counting circuit was designed with a 1-2-4-6 output which could
be steered through the sequence circuit to the appropriate register.
Furthermore, it was decided that the relays of a register should follow
the counting relays during the reception of a digit, and lock-in the final
pattern simultaneously with the release of the counting circuit. The
initial locking path is obtained from a back contact on relay (PTD)
carried through the sequence circuit to the register.

Several basic relay registration schemes taken from Chapter 20
are shown in Fig. 24-7. The simplest of these, the circuit of Fig. 24-7A,
is unsuitable since it includes a back-up path over the register-locking
lead among the four relays. Ground signals over the input leads from
the counting circuit overlap as the count advances, with the result that
a legitimate operating path to one relay could falsely hold, via the lock-
ing path, a relay that should otherwise release.

The circuit of Fig. 24-7B could conceivably get around this dif-
ficulty if a highly marginal hold adjustment were placed on the relays.
The Design of Switching Circuits

However, the working margins might well be so close that it would be extremely difficult to achieve a practical circuit. Furthermore, the cost of the low-tolerance resistors required would probably more than offset the cost of the secondary relay windings of the circuit of Fig. 24-7C, which completely isolate the operating paths from the locking paths. Since there is no reason to attempt designing a circuit based on Fig. 24-7B, the double-winding circuit will be chosen.

The output circuit of the register is very simple since there is a one-to-one correspondence between each relay and an output lead. Each output lead can be taken from an individual make-contact on the corresponding relay, or it may be connected directly to the locking winding. On the assumption that the external circuit utilizing the registered information does not cross or ground unconditioned register leads, the latter circuit will be used. It is shown in complete form in Fig. 24-8.

24.4 THE SEQUENCE-CONNECTING CIRCUIT

The last major functional unit of the dial-pulse register to be designed is the sequence-connecting circuit. This circuit must satisfy the primary requirements of controlling a connection between the counting circuit and each of the register circuits in turn, and also of establishing the locking path for each register at the appropriate time.

Fig. 24-8 Complete Register Circuit
In addition, certain minor requirements, such as control of dial tone, have been tentatively assigned to the sequence circuit. The sequence circuit itself is under the control of a back contact (if necessary, a front contact) of the (PTD) relay.

Fig. 24-9 Connecting Paths through Sequence Circuit
In establishing the connecting paths between counting circuit and register circuit, the sequence circuit may either operate separate connecting relays or carry the connecting contacts on its own relays. Since only four leads are involved, it is feasible to place the contacts on the sequence relays, and the attempt should be made in order to save relays. The connecting paths can therefore take either of two general forms depending upon the method of operation of the sequence circuit. If the relays carrying the connecting paths overlap in operation during the registration of a digit, some form of transfer chain such as shown in Fig. 24-9A is necessary to exclude interference among registers. On the other hand, if each connecting sequence relay operates exclusively during the registration of its associated digit, the simpler make-contact network shown in Fig. 24-9B is satisfactory. The latter circuit is preferable, other factors being equal, since there are fewer contacts and the paths are less susceptible to trouble from dirty contacts.

![Fig. 24-10 Relay Sequence during Pulsing of a Digit](image)

The method by which the sequence circuit establishes a locking path for each register at the end of a digit requires close co-ordination of relay operations and should be examined upon the basis of the known sequence of events. A time chart of the pertinent relay actions during the reception of a digit is shown in Fig. 24-10. The register relays follow the counting relays over connecting paths established by a sequence relay (K1). At the conclusion of the digit, relay (PTD) releases, releasing the counting circuit relays. At this same time, a locking path for the register relays must be closed and maintained for the duration of the call. It has already been decided that a continuity combination on relay (PTD) will release the counting circuit through a front contact and close a temporary register holding path through the back contact. However, the temporary path must be supplemented by a permanent ground before (PTD) re-operates on the next digit. If a sequence relay, shown in Fig. 24-10 as (K2), operates when (PTD) releases, its contacts may
close the permanent locking path. The contacts that perform this action must not permit a ground backup on the lead from (PTD) which can interfere with other register relays. A simple circuit designed on this basis is shown in Fig. 24-11.

Two major types of sequence circuits have been discussed in detail in Chapter 11. One type, based upon the prime counter, requires two relays per stage and a simple control network. The other type utilizes only one relay per stage, but requires in addition an elaborate control network. It is a reasonable assumption that either of these types of circuits can be adapted to the register requirements.

Analysis of the requirements of the sequence circuit indicates that in all probability a stage for each of the four digits is necessary even though no units register is required. The principal reason for this is that the end of the fourth digit must be detected in order to generate an output start signal at the right time. With the prime-counter sequence circuit, this requires a total of eight relays. A preliminary analysis of the second type of sequence circuit indicates that, although a lesser number of the individual stage relays is necessary, the contact spring load per relay is probably excessive and may require supplementary relays. The choice between the two circuits is primarily a designer's choice, and in the present case it is felt that the prime-counter circuit is more suitable.

There are two principal variations of prime-counter sequence circuit, the sequence diagrams of which are shown in Fig. 24-12. The relays of the upper diagram remain locked up for the duration of each call and thus require the less desirable transfer chain of Fig. 24-9A for connecting. The operated relays of the lower diagram of Fig. 24-12 are released in pairs by the next prime relay at the end of each successive control impulse from a back contact of (PTD). However, there is sufficient overlap during the dial-pulsing period to require the use of a transfer-connecting chain to prevent interference with an earlier registration. This could be taken care of if the circuit could be modified to release a preceding prime pair by the operation of the next non-prime relay during the interdigital interval. However, this cannot be
accomplished without upsetting the essential characteristics of the prime counter.

Another and more practical modification of the prime counter circuit is a compromise between the two diagrams of Fig. 24-12. It consists of releasing a non-prime relay by the operation of the next non-prime relay, and at the same time holding each prime relay as it operates for the duration of the call. A sequence diagram illustrating this assumption is shown in Fig. 24-13. It has the advantage that a single non-prime relay (which can carry the connecting contacts) is operated during the reception of each digit, and also that a unique condition of prime relay operated and non-prime relay released exists at the end of each digit to establish and maintain the permanent register.
locking paths. Although this is different from the hold condition contemplated in Fig. 24-11, it offers no design difficulties. The sequence circuit will therefore be designed around the chart of Fig. 24-13.

In Fig. 24-14A is shown an intermediate stage of a prime counter of the lock-up type (corresponding to the top sequence of Fig. 24-12). In order for the (X) relay to release when the next non-prime relay operates, the locking ground must be carried through a break-contact on the latter relay. However, this arrangement will also release the (X') relay unless a separate locking circuit is added to (X'). The most practical method of locking up (X') without interfering with (X) is to add a second winding to (X') as is shown in Fig. 24-14B. A sequence circuit constructed of relay pairs wired in this manner will conform to the time chart of Fig. 24-13.

![Fig. 24-14 Locking Control for Prime Counter](image)

The next step in designing the sequence circuit is to construct the locking path for each register circuit. The sequence diagram for two pairs of prime relays, each pair redesignated (C) and (L) for "connect" and "lock", is shown in Fig. 24-15A. The period during which the register associated with the (C) and (L) pair must be locked is bracketed in two parts labelled (1) and (2). The algebraic equations expressing the locking path can be written:

\[
\begin{align*}
[(PTD)' + (C) + (L)] [ (C)' + (L)] & \\
(1) & \\
(2) &
\end{align*}
\]
This reduces to

\[(L) + (C)' (PTD)'\]

which is drawn in circuit form in Fig. 24-15B. It can be seen that the (C) break-contact ground backs up onto the lead from (PTD) which is common to all sequence stages, and will cause mutual interference among the register circuits. Therefore, this path must be made disjunctive by placing a make-contact on (C) in the back-up path. This can be expressed as

\[(L) + (C)' [(PTD)' + (C)]\]

and is shown in Fig. 24-15C. The transfer on relay (C) permits a momentary open in the register locking path and must be converted to the continuity type. Further analysis of this circuit indicates that the make-contact on relay (L) contributes nothing useful, since the associated register relays do not operate until (L) has operated. Therefore

---

Fig. 24-15 Development of Register Lock Circuit
this contact can be omitted. The complete circuit for one sequence stage is shown in Fig. 24-15D.

The complete sequence circuit as it has been designed up till now appears in Fig. 24-16. The relationship with the counting circuit, the register circuits, and the (PTD) relay is shown in detail. To this circuit must now be added the group of minor functions which the preliminary planning and analysis indicated could best be performed by the sequence circuit.

24.5 THE MINOR FUNCTIONS

Circuits to fulfill the group of minor functions have been deferred until the more important functions have been accommodated. These minor functions comprise the following:

1. A "start" signal to an associated circuit at the end of the units digit, the signal to consist of ground on an "ST" lead.
2. Means to lock the counting circuit during or at the termination of the units digit so that the counting circuit may be used as the units register.
3. Means to open the dial-tone path during or at the end of the first digit.
4. Means for disregarding a single pulse for the first digit.
5. Means to prevent interference with the units registration on the counting circuit when an extra digit is dialed.

Since each of these functions must be performed at a certain time with respect to the dialing of the digits, the appropriate circuits can best be determined by examining the sequence-circuit time chart of Fig. 24-13. The "start" signal must occur at the termination of the units digit. From Fig. 24-13 it can be seen that a unique relay combination at that time is (PTD) released and (U'), now designated (UL), operated. Therefore ground in series with a back contact on (PTD) and a front contact of (UL) can provide the start signal. Examination of the circuit of Fig. 24-16 indicates that the equivalent of this circuit is available on relay (UL) at the unused front contact of the chained control transfer extending back to (PTD). Use of this contact for the start signal is shown in Fig. 24-17.

The supplementary locking path for the counting circuit must be closed during or after dialing of the units digit. The ground applied to the start lead in Fig. 24-17 occurs at the correct time and could be used for this additional purpose. However, in the absence of specific information about the associated circuit which utilizes the start signal,
Fig. 24-16 The Sequence-Connector Circuit
it is better to keep the counting circuit lock-up path independent of the start lead to prevent possible back-ups or interference. This can be done by supplying supplementary locking ground to the counting circuit from a simple make-contact on relay (UL) as shown in Fig. 24-18. At the end of the units digit, the (PTD) ground drops off while the (UL) ground remains until the circuit is dismissed.

A convenient method of removing dial tone after the dialing of the first digit has started is to pass the dial tone lead through a break contact on the (THL) relay. Relay (THL) does not operate until relay (PTD) operates at the beginning of dialing, and it remains locked up for the duration of the call. The dial-tone circuit on this basis is shown in Fig. 24-19.
Some means for preventing the dial-pulse register from accepting a single pulse for the thousands digit must now be developed. Basically, this consists of preventing a permanent registration in the thousands register, cancelling the removal of dial tone, and preventing the advance of the sequence circuit, unless more than one pulse is received. Fortunately, these functions, with the exception of register locking, are controlled by operation of the (THL) relay. Therefore, if (THL) can be prevented from operating on the first pulse of the thousands digit (as it does now), and the operation can be deferred until two consecutive pulses have been received, the desired objective will have been attained. Although the digit "one" will be locked in the thousands register, it will be released when (PTD) re-operates.

With the present circuit, (THL) operates when (PTD) opens a back contact on the first pulse to remove the shunt around the winding of (THL). If a supplementary shunting ground is applied from a break-contact on relay (2) of the counting circuit, relay (THL) cannot operate until two pulses of the thousands digit have been counted. This supplementary ground must pass through a back contact of (THL) in order to prevent subsequent ground closures from relay (2) interfering with the sequence control lead from (PTD). The circuit is shown with dashed lines in Fig. 24-20. The ground path must be extended through a make-contact on the (HD) relay so that it will not be present when the circuit

![Circuit Diagram](image-url)
is idle. Under these conditions, when the circuit is first seized by closing the switchhook connected to the loop, the dashed ground path will operate relay (THC). This makes unnecessary the (THL) back contact path of the "SA" lead to the winding of (THC), as indicated in Fig. 24-20.

The final circuit function that must be added to the dial-pulse register is to prevent any circuit reaction to the dialing of a fifth digit. As the circuit now stands, additional pulses after the units digit will change the setting of the counting circuit and will open the output start lead by operating relay (PTD). In order to eliminate this action, the P or pulse lead to relay (PTD) and the counting circuit must be opened when the units digit has been completely recorded by the counting circuit. To determine an appropriate circuit to perform this action, the sequence chart of Fig. 24-13 should be examined. The chart indicates that, at the conclusion of the units digit, a unique path that is closed at no other time is (PTD)' + (UL). The negative of this, expressed as (PTD) (UL)', is a path only opened at this time. If this pair of contacts is inserted in the pulsing lead, dial pulses after the units digit will be completely ineffective. The circuit is shown in Fig. 24-21.

One caution is necessary with this circuit. At the start of the units digit, the make-contact of (PTD) must close before the break-contact of (UL) opens. The (UL) relay in turn is operated when the break-contact of (PTD) opens. Slow relays like (PTD) are subject to considerable contact stagger, and conceivably there could be sufficient delay between opening the back contact and closing the front contact of (PTD) to cause the circuit to fail on this account. Therefore the extreme operating times of the relays chosen for (PTD) and (UL) and the possible stagger must be closely scrutinized before the circuit of Fig. 24-21 is approved.
Fig. 24-22 Dial-Pulse Register Circuit
24.6 CONCLUSION

The design of the dial-pulse register is now essentially complete. Circuits to perform all planned functions have been individually designed, and it remains only to co-ordinate them all in a single diagram. During this process certain circuit simplifications may appear and these should be incorporated in the design. Two examples of this develop in the sequence circuit of Fig. 24-16. Analysis of the sequence circuit and associated circuits discloses that there is no reason why the two back contact paths, "SA" and "RL", from relay (PTD) should not be connected to a single contact. Eliminating one break-contact pair from the relay may actually make easier the choice of a relay to meet the particular requirements of (PTD). The other example is the secondary locking winding of relay (UL). As the circuit has developed, the operating path for (UL) is not opened as it was for the other sequence relays, and the locking winding is unnecessary.

When the loop path is opened at the conclusion of a call, or, in fact, at any time during the call, the entire dial-pulse register circuit
must release. In designing the circuit, particularly the sequence circuit of Fig. 24-16, certain holding grounds were shown with dashed lines to indicate that they were temporary. All such grounds must be taken from make-contacts on the (HD) relay which remains operated for the duration of the call. In addition, any other permanent grounds which conceivably could result in falsely locking up the circuit should be shifted over to (HD) grounds.

The complete circuit is redrawn in Fig. 24-22. An over-all sequence diagram, also very useful in circuit analysis, is shown in Fig. 24-23. Both these drawings must be analysed very carefully to insure that all the requirements listed in Chapter 23 have been met and that no trouble conditions have crept in during the design process. A point for checking that was mentioned earlier in the chapter was the possible effect of the pulse to the counting circuit that occurs when the switch-hook contact at the calling device is opened. Introduction of the circuit to reject a fifth digit eliminates any possible effect of this pulse at the time of normal release. Analysis of the over-all circuit indicates that there is no undesirable effect of this pulse when release is premature.

This concludes the design of the dial pulse register. As an example of practical switching circuit design, the problem has been relatively simple and straightforward, and yet it has embodied most of the principles that apply to the design of highly complex circuits or systems. Starting with a statement of the purpose of the circuit and the specific requirements which it must meet, the design progressed through the devising of a general plan, the refining of the plan into a complete functional block diagram which constituted an itemized program for the detailed circuit design work, and the final development of circuits to fill and interconnect the functional blocks. By following this general procedure and applying with intelligence and ingenuity the principles and techniques discussed in the body of this book, circuits can be designed to perform any function expressible in logical terms.
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